Who Knows Mathematics Well Enough to Teach It?

Deborah Loewenberg Ball



Current Research Group: Hyman Bass, Heather Hill, Mark Thames, Laurie Sleep, Jennifer Lewis, Imani Goffney, Seán Delaney, Geoffrey Phelps, Deborah Zopf, Yaa Cole

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List these fractions in order from least to greatest.

$$\frac{4}{5} \qquad \frac{7}{9} \qquad \frac{6}{11} \qquad \frac{9}{18}$$

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Which list of fractions would be best to challenge students to put fractions in order without using common denominators?

(a) $\frac{3}{8}$ $\frac{2}{7}$ $\frac{3}{10}$

(b) $\frac{1}{3}$ $\frac{4}{11}$ $\frac{3}{5}$

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What mathematics do teachers need to know?

- Where the question came from (for me and for the field) and how it has evolved
- Why the question matters
- Different ways to answer the question
- How different answers inform practice and policy

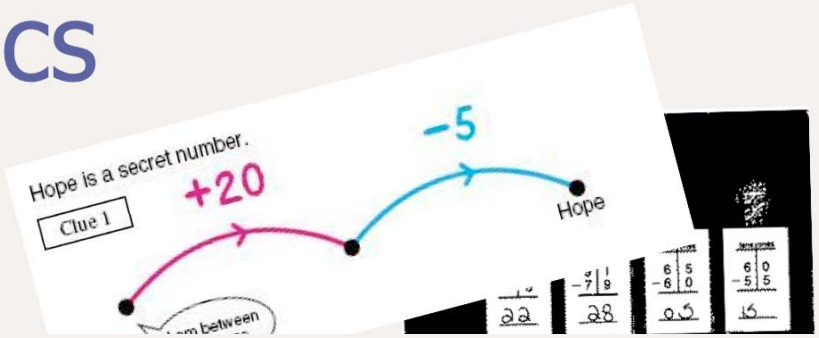
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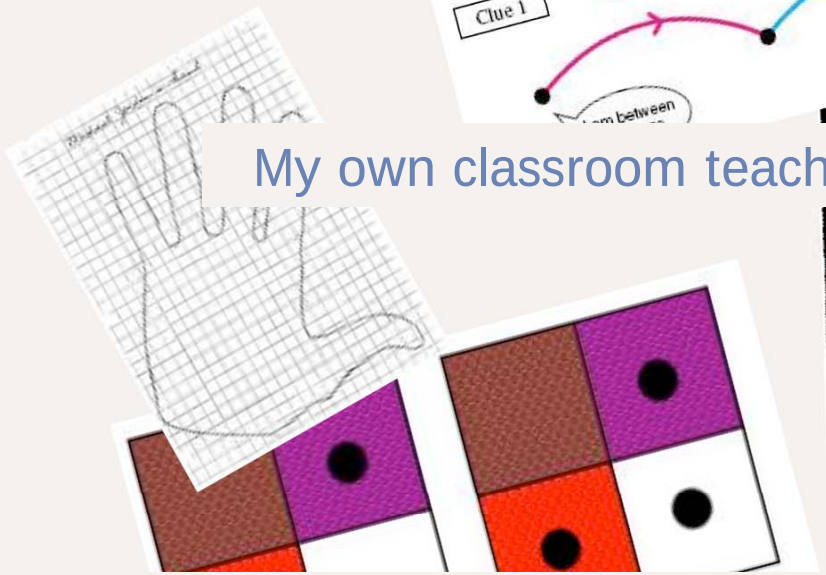
Overview

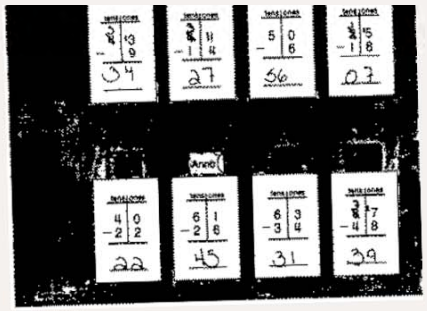
- Defining “mathematical knowledge for teaching”
- How well do YOU know mathematics for teaching?
- Measuring mathematical knowledge for teaching: Results

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Studying the work of teaching mathematics







My own classroom teaching (1975 - 1988, 1988 -)

Other classrooms

Deliberate design of laboratory classes (PCMI, others)

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Mathematical knowledge for teaching (MKT)

- What do we mean when we use this term, "mathematical knowledge for teaching"?
  - Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching
- What do we mean by the "work of teaching"?
  - The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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Other work of teaching

