What is Special About Knowing Mathematics for Teaching and How Can It Be Developed?

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Overview

1. What does it mean to "know" mathematics for teaching?
2. Measuring mathematical knowledge for teaching: Results
3. Developing opportunities for teachers to learn MKT: A new collaboration between the UM research group and Albert Shanker Institute

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Clarifying the Problem

The quality of mathematics teaching and learning

Teachers' knowledge of mathematics
and their ability to use it in teaching

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But: U.S. Teachers Often Lack Sufficient Mathematical Knowledge

- What's the evidence?
  - Anecdotes
  - Research
- What does it tell us and what is not well understood?
  - Lack of a system
  - This is a societal problem, widespread

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Some Approaches to Solving the Problem — and the Unresolved Question

- Require more mathematics for certification
  - More mathematics courses
  - A major (or minor) in mathematics
  - Mathematics test
- Recruit mathematically trained people into teaching
  - Engineers, accountants, mathematicians, …
- Fund mathematically focused professional development

But -- these approaches assume we know what mathematical knowledge and skill is needed for high-quality teaching.

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Toward A Practice-Based Theory of Mathematical Knowledge for Teaching

1. Study instruction, and identify the mathematical work of teaching
2. Analyze what mathematical knowledge is needed to do that work effectively, and how it must be understood to be useful for the work
3. Develop and evaluate approaches to helping teachers learn mathematical knowledge for teaching
4. Develop, test, and refine measures of mathematical knowledge for teaching using multiple methods as a means to evaluate professional education, investigate effects on students' learning, and improve theory

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Mathematical Knowledge for Teaching

What do we mean when we use this term, "mathematical knowledge for teaching"?

- Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching

What do we mean by the "work of teaching"?

- The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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Knowing Multiplication

Multiply:

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline \end{array}$$

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Knowing Multiplication for Teaching: Analyzing Student Errors

What mathematical steps produced each of these answers?

(a)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 405 \\ 108 \\ \hline 1485 \end{array}$$

(b)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 225 \\ 100 \\ \hline 325 \end{array}$$

(c)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 1250 \\ 25 \\ \hline 1275 \end{array}$$

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Knowing Radical Expressions

Simplify:

$$\sqrt{150}$$

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Knowing Radical Expressions for Teaching: Choosing Numerical Examples

Which of the following is best for setting up a discussion about different solution paths for simplifying radical expressions?

(a) $\sqrt{54}$

(b) $\sqrt{156}$

(c) $\sqrt{128}$

(d) These examples all work equally well.

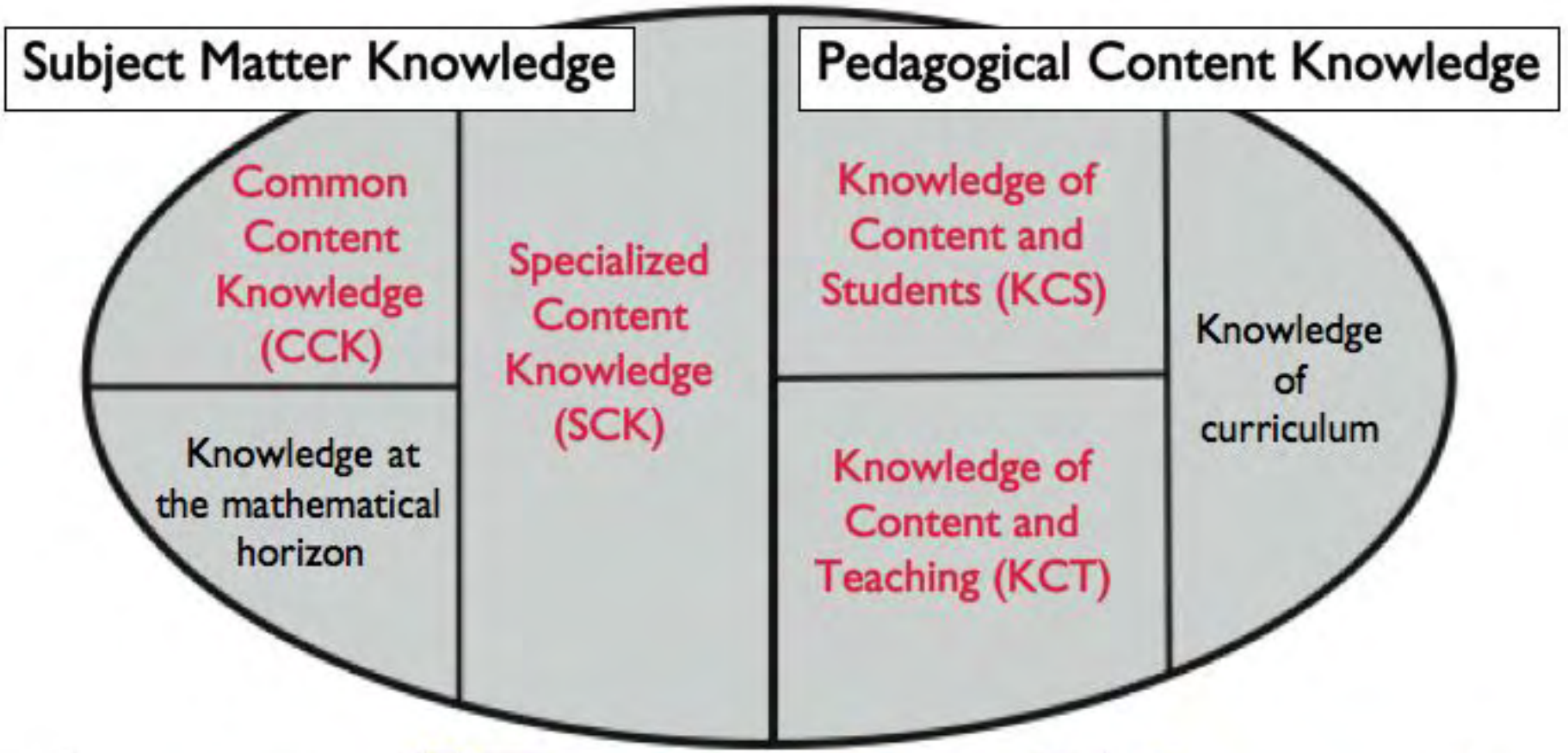
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Other Work of Teaching

