




Supporting High-Quality Teaching of Mathematics for All Students

Deborah Loewenberg Ball

Utah Leadership Academy

Salt Lake City, UT • June 10, 2010

SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN

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A short pre-test

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Which are myths? Which are true?

Mark M (myth) or T (true).

a. The U.S. mathematics education system used to work better than it does now.

b. The number of mathematics courses that a teacher has taken is a good predictor of how effective he or she will be.

c. Societal problems — e.g., inequality, poverty, the declining family — are so overwhelming that schools cannot do their job.

d. The U.S. math curriculum is similar to the one taught 50 years ago.

e. The most important characteristic of effective professional development is that it focus on instruction.

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Goals for today

1. Understand concisely the problem to be solved
2. Articulate the goals for kids
3. Specify what instructional practices are needed for students to attain those goals
4. Detail the resources teachers need to be able to practice effectively
5. Identify ways that will enable teachers to learn to be successful with all students
6. Develop clear ideas about how you can build the supports for high-quality instruction in your setting

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1. The problem

The mathematical education that young people receive in the U.S.

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Why are so many people in our country "not good at math"?

Are you "good at math"?

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What's the problem?

- http://www.nsf.gov/news/special_reports/math/classroom.jsp

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The urgency

1. Enormous gaps in learning opportunities and disparities in achievement (within U.S. and in international comparisons)
2. Rapidly changing school population
3. Higher, more complex academic goals
4. High expectations for all students



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2. Goals for students

What does it mean to be "mathematically literate"?

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What is the chance of rain?

Weather forecast:

Saturday: 50% chance of rain

Sunday: 50% chance of rain

What a bummer! I wanted to play golf, but there is a 100% chance of rain this weekend!

Is this correct? Explain.

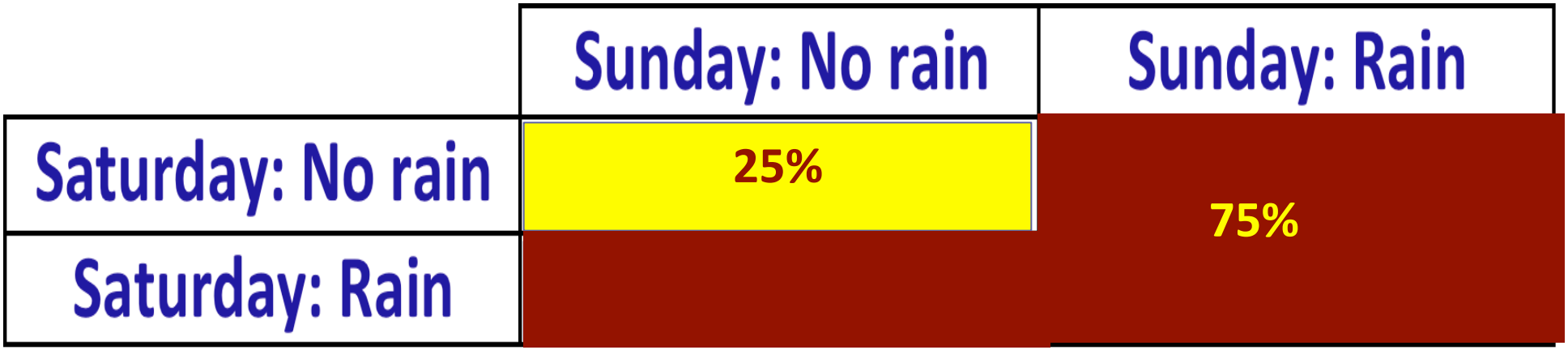
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What is the chance of rain on the weekend?

	Sunday: No rain	Sunday: Rain
Saturday: No rain	25%	25%
Saturday: Rain	25%	25%

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What is the chance of rain on the weekend?



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Being mathematically literate involves being able to . . .

- Deploy mathematical skill and understanding
- Frame and solve problems involving quantity, space, and probability
- Interpret and reason about quantitative, probabilistic, and spatial information and situations
- Use mathematical language, symbols, and representations to communicate about mathematical ideas
- Think with and use data

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Mathematical proficiency

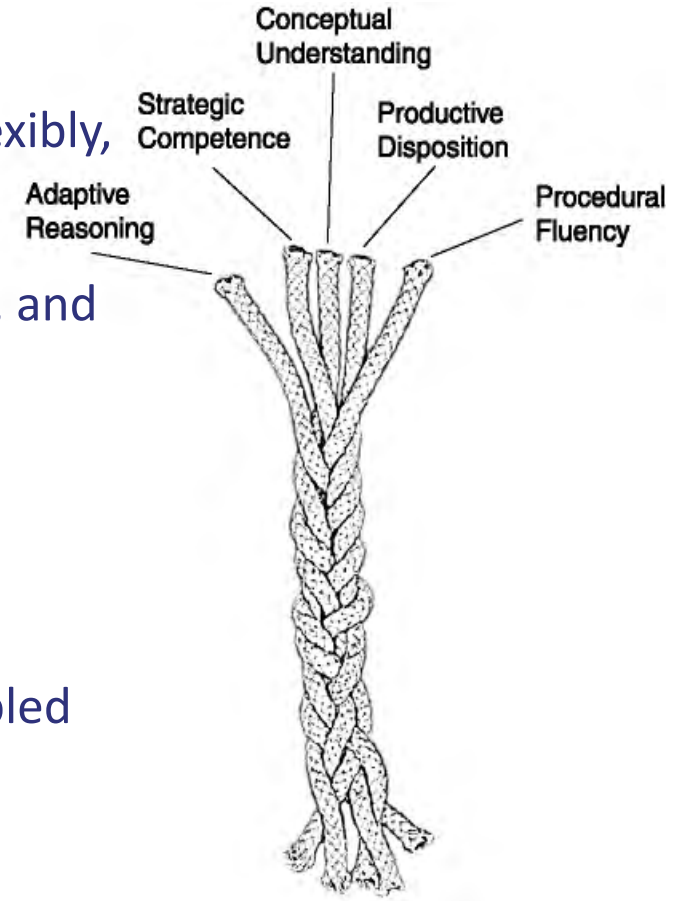
Conceptual understanding - comprehension of mathematical concepts, operations, and relations

Procedural fluency - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

Strategic competence - ability to formulate, represent, and solve mathematical problems

Adaptive reasoning - capacity for logical thought, reflection, explanation, and justification

Productive disposition - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy



Kilpatrick, J., J. Swafford, and B. Findell. (2001). *Adding It Up: How Children Learn Mathematics*. Washington, DC: National Academy Press.

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The 8's problem

Write as many 8s as you like. Insert plus signs so that the expression equals 1000.

How many different equations can you write following this rule?

What is your approach?

Can you find all of them? How do you know?

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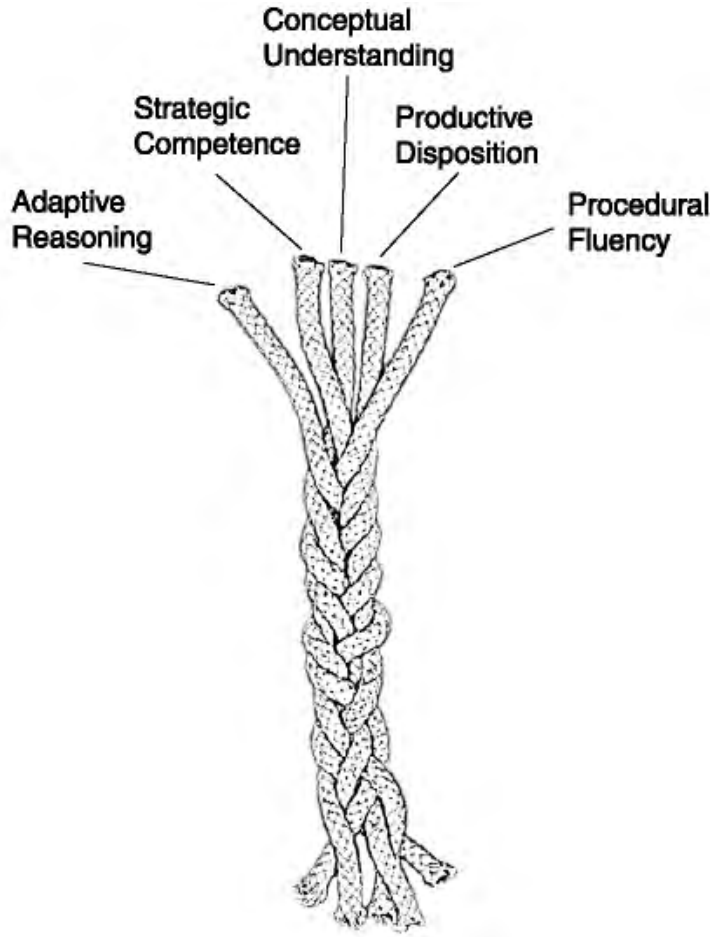
Can you find all of them? How do you know?

There are exactly 14 solutions. Can you find them all and prove that there are no more?