- Examining the mathematical quality of a textbook treatment of a topic
- Using and defining terms
- Supporting the linguistic demands for English language learners (ELLs)
- Producing and evaluating mathematical explanations
- Using notation
- Interpreting and evaluating alternative solutions and thinking
- Choosing contexts with care for mathematical integrity, diversity, and transparency for learning
- Explaining goals and mathematical purposes to others
- Designing homework and quizzes; Selecting and modifying mathematics problems
- Choosing and using representations
- Building correspondences between a model and a concept or procedure
- Evaluating students' work and statements (often quickly)
- Posing questions
- Choosing examples

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How well do you know mathematics for teaching?

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Overview: Trying your hand at using mathematics in teaching

1. Noticing and identifying the mathematics in students' productions
2. Posing strategic mathematical problems to students
3. Choosing examples for student tasks

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Case 1:
Noticing and identifying the mathematics in classroom discussion

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Classroom Context

- Third graders (ages 8 - 10)
- Mathematics: division with remainders, fractions as parts of whole, fractions on the line
- Multi-lingual and multi-cultural class

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What do you hear and notice mathematically?

- What is going on mathematically?
- What mathematical issues are there for the teacher to contend with?

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videoclip

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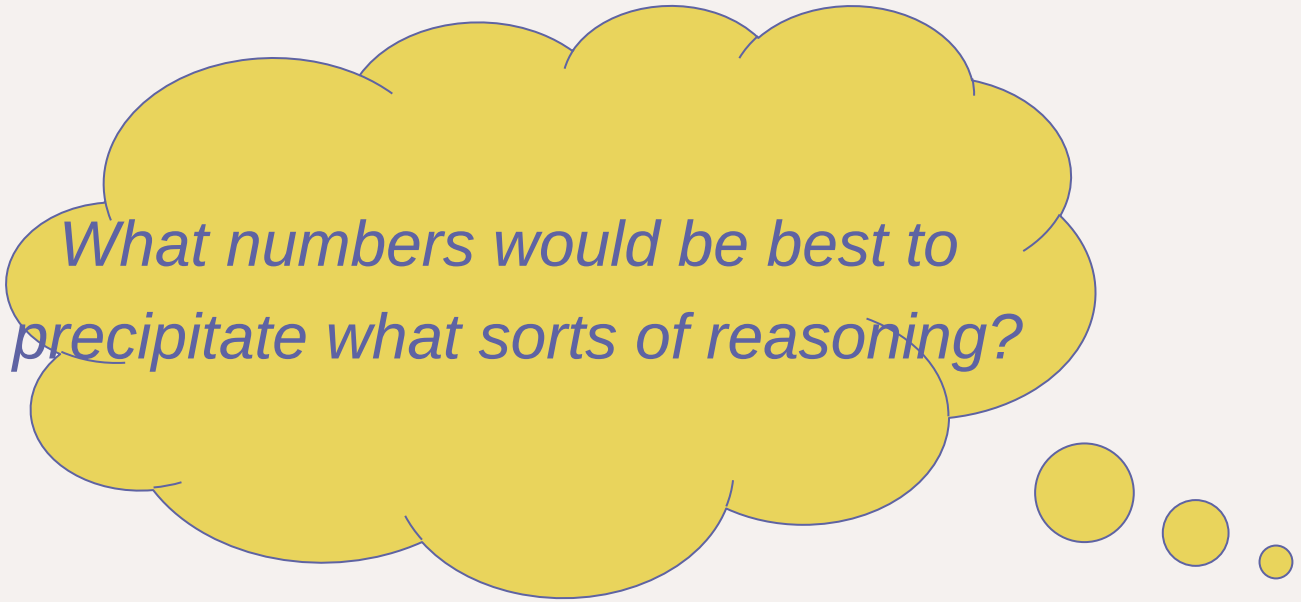
Case 2:
Posing mathematical problems to students

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Instead of asking what is three-fourths of 12,

"What is _____ of _____?"



