

Challenges of Knowing Mathematics for Teaching in the United States

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Mathematical knowledge of teachers in the U.S.

- Has often been overlooked by curriculum developers, policymakers
- Has been an increasing concern
  - The failure of the "New Math" in the 1960s
  - Mathematicians' critiques
  - Research on teachers' mathematical knowledge in the 1980s and 1990s

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What is known about teachers' mathematical knowledge

- Teachers often lack basic understanding of mathematical ideas and procedures
- The number of mathematics courses taken does not predict student achievement below secondary school; weak predictor at secondary school level
- The number of "mathematics methods" courses is a better predictor (CLUE)

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Asking a better question

- Not "how much" does a teacher know, but "in what ways" and "how well" does a teacher know mathematics
- Must be related to the use of mathematics in practice – teachers' use of mathematics in teaching

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Teaching is mathematically intensive work

- Studies to investigate the kinds of mathematical knowing needed in teaching: initial evidence related to student achievement
- Many mathematical demands arise in the course of teaching
- Mathematical skills involved in planning and assessing

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Examples of the mathematical work involved in teaching

- Understanding and interpreting students' mathematical talk

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Ofala: I just wanted to say if you wanted an odd number, you just have to take one- one off of it.

Teacher: Why would that work?

Ofala: Because usually odd numbers are like-

Teacher: Cassandra, can you hear Ofala okay?

Ofala: This. That is kind of like have ones in the middle? Like five.

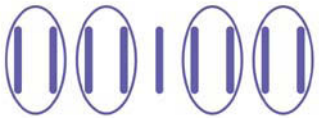


And there's a one in the middle, or nine.

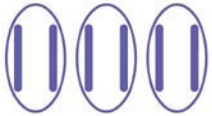


Mei: I think it's better with circles.

Ofala: This two together, this two together, this two together, and this two together.



There's one left. And even numbers, like six-



You can't get anything in the middle. There isn't one left.

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Other examples

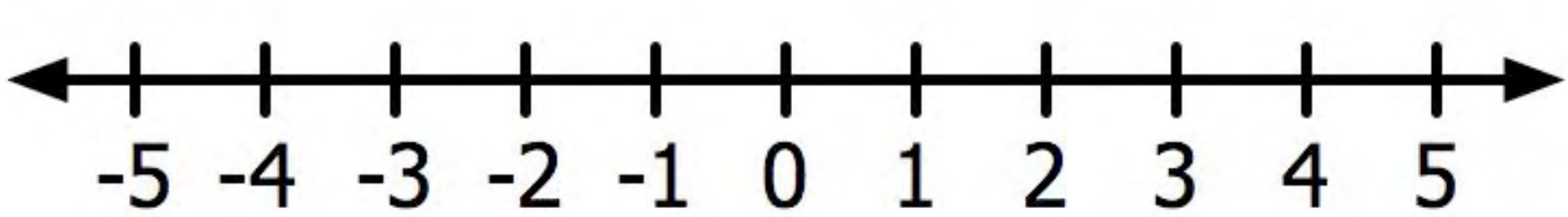
- Choosing and using representations with care

2 + 3

2 - 3

2 + (-3)

2 - (-3)



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A second example

- Figuring out the mathematical point of a task or a lesson in the curriculum and being able to use it in real time with students to achieve that point (Sleep, 2010)
- This more than being able to do the task oneself

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A primary grade task





Using exactly one positive and one negative checker, find all the numbers that can be represented on this Minicomputer board.

Prove your answer and explain why.

Papy Minicomputer

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A primary grade task

What is the largest number that can be made? What is the smallest?

Can you get a number in more than one way?



Using exactly one [illegible] and one negative checker, find all the numbers that can be represented on this Minicomputer board.

Prove your a[illegible] in why.

What numbers inbetween can be made? How and why?

Might some numbers not be possible? Why might that be?

Papy Minicomputer

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Mathematical ideas and practices involved

- Even and odd numbers
- Powers of two
- Properties of integer addition
- Constructing and critiquing mathematical arguments
- Symmetry

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What are the mathematical purposes of the task?