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Mathematical proficiency with the 8's problem

- **ADAPTIVE REASONING:** How do you know that your answer are solutions to the problem? How do you figure out if you have all the possibilities?
- **STRATEGIC COMPETENCE:** What are different ways to represent the set of solutions?
- **CONCEPTUAL UNDERSTANDING:** What happens each time you use one more 88? Why?
- **PROCEDURAL FLUENCY:** What is 88 x 5?
- **PRODUCTIVE DISPOSITION:** Could you solve the problem if you had enough time?



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3. Effective instruction

Making mathematical proficiency attainable by all

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How teaching is typically understood

1. Child-centered versus teacher-centered
2. Reform teaching versus traditional teaching

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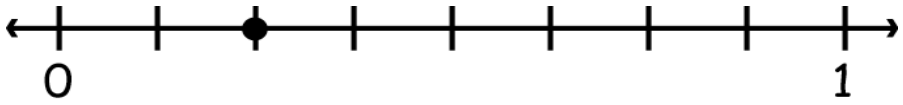
Is the teaching in this segment . . .

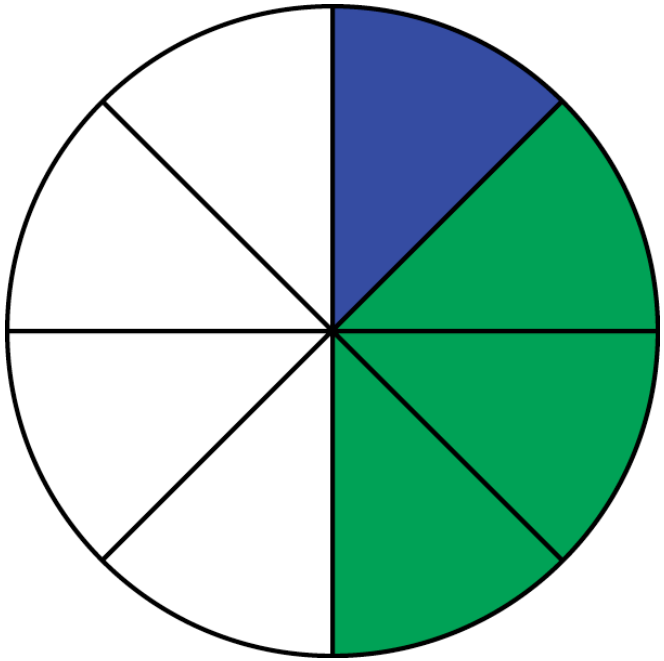
- Child-centered or teacher-centered?
- Reform or traditional teaching?

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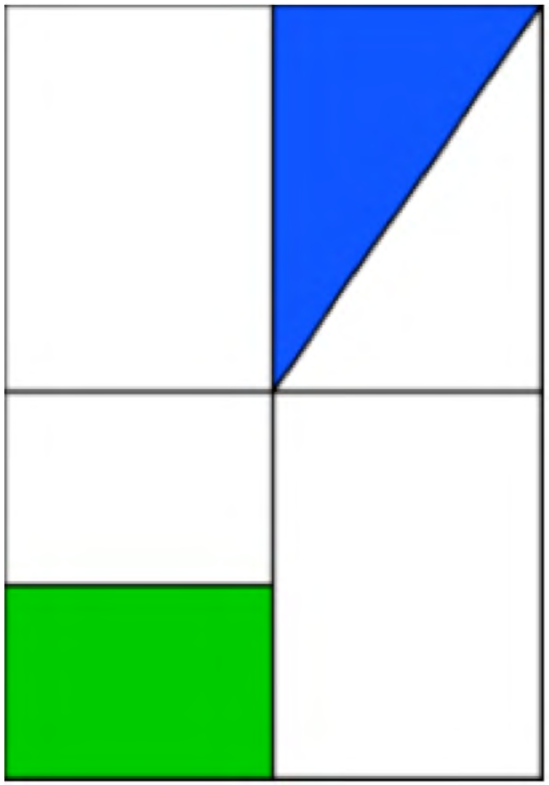
How are fractions typically taught at the upper elementary level?

What difficulties do students frequently have?





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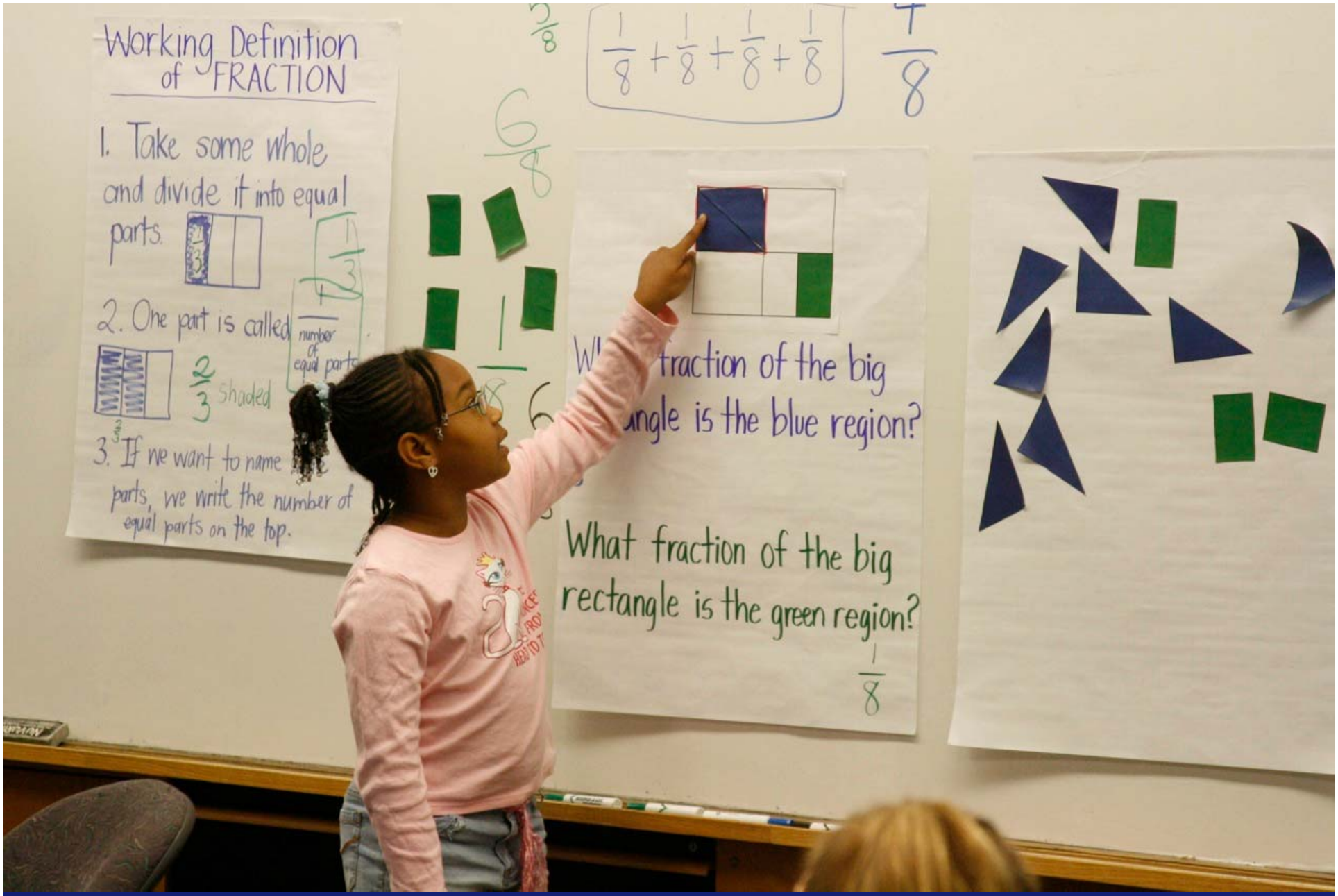


What fraction of the big rectangle is shaded blue?