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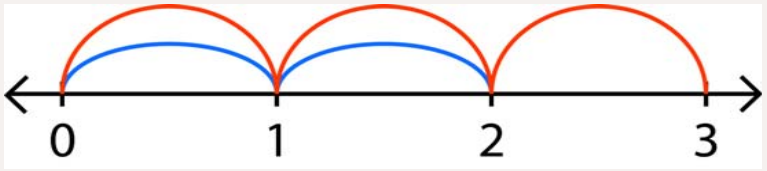
Case 3: Evaluating examples

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Making representations

Make a representation that shows the meaning of $2 \div \frac{2}{3}$

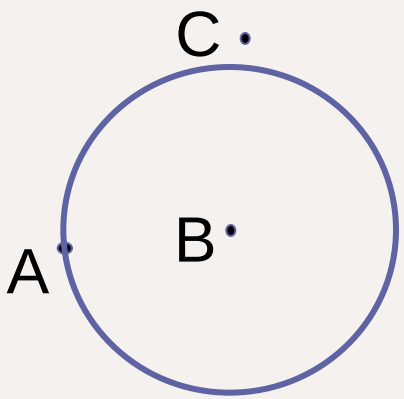


Does this represent $2 \div \frac{2}{3}$?

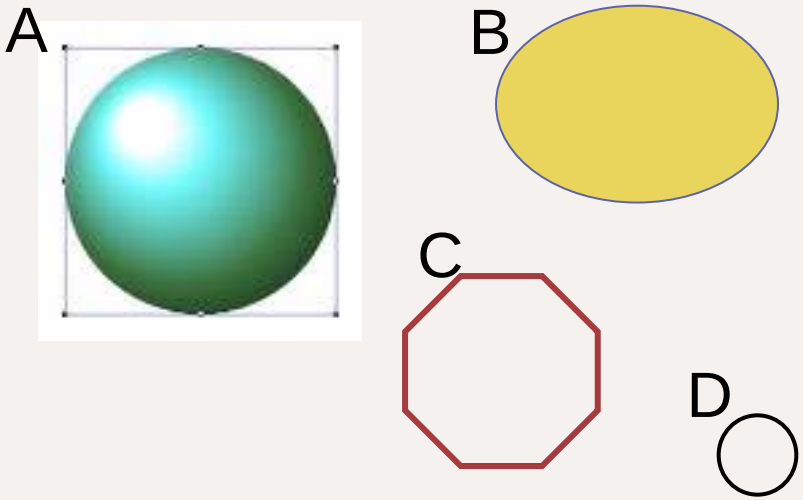
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What is the core mathematical issue in each question? Which test question will be more difficult for students and why?

1. Which point is on the circle?



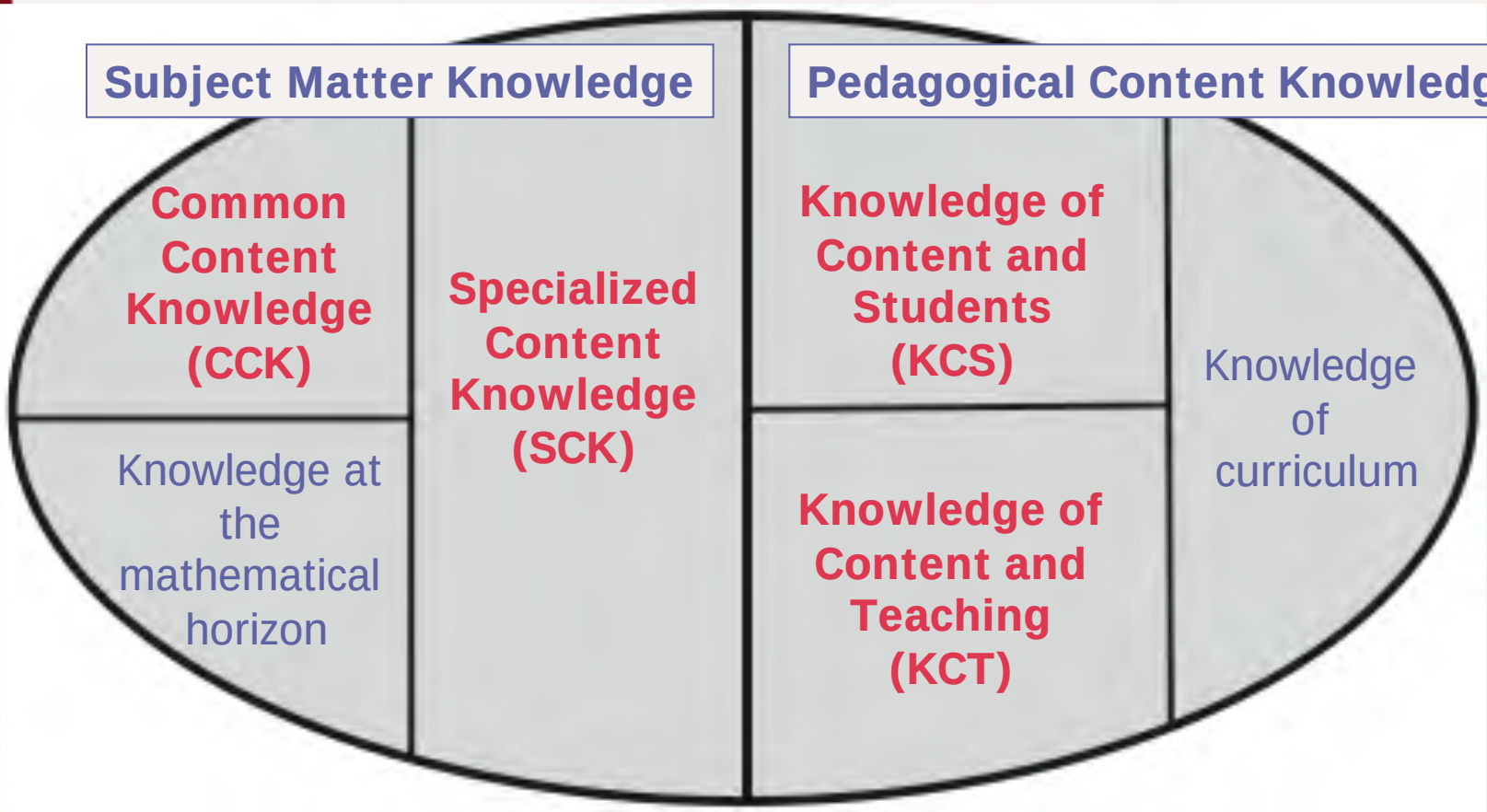
2. Which shapes are <u>not</u> circles?