1. Noticing and using the structure of the solution space:
   - Symmetry: Realizing that reversing the checkers gives the opposite number (so it suffices to consider positive numbers).
   - All numbers you get are differences of powers of two.
   - Finding that to get an odd (positive) number, the negative checker must be on the 1-square, so you can only get a power of two minus one (i.e., this rules out 5.)
2. Reasoning and proving:
   - Developing habit of asking whether a solution exists and that its existence (or lack thereof) can be proved.
   - Developing habit of finding all the solutions, and proving completeness.
3. Practice with adding integers and explaining answers.

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Many other ways in which teaching is mathematical work

Examples:

- Explaing mathematical ideas and procedures clearly and correctly, attuned to learners
- Choosing pedagogically strategic examples
- Devising and asking mathematical questions in class
- Managing a discussion of a set of solutions to a problem
- Figuring out how to fill in learners' gaps while also advancing their progress
- Writing and evaluating quizzes and tests

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New evidence

- The specialized knowledge needed for teaching (MKT) may be more predictive of student learning than courses taken or a major in mathematics (Hill, Rowan, Ball, 2005)
- Mathematical knowledge for teaching includes "content knowledge," as well as more specialized mathematical knowledge, and "pedagogical content knowledge" (e.g., Ball, Thames, Phelps, 2008, but several other relatively similar schemes)

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Despite this progress, U.S. teachers' mathematical knowledge for teaching remains weak.

Typical explanations:

- Low selectivity into teaching
- Low quality of entrants to teaching
- Lack of sufficient mathematical requirements
- Weak emphasis on content knowledge in teacher education

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Yes, but . . .

- Other features of the U.S. system create challenges to building a profession of mathematically skillful teachers
- Are these unique to the U.S. context?
- How could the United States solve these challenges? Is there the capacity (will, knowledge, and skill) to do so?

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Why are we surprised?

- U.S. students perform poorly compared with our peers internationally.
- U.S. teachers received their mathematics education from the very system that is underperforming.

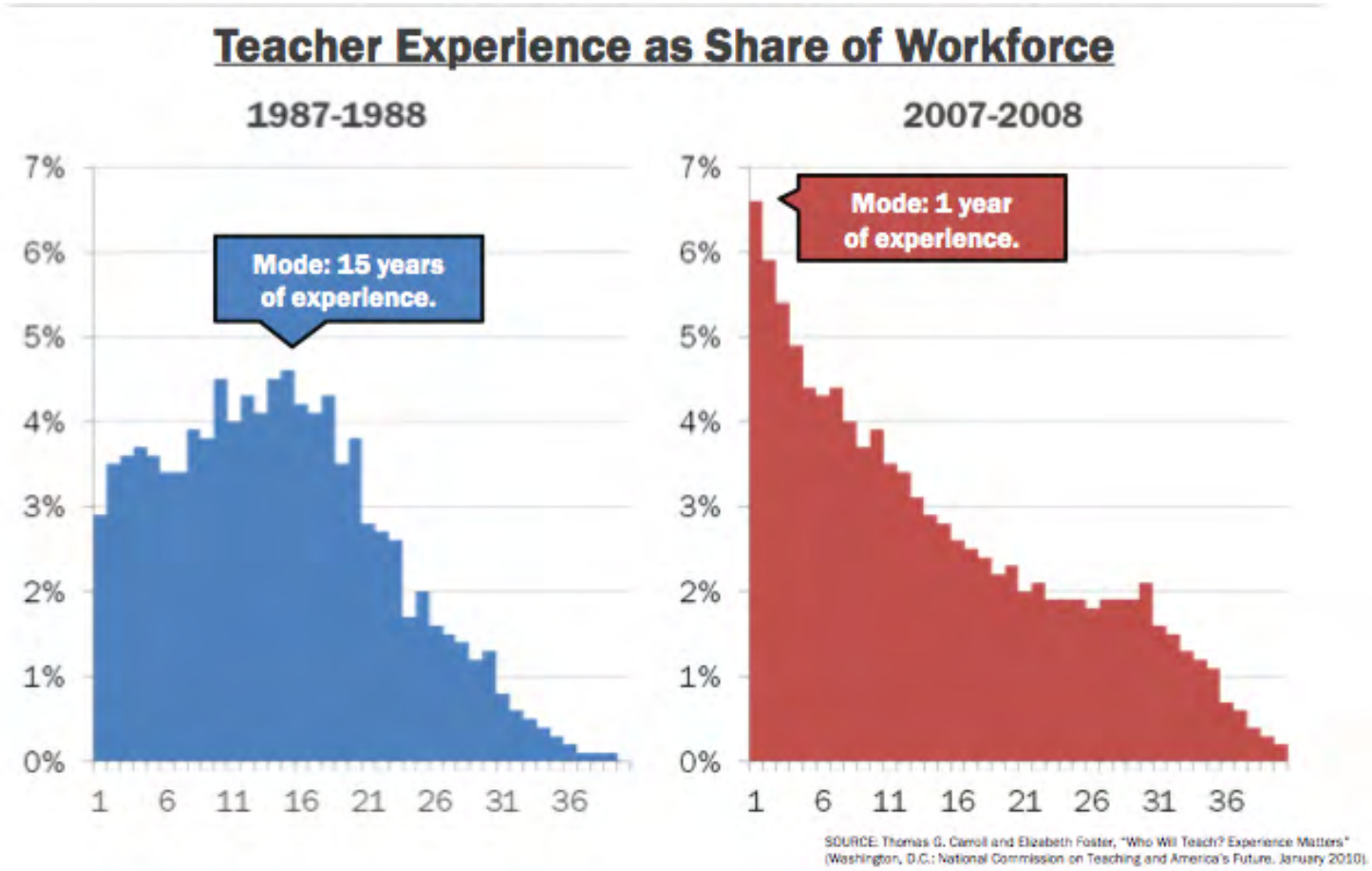
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Scale and dispersion of U.S. educational "system"