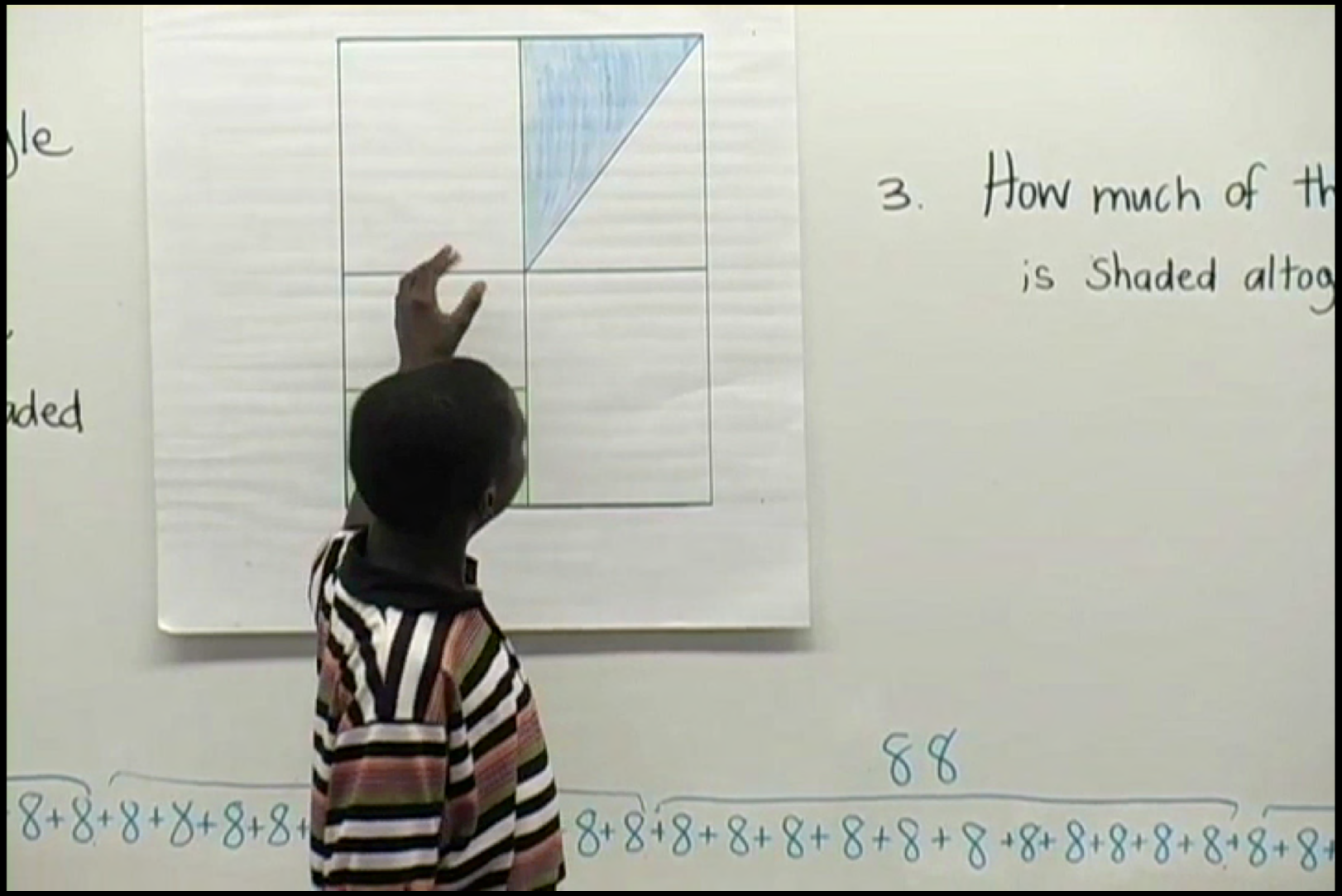
What fraction of the big rectangle is shaded green?

What fraction of the big rectangle is shaded altogether?

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Was the teaching in this segment . . .

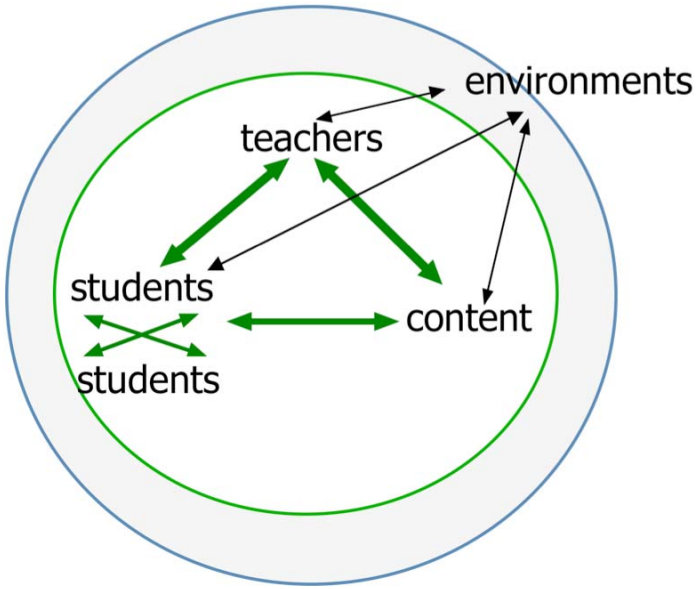
- Child-centered or teacher-centered?
- Reform or traditional teaching?

If our purpose is to enable teachers to learn to practice effectively, what is important to identify and name in this segment?

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What is “teaching”?

- Teaching is what is co-produced by students and teachers in contexts, around specific content and curriculum
- Teachers are the ones uniquely responsible for increasing the probability that students learn important content and skills, and gain at least a school year’s worth each school year



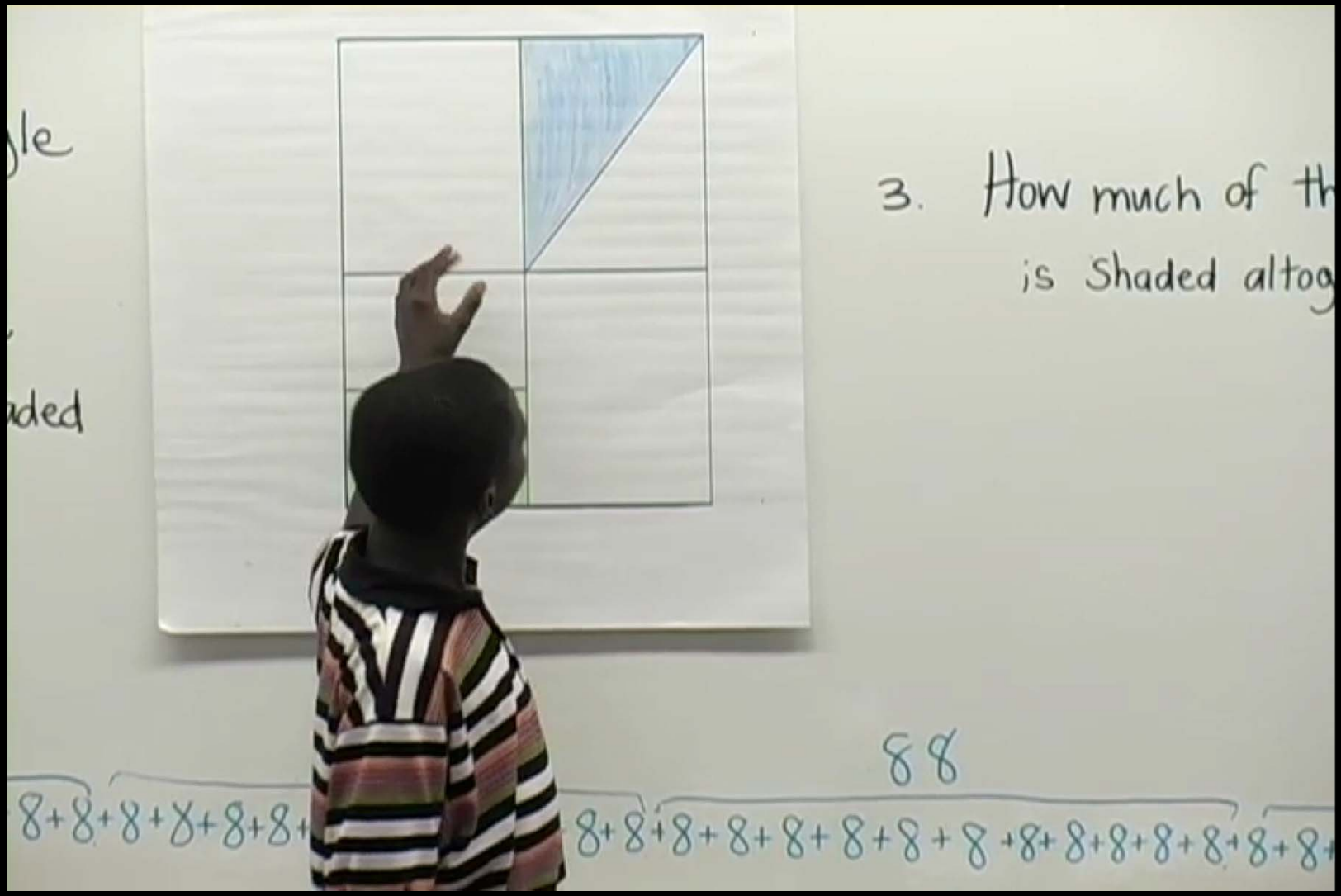
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Teaching as intricate and deliberate work

1. "Off camera": Before this episode
2. During these 5 minutes

What are some specific tasks required in order to teach this segment?

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Teaching as intricate and deliberate work

"Off camera": Before this episode

1. Learn about individual children and what they know, care about, are worried about, can do, etc.
2. Establish the environment to manage behavior
3. Teach intellectual habits (e.g., drawing, speaking to peers, knowing and being able to choose and make different kinds of mathematical moves)
4. Choose the specific problem: Why that diagram? Why that wording? Why the three specific questions?

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During these 5 minutes

00:00—Open the discussion: encourage participation by more students; establish expectation for explanation; use wait time; choose whom to call on; call on that child.

00:26—Mamadou gives answer of ½. Ask Mamadou to explain his reasoning; make sure other students can hear and are listening; interpret Mamadou's explanation; recognize relationship to key mathematical idea; determine how to respond (whether to take up, whether to clarify question, whether to call on different student).