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Mathematical knowledge for teaching



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lmt
Learning Mathematics for Teaching (LMT) Project

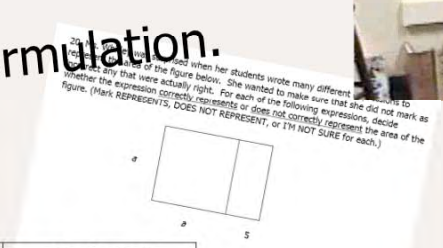
Developing measures of mathematical knowledge for teaching

Write a story or a situation or make a diagram for which

$$1\frac{3}{4} \div \frac{1}{2}$$

is the mathematical formulation.





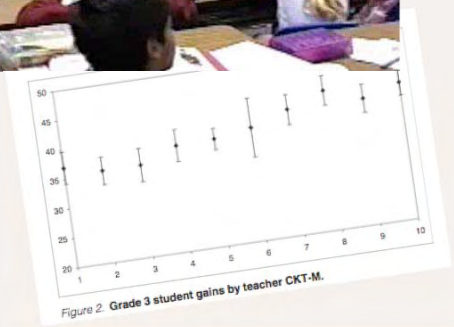


Figure 2. Grade 3 student gains by teacher CKT-M.

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

Which of the students is using a method that would work to multiply any two whole numbers?

American Educational Research Journal
Summer 2005, Vol. 42, No. 2, pp. 371–406

Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement

Heather C. Hill, Brian Rowan, and Deborah Loewenberg Ball
University of Michigan

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Developing measures of MKT

1. To assess the quality of different approaches to professional development
2. To investigate the relationship between teachers' MKT and their instruction and their students' learning
3. To test the theory

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Which student is using a method that would work for any two whole numbers?

Student A	Student B	Student C
35 x 25 125 +75 875	35 x25 175 +700 875	35 x 25 25 150 100 +600 875