- 3.5 million teachers
- 50,000,000 schoolchildren
- 15,000 separate school districts
- Very weak educational infrastructure

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High teacher turnover → lack of a profession



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1. Local control of education and curricular variability

- No national curriculum
- Wide variation in curriculum and curriculum materials
- No examinations referenced to what is taught (although lots of testing)
- Waves of reform, over and over (consider the curriculum experience of someone who enters teacher preparation in 2012)

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2. Enormous inequity among schools and learners' opportunities and experiences

All highly correlated with income and race:

- Availability of advanced courses
- Teacher skill and expertise
- Quality of curriculum
- Amount and coherence of instruction

Rapidly increasing diversity of U.S. school population (language, race, religion, ethnicity)

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Big variability in what people learn in school leads to big variability in the pool of prospective teachers.

This is a big problem because we need:

- Mathematically skillful teachers
- A more diverse workforce (only 14% teachers of color)
- Many teachers (1 million new teachers needed in the next 4- 5 years)

Is teacher education the solution?

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3. But teacher education in the U.S. also highly varied

- Over 2200 independent programs; huge variability in size, quality, curriculum, instructors, length, etc.
- No common standards for content preparation: Most mathematical preparation in university mathematics departments, not coordinated
- No common core of professional knowledge and skill
- Highly varied instructors in mathematics and practice
- No common examinations of teachers' actual readiness for the classroom

Big changes would be needed in teacher education.

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A big question

- Does the United States have the capacity (will and resources) to meet these challenges, and start building a mathematically skillful workforce of teachers?

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Good signs

- More attention to instruction (the technical core) than ever before
- The Common Core State Standards in English language arts and mathematics
- Some signs of systems developing, even if not yet common

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Big challenges

- Growing economic inequality and local governance: teacher distribution, other resources for instruction
- Emphasis on teachers (instead of teaching)
- No professional ministry of education, or education inspectorate for improving instruction

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Is the U.S. unique?

- Do other countries face similar challenges?
- Have other countries managed to solve problems such as the U.S. faces?

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Thank you!



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