00:47—Ask Mamadou to come to the board and explain using the diagram. Orient class toward Mamadou's explanation: get student to repeat what Mamadou said without explaining the error; comment about listening carefully; focus students on understanding reasoning.

01:37—Mamadou uses diagram to explain his answer. Invoke the working definition of fractions: ask Mamadou what he is calling the whole, how many equal parts, and how many are shaded.

02:26—Check class' understanding and manage risk of losing rest of class: call on student to explain Mamadou's solution; trace whole on diagram; establish correctness of Mamadou's answer given his selection of the whole.

03:00—Clarify the whole in the original question. Validate Mamadou's work, while establishing that the problem is asking something different. Ask student to read question and show on the diagram what is meant by "big rectangle"; ask Mamadou if he is watching; restate question and trace whole on a new copy of diagram.

04:06—Ask Mamadou to explain the difference between this question and the one he answered. Decide how to handle language "whole square" and "half the rectangle." Elicit answer to original question.

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Key practices of teaching to enable achievement for all students

1. Expecting and enabling complex work of each student
2. Coordinating subject-specific, school, and students' language
3. Using "real world" connections with sensitivity to students' funds of knowledge and to the integrity of the content
4. Creating a respectful academically-focused learning environment

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Key practices of teaching to enable mathematics achievement

1. Expecting and enabling complex mathematical work of each student
2. Coordinating mathematical, school, and students' language
3. Using contexts with sensitivity to students' funds of knowledge and to the integrity of the mathematics
4. Creating a respectful mathematically-focused learning environment

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Key practices of teaching to enable mathematics achievement

1. Expecting and enabling complex math… work of each student
2. Coordinating … ts' lang…
3. Usin… students' funds of knov… integrity of the mathematics
4. Creat… a respectful mathematically-focused learning environment

Expecting and enabling complex mathematical work of each student,

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Enacting high expectations in mathematics instruction

1. Broaden what it means to be successful in math class
2. Make mathematical practices explicit
3. Support students' mathematical work both publicly and privately
   - Listen carefully to students' talk
   - Notice and improve ambiguous talk
   - Ask mathematical questions
4. Teaching students to be "people who study mathematics"

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Enacting high expectations in mathematics instruction

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4. Teach students to be "people who study mathematics"

What is the work of teaching?

1. SCAFFOLDING STUDENTS' EXPECTATIONS AND STANDARDS
2. CHOOSING MATHEMATICAL TASKS
3. ENACTING HIGH EXPECTATIONS DURING CLASS

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Effective teaching as unnatural and intricate work

1. Teaching requires teachers to do things that are more than commonsense.
2. Teaching involves some moves or orientations that are unnatural in regular adult life.
3. Teaching practice is intricate, requiring a complex combination of knowledge, skill, timing, and relational work.

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"High-leverage" practices

- Have significant power in teaching because they:
  - Are central to the daily work of teaching
  - Attend to considerations of equity
  - Are most likely to support student learning

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Examples of considerations

- *Considerations central to the practice of teaching:*
  - High probability of making a difference in teaching quality and effectiveness
  - Effective in using and responding to differences among pupils
  - Useful broadly across contexts and content

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Examples of high-leverage practices

- Explaining ideas and processes
- Choosing and using representations, examples, and models of core content
- Setting up and managing small-group work
- Recognizing and identifying common patterns of student thinking
- Selecting and using specific methods to assess students' learning on an on-going basis

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Your comments so far?

1. What was particularly helpful? What worked well for you today?
2. What was less helpful today? What did not work for you?

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4. "Teacher quality"

What do teachers need to be able to do, and therefore, know?

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Teachers matter — a lot

- Persistent evidence that a large proportion of the variability in student achievement gains is due to the teacher
- So one obvious strategy for improving students' opportunities and learning is to ensure that they have teachers who are able to help them learn
  - ❑ Recruitment
  - ❑ Professional training

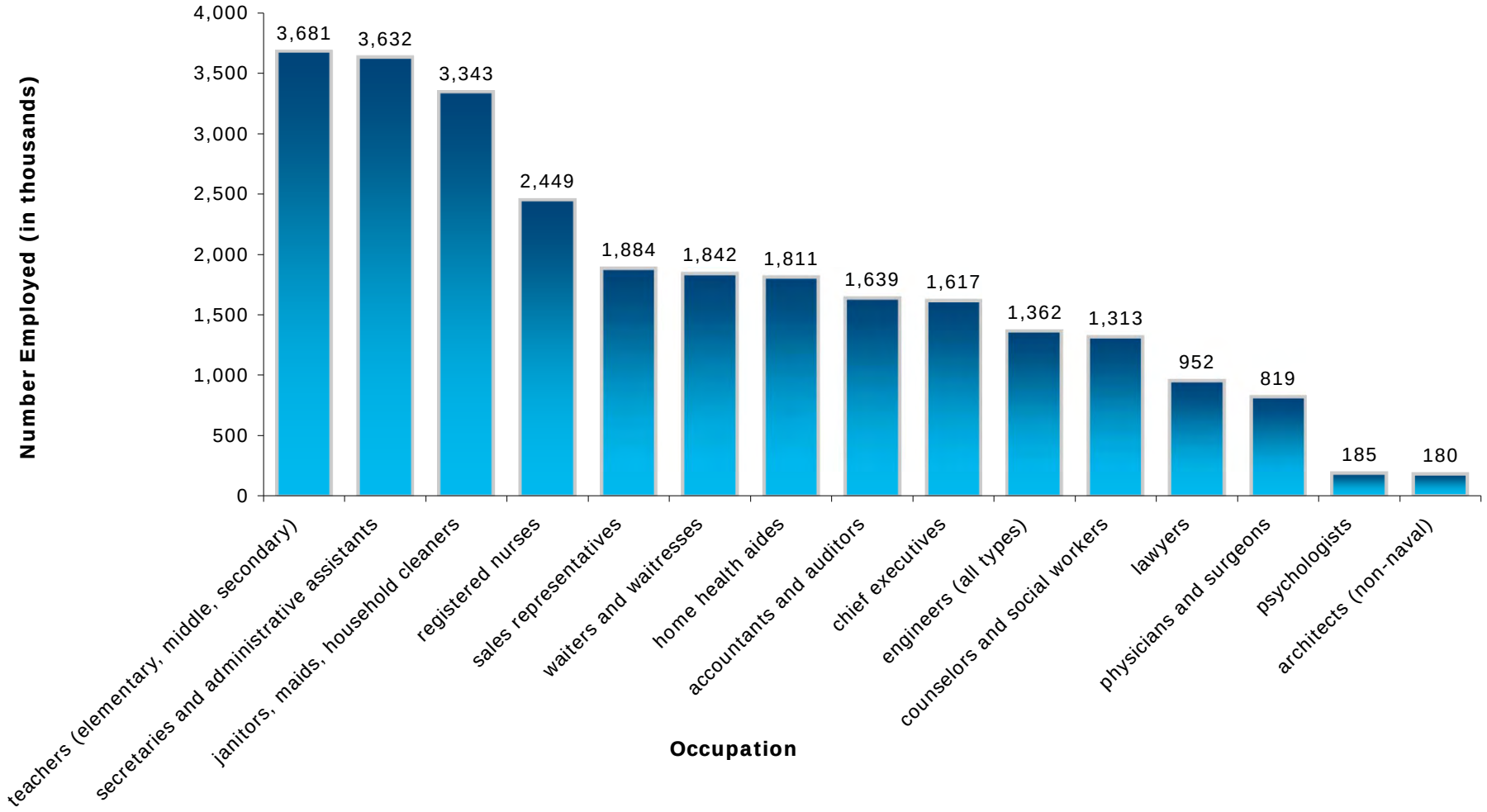
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Why the problem is one of professional training

- The scale of the need
- Teaching as unnatural, intricate, and deliberate work

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The realities of scale



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What *is* "mathematical knowledge for teaching"?

Knowing multiplication

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline \end{array}$$

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Knowing multiplication *for teaching*

What steps produced each of these answers?

(a)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 405 \\ 108 \\ \hline 1485 \end{array}$$

(b)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 225 \\ 100 \\ \hline 325 \end{array}$$

(c)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 1250 \\ 25 \\ \hline 1275 \end{array}$$

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Teaching involves special kinds of mathematical work

1. Solving special kinds of mathematical problems
2. Engaging in specialized mathematical reasoning
3. Using mathematical language precisely but accessibly

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A problem in teaching

You ask your fourth graders to explain what a rectangle is. One child offers a definition:

A rectangle is a flat shape. It has four square corners, and it is closed all the way around.

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What is the work for the teacher?

A rectangle is a flat shape. It has four square corners, and it is closed all the way around.

1. To see that something is missing
2. Decide what to do or say
3. Offer a counterexample

What is a shape that satisfies this definition and yet is <u>not</u> a rectangle?

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What is the work for the teacher?

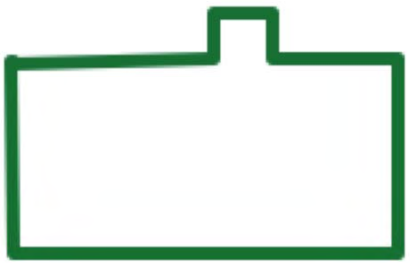
A rectangle is a flat shape. It has four square corners, and it is closed all the way around.