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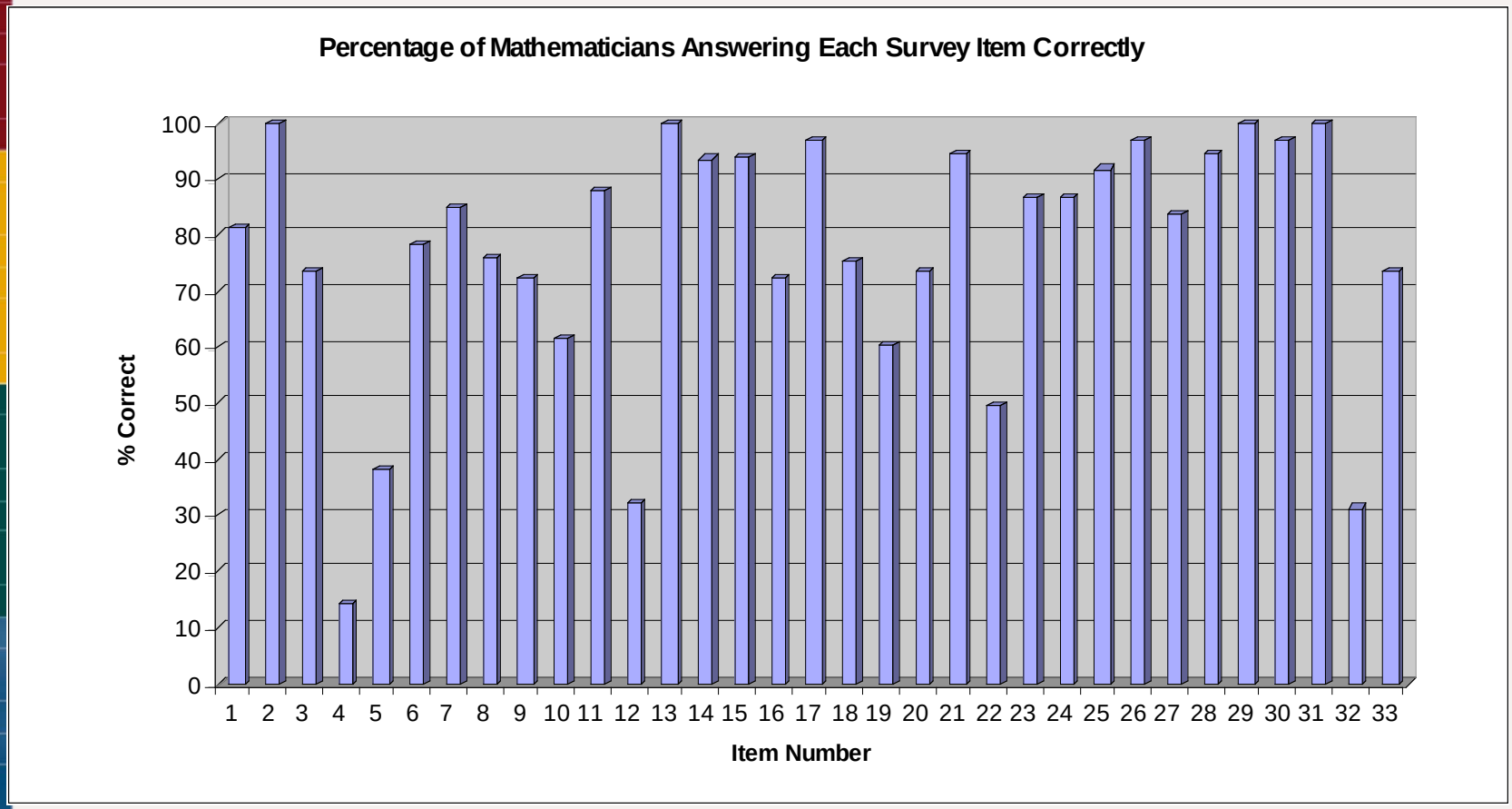
Validating our measures

How do we interpret teachers' performance on our questions?

1. Their score reflects their mathematical thinking
   - Cognitive interviews
2. Higher scores mean higher-quality mathematics instruction
   - Videotape validation study
3. Scores reflect common and specialized knowledge of content
   - Mathematician and non-teacher interviews
4. Higher scores related to improved student learning
   - Study of Instructional Improvement student gains analysis

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Percentage of mathematicians answering each survey item correctly



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Why did mathematicians get items wrong?

- Items had mathematical flaws.
- Items required knowledge of learners.
- Items demanded mathematical knowledge unique to the work of teaching:
  - Making sense of non-standard solutions or ideas
  - Choosing numerical examples
  - Choosing representations

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Linking teacher knowledge and student achievement

- Questionnaire consisting of 30 items (scale reliability .88)
- Model: Student Terra Nova gains predicted by:
  - Student descriptors (family SES, absence rate)
  - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significant
  - Small effect (< 1/10 standard deviation): 2 - 3 weeks of instruction
  - But student SES is also about the same size effect on achievement

(Hill,Rowan, and Ball, AERJ, 2005)

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Conclusions

1. Content knowledge is crucial to teaching effectiveness.
2. The mathematical knowledge for teaching needed is specialized and is more than common content knowledge known by any well-educated adult.
3. This kind of content knowledge in mathematics can be assessed.
4. This is a professionally-shaped way of knowing the subject. Who knows this content, and who can teach it to teachers?

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Slides will be available at

http://www-personal.umich.edu/~dball/

Easier: Google “Deborah Ball”

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