- Making mathematical practices explicit
- Attending to issues of equity (e.g., language, contexts, mathematical practices)
- Using and defining terms
- Using notation
- Explaining and guiding explanation
- Interpreting and evaluating alternative solutions and thinking
- Choosing contexts with care for mathematical integrity, diversity, and transparency for learning

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Shulman's Original Category Scheme (1985) Compared with Ours



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Opportunity: Study of Instructional Improvement

- Study of three Comprehensive School Reforms; teacher knowledge a key variable
- Instrument development goals:
  - Usable with 5000 teachers: survey, multiple choice
  - Measure content knowledge teachers *use* in teaching – not just what they *know*
  - Differentiate among teachers
  - Non-partisan

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Sample Item: Operations SCK

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

Which of these shows a method that could be used to multiply any two whole numbers?

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Overarching Findings: Factor Analyses

- Multidimensionality of mathematical knowledge for teaching
  - Knowledge of students and content different from "pure" content knowledge
  - Knowledge differentiable by topic domain (e.g., algebra, geometry)
  - Number and and operations mathematical knowledge for teaching
    - Specialized
    - Common
- Tentative conclusion: existence of a kind of "professional" knowledge for teaching

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Validating Our Measures

How do we interpret teachers' performance on our questions?

1. Their score reflects their mathematical thinking
   - Cognitive interviews
2. Higher scores mean higher-quality mathematics instruction
   - Videotape validation study
3. Scores reflect common and specialized knowledge of content
   - Mathematician and non-teacher interviews
4. Higher scores related to improved student learning
   - Study of Instructional Improvement student gains analysis

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Linking Teacher Knowledge and Student Achievement

- Questionnaire consisting of 30 items (scale reliability .88)
- Model: Student Terra Nova gains predicted by:
  - Student descriptors (family SES, absence rate)
  - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significant
  - Small effect (< 1/10 standard deviation): 2 - 3 weeks of instruction
  - But student SES is also about the same size effect on achievement

(Hill,Rowan, and Ball, AERJ, 2005)

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Conclusions from Our Research

- Mathematical knowledge for teaching is the knowledge needed for the work of teaching.
- A kind of content knowledge distinct from that required by other mathematically-intensive professions
- Importance of building this theory IN practice
- Possible to write valid and reliable survey measures of MKT
- Credibility of this approach with teachers
- This kind of testing is professional

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A New Problem: How to Create Opportunities for Teachers to Learn MKT

- Learning in most mathematics courses is not geared for this kind of knowledge, skill, and problem solving
- Much professional development is content-free
- Lack of materials for learning this sort of math for these applications

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How Can Professional Development Help Teachers Learn Mathematical Knowledge for Teaching?

- Focus professional development on mathematical content, and build explicit connections to the work of teaching
  - Study content and do application problems that apply that knowledge to teaching
  - Examine student work and analyze it mathematically
  - Keep a list of mathematical terms that have other uses, and learning to distinguish which ones require careful attention, and how
  - Analyze and write mathematical definitions
  - Analyze a textbook lesson for its mathematical purposes, connect to grade level standards
  - Write worksheets and quizzes
  - Analyze assessment items for their mathematical language demands
- Provide opportunities to teachers to develop fluency with mathematical knowledge for teaching
- Connect direct work on achievement disparities with mathematical resources
  - Mathematical language
  - Care with use of contexts
  - Opportunities to broaden what it means to be "good at" mathematics

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New R&D Project : Collaboration Between UM and Albert Shanker Institute

- Develop coherent, specific course outline for teaching MKT (leading to algebra)
- Core curriculum keyed to core curriculum for students, plus more (linked to Achieve benchmarks)
- Topics, problems, applications, materials
- "Teachers' guides"
- Assessments
- Cycles of design and development, piloting, revision
- Link to other TE reforms
- Collaboration between arts and sciences and education
- Informed by research and practice in U.S. and in high-performing countries
- Research on teachers' learning and on instructors' use of the material
- Opportunities for the professional development of those who teach teachers

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Teaching Mathematics for Teaching: Goals and Challenges

- Tackling issues of "coverage" and scope
  - Choosing and justifying mathematical content (topics and practices) for in-depth work
  - Choosing and justifying content for brief treatment
  - Choosing and justifying content to omit
- Understanding and developing mathematical fluency for teaching
- Developing tasks and pedagogy appropriate for mathematical knowledge for teaching: Courses/workshops should not necessarily be taught as teachers would teach their own students
- Developing performance assessments: knowing and using mathematics (e.g., explaining, representing)
- Rethinking the intersection of "content" and "methods"
- The professional development of professional developers

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Final Remarks

1. Mathematical knowledge is crucial to teaching effectiveness.
2. The mathematical knowledge needed is specialized and is more than common content knowledge known by any well-educated adult.
3. Teachers need opportunities to learn mathematics for teaching and to practice using it to teach effectively and to attend to issues of achievement disparities.
4. Professional development providers need professional development, too.
5. Professional development should be evaluated by its effects on instruction and student achievement.

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