1. To see that something is missing
2. Decide what to do or say
3. Offer a counterexample

"Is this a rectangle?"

(straight sides)





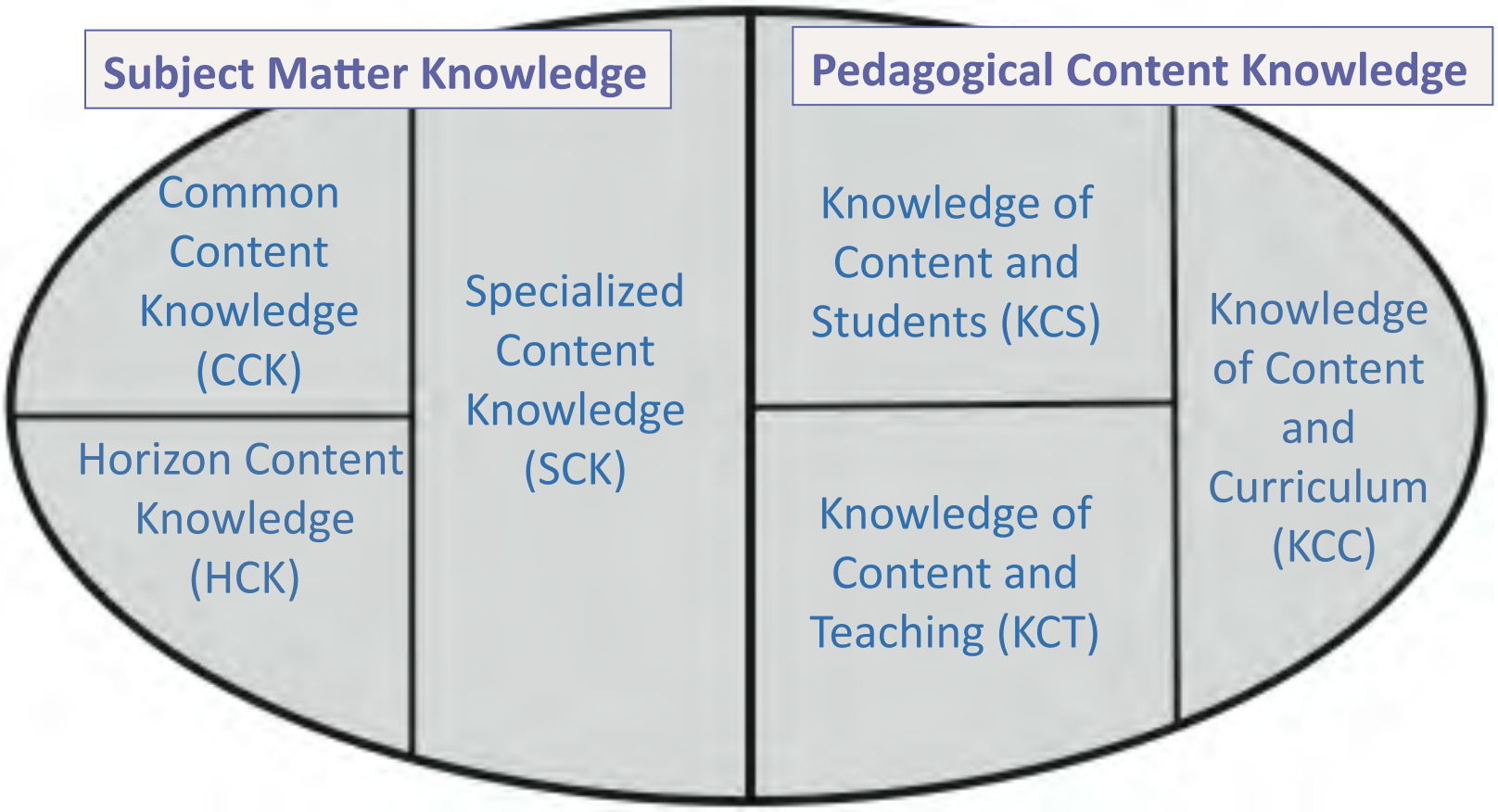
(exactly four square corners)

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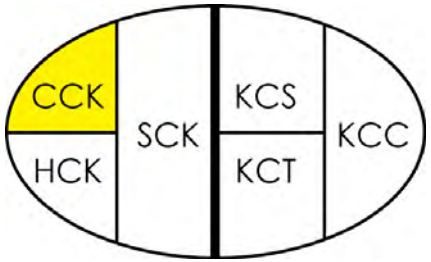
A rectangle is a flat shape with **straight sides that are connected at exactly** four square corners. It is closed all the way around.

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Mathematical Knowledge for Teaching (MKT)



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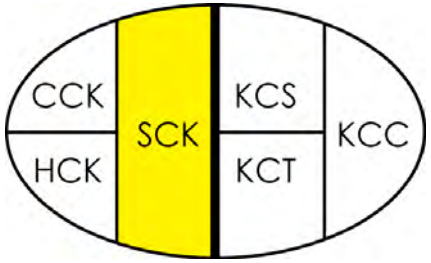


Common content knowledge (CCK)

Draw a **rectangle**.

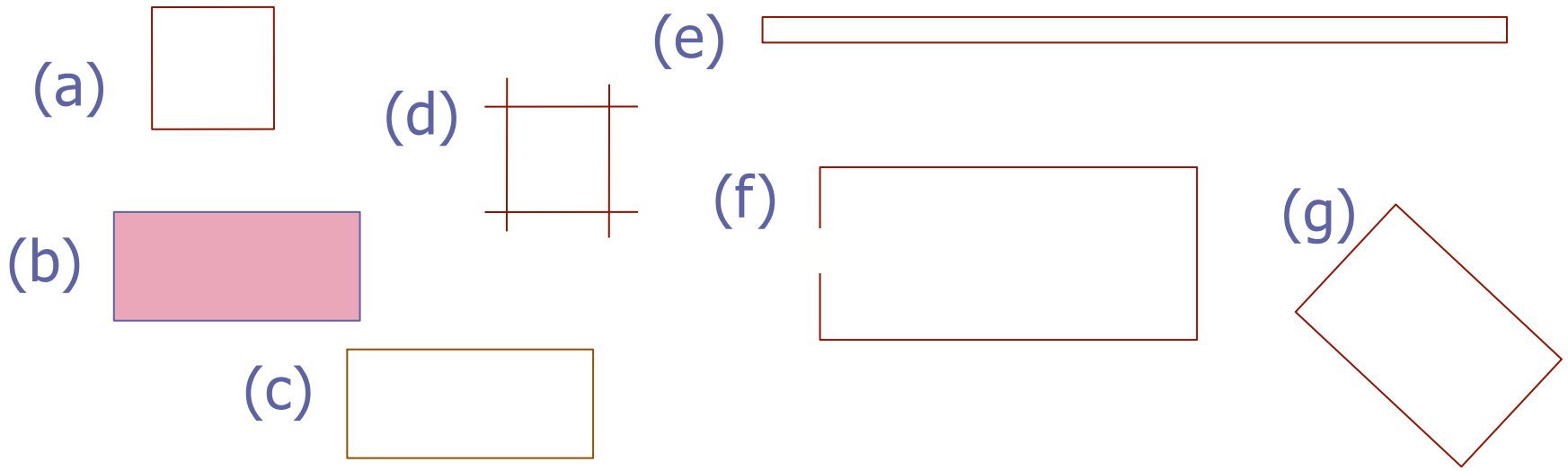


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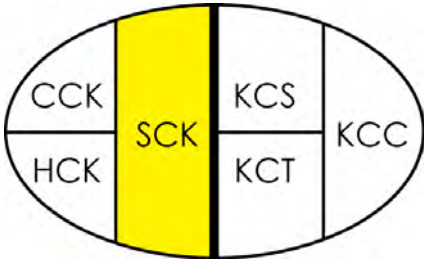


Specialized content knowledge (SCK)

Which of these figures would be good to present to assess whether students understand what a rectangle is, and why?



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Specialized content knowledge (SCK)

Which of these is a mathematically accurate definition of "rectangle"?

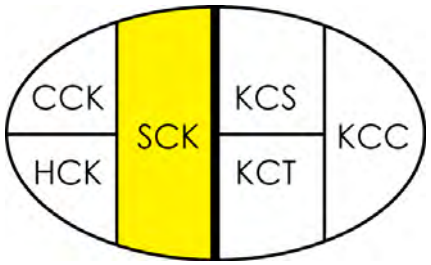
① A rectangle is a figure with four straight sides, two long and two shorter.

② A rectangle is a shape with exactly four connected straight line segments meeting at right angles.

③ A rectangle is flat, and has four straight line segments, four square corners, and it is closed all the way around.

For any that are not mathematically accurate, give an example that shows what is wrong.

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Specialized content knowledge (SCK)

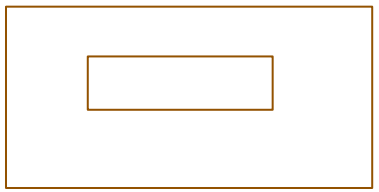
① A rectangle is a figure with four straight sides, two long and two shorter.



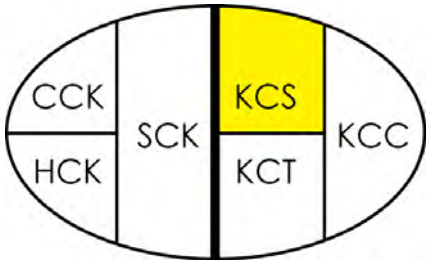
② A rectangle is a shape with exactly four connected straight line segments meeting at right angles.



③ A rectangle is flat, and has four straight line segments, four square corners, and it is closed all the way around.



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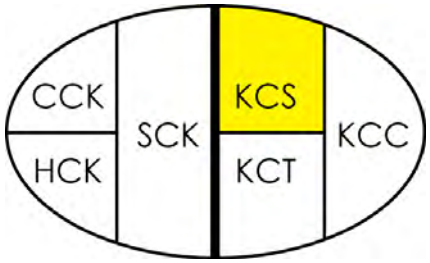


Knowledge of content and students (KCS)

- Write a mathematically accurate definition of "rectangle" that is usable by second graders.

- How can the notion of "simple closed curve" be expressed in a way that is both mathematically accurate and usable? Which part of this phrase is most challenging for children?

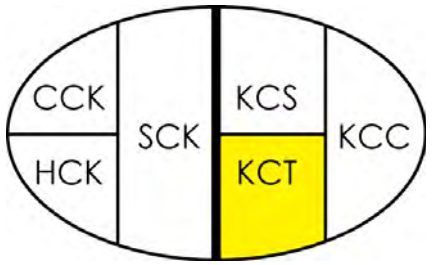
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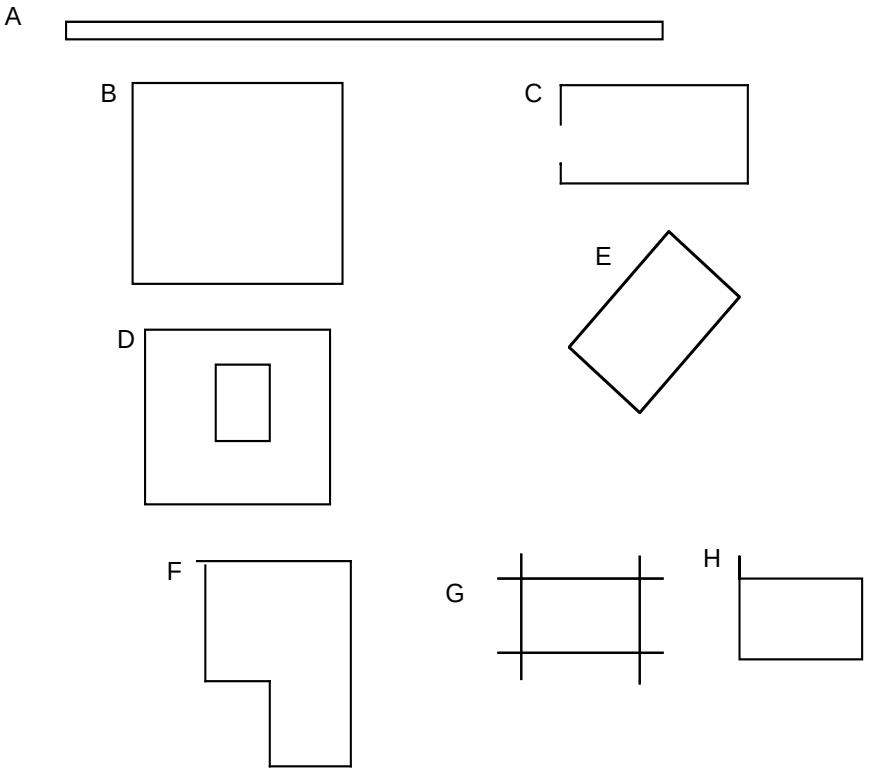
Knowledge of content and students (KCS)

- What are students likely to know about what a rectangle is?
- What do students typically have difficulty with in learning about rectangles, and why?

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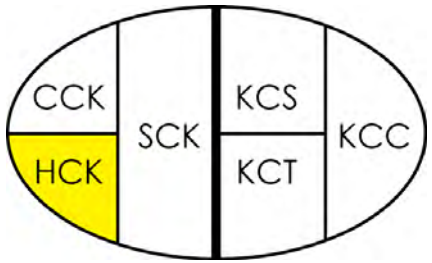


Knowledge of teaching and content (KCT)



- How would you sequence these figures to discuss the concept of a rectangle?
- What task would you create using these figures (or others) to set up a productive discussion aimed at developing a definition?
- In a whole-class discussion, which one would be good to discuss first?

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Horizon content knowledge (HCK)

- Is it okay to shade in the figures shown to students – e.g., 
- What are the issues involved with the fact that children learn about rectangles before polygons?

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5. Professional training and development

How can teachers learn to do the work of teaching effectively?

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Main argument

We need a system that can reliably equip "regular" people for effective professional practice in teaching.

- Doing that requires:
  - Focusing on the *work of teaching*;
  - Zooming in developmentally on the highest-leverage practices;
  - Emphasizing the *performance of teaching* in the curriculum and in assessments.

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We lack a reliable system to prepare professionals for practice

- A professional curriculum off-center, emphasizing knowledge and beliefs rather than practice
- An unwarranted degree of faith in "learning from experience"
- Assessments that do not measure actual professional effectiveness
- Inappropriate subject matter preparation
- Inadequate preparation for diversity of U.S. classrooms

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The challenge

1. Lack of common or sufficiently precise language for the work of teaching (Grossman, et al..)
   - Teaching practice has not been decomposed into practically-sensible parts
   - Lack of useful terms for the parts
   - Where there is language, it is often not shared
2. Incomplete knowledge about the problems of designing and delivering more explicit clinical training in teaching

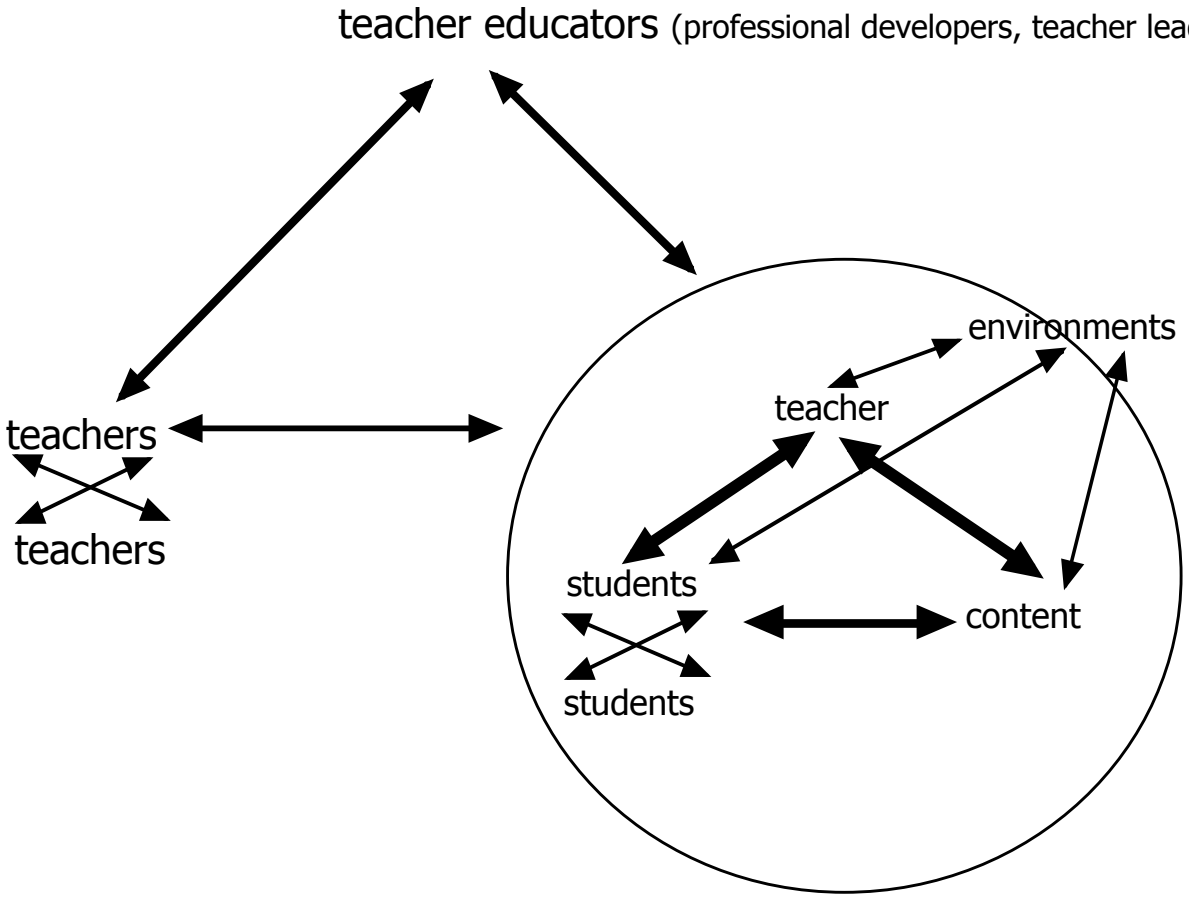
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Why is our system of teacher development so impoverished?

1. Lack of attention to teaching
   - Value-added measurement that fails to investigate variation in teaching practices
   - Orientation to "constructivist" or "student-centered" classrooms
   - Belief in teaching as highly improvisational and largely unspecifiable
2. A confounded view of "professionalism"
   - Agreement on the complexity of practice
   - Disdain for "prescriptiveness" and detail: "de-skilling" teaching (an irony)

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Teaching practice as the content of teacher development



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Identifying what to work on

- Decompose the work of teaching into smaller practices that:
  - can be articulated, unpacked, studied, and rehearsed
  - can be reintegrated into real-time teaching
- Choose practices that are "highest-leverage"
  - Occur frequently in teaching
  - Crucial to improve the learning and achievement of all students
  - Can be articulated and taught

Grossman and Shahan, 2005; Boerst, Sleep, and Ball, 2006; Ball, Sleep, Bass, and Boerst, in press

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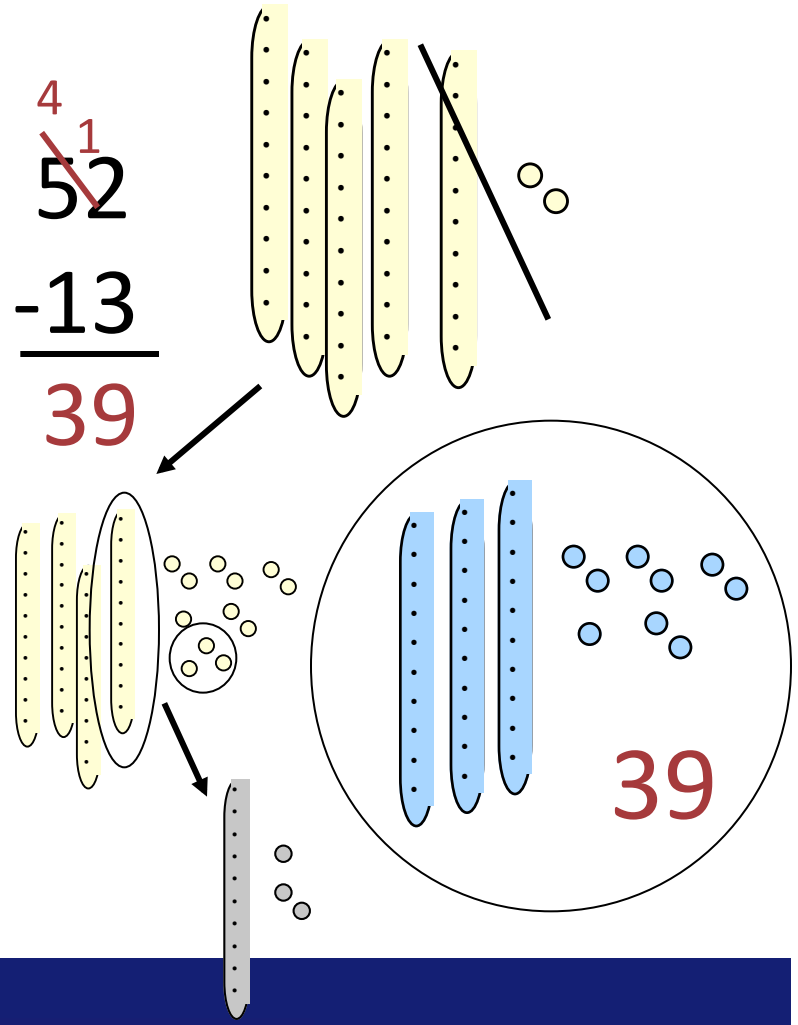
A few examples . . .

- Communicating productively with parents about mathematics
- Posing well-framed questions
- Launching a task
- Explaining mathematical ideas and procedures clearly
- Assessing and diagnosing pupils' skills and knowledge
- Leading a whole-class discussion of a math problem

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Using specialized content knowledge:
Modeling subtraction with base-ten materials

- Mapping closely between actions with the materials and the symbolic representation
- Showing that "borrowing" is actually regrouping, or "re-writing" a number
- Developing fine attention to details of doing the representing



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Teaching practice

1. Outside the classroom (but still "in practice")
2. Subject-matter focused
3. Real pieces of the work
4. Roles of colleagues

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What are possible candidates for "high leverage practices" of teaching?

(constructed collective list)

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What are ways to support teachers' learning of practice?

1. Situate teachers' opportunities to learn in the curriculum they teach
   - Work on the actual content and ways to represent and explain it
   - Study students' work and thinking on the specific content
   - Work on detailed lesson design
2. Work on specific instructional practices
   - Choose a small set of high-leverage practices
   - Provide focused time to study, rehearse, analyze, and practice these HLPs

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Practicing the mathematical work of teaching

- Working on examples like we have done, including practicing providing reasons
- Developing fluency and speed
- Developing sensibilities about mathematical language

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Choosing pedagogically-strategic examples

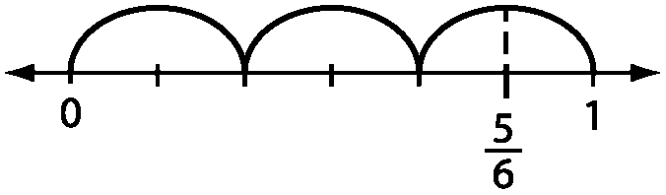
Which of the following lists would be best for assessing whether your students understand decimal ordering? Justify your choice.

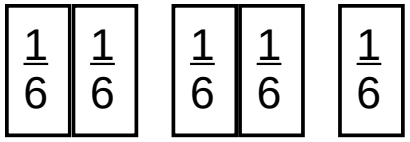
A.	**.5**	**7**	**.01**	**11.4**
B.	**.60**	**2.53**	**3.12**	**.45**
C.	**.6**	**4.25**	**.565**	**2.5**

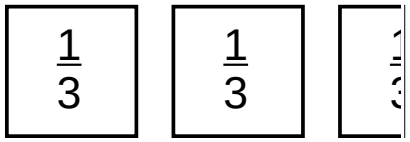
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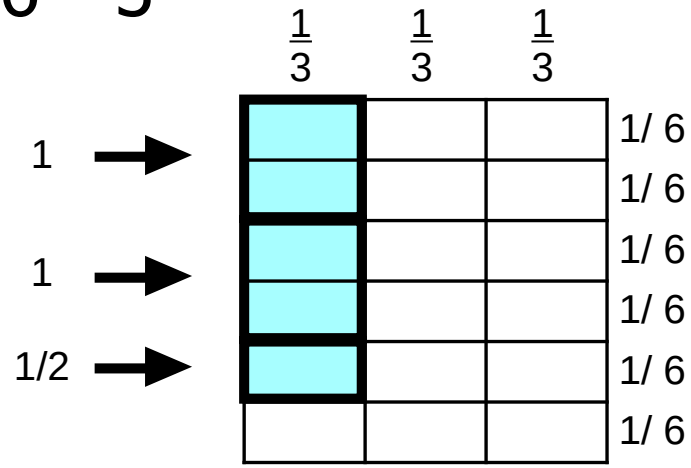
Analyzing — and "talking" — representations

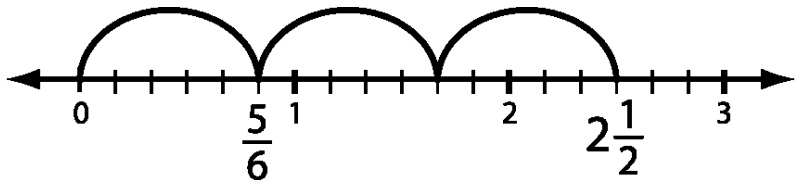
Which of these can be used to represent $\frac{5}{6} \div \frac{1}{3}$? Explain with reference to all parts of the expression.











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Analyzing non-standard (but correct) responses

Which student is using a method that could be used to multiply any two whole numbers?

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

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Using language precisely and accessibly #1

A polygon is a simple closed plane curve composed of finitely many straight line segments.

OUR DEFINITIONS

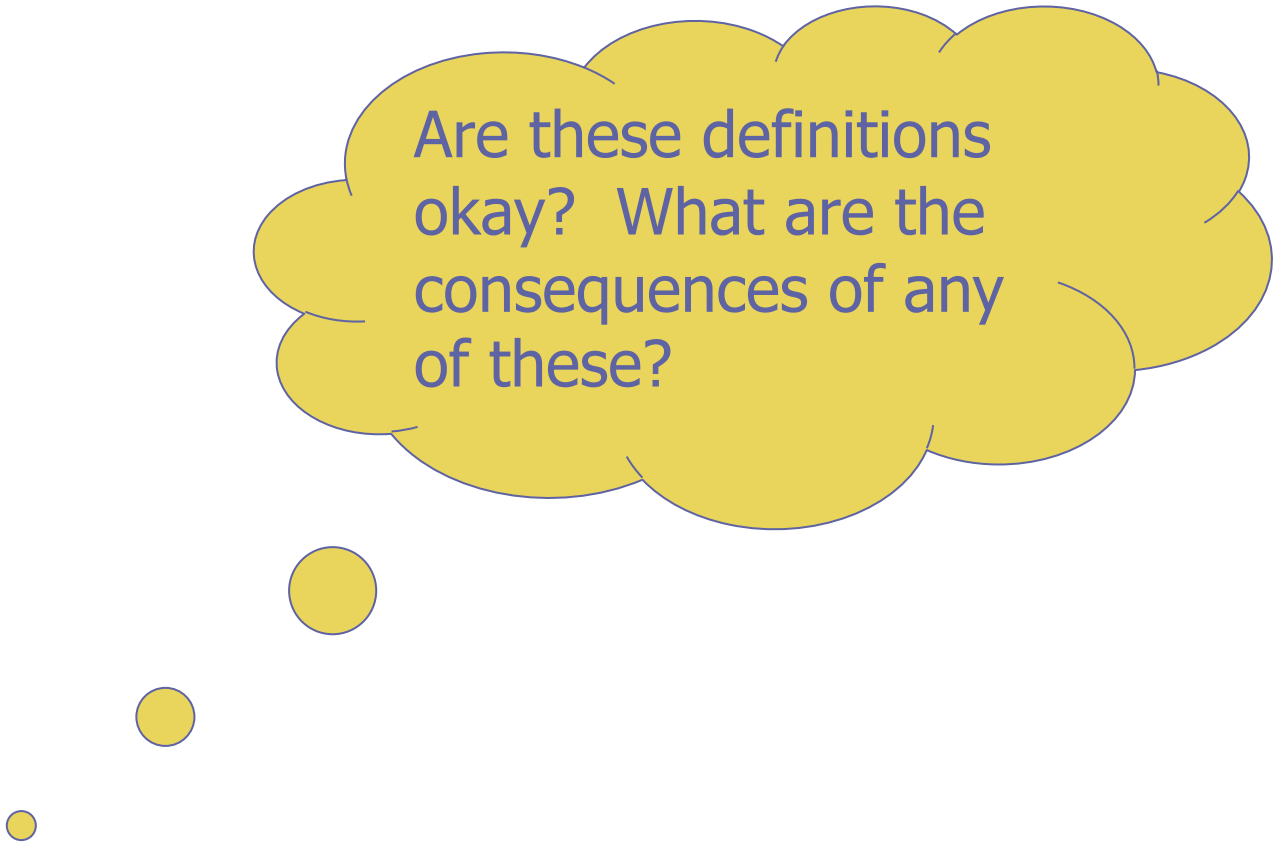
Polygon:

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Using language precisely and accessibly #2

a) An even number is a number that can be divided into two equal parts.
b) An even number is any multiple of 2.
c) An even number is any integer multiple of 2.
d) An even number is any number whose unit digit is 0, 2, 4, 6, or 8.
e) A whole number is even if it is the sum of a whole number with itself.

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a) An even number is a number that can be divided into two equal parts.

b) An even number is any multiple of 2.

All numbers, for example 7, 3/5, $\sqrt{2}$, π, are even!

c) An even number is any integer multiple of 2.

This is a correct definition of even number.

d) An even number is any number whose unit digit is 0, 2, 4, 6, or 8.

In this case, 36.7 is an even number!

e) A whole number is even if it is the sum of a whole number with itself.

This is a correct definition of evenness for whole numbers, and is consistent with the general definition for integers that will arrive later.

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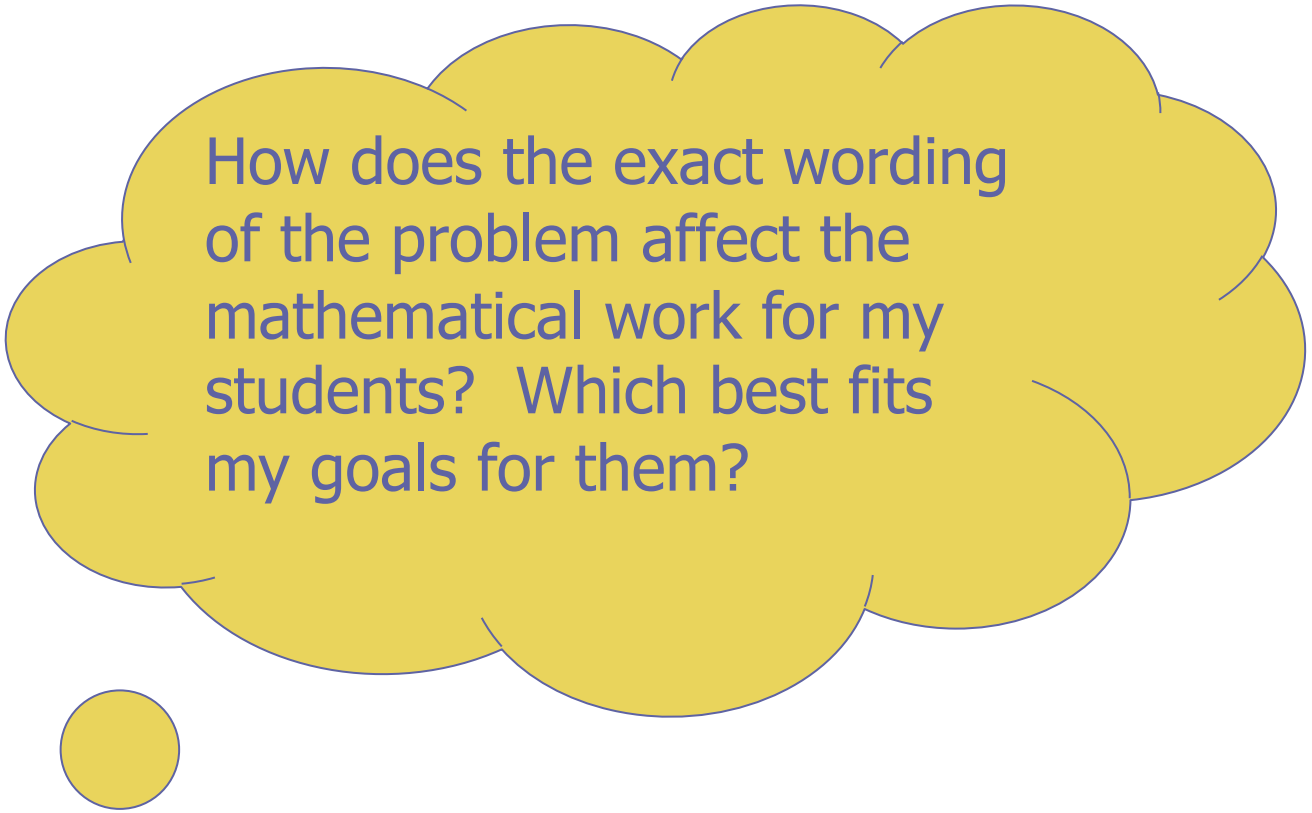
Posing mathematics problems

POSSIBLE PROBLEM:

I have pennies, nickels, and dimes in my pocket. If I pull out two coins, what amounts of money might I have?

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Reasoning about different wording



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Reasoning about different wording

1. I have pennies, nickels, and dimes in my pocket. If I pull out two coins, what amount of money might I have?

2. I have pennies, nickels, and dimes in my pocket. If I pull out two coins, how many combinations are possible?

3. I have pennies, nickels, and dimes in my pocket. If I pull out two coins, how many different amounts of money are possible? Prove that you have found all the amounts that are possible.

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Resources for focusing on practice

- Time
- Specificity of what to work on
- Courage to see teaching as highly skilled, precise work
- Records of practice

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6. Next steps

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Wrapping up

1. Your post-test
2. Next steps
3. Evaluation of the day

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The post-test
Checking your answers

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Which are myths? Which are true?

Mark M (myth) or T (true).

a. The U.S. mathematics education system used to work better than it does now.

b. The number of mathematics courses that a teacher has taken is a good predictor of how effective he or she will be.

c. Societal problems — e.g., inequality, poverty, the declining family — are so overwhelming that schools cannot do their job.

d. The U.S. math curriculum is similar to the one taught 50 years ago.

e. The most important characteristic of effective professional development is that it focus on instruction.

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Which are myths? Which are true?

M The U.S. mathematics education system used to work better than it does now.

M The number of mathematics courses that a teacher has taken is a good predictor of how effective he or she will be.

M Societal problems — e.g., inequality, poverty, the declining family — are so overwhelming that schools cannot do their job.

T The U.S. math curriculum is similar to the one taught 50 years ago.

T The most important characteristic of effective professional development is that it focus on instruction.

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Evaluation

1. What was particularly helpful? What worked well for you today?
2. What was less helpful today? What did not work for you?
3. What supports or follow-up would you like to have?

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Thank you!

Google “Deborah Ball”
for slides from this talk.

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