

What Do Teachers "Need to Know"?

The Evolution of a Research Problem

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Overview

1. Why worry about mathematical knowledge of teachers?
2. What's the question?
3. Investigating the mathematical demands of practice
4. Making progress on the problem, seeing new problems

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Why ask ?

- What to include in requirements for certification
- What to teach teachers
- What factors affect teachers' effectiveness
- Whom to hire

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Different questions

1. What mathematics do teachers need to know?
2. What mathematics do teachers know?
3. What mathematics do teachers use?
4. What mathematics does teaching entail?

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What do we mean by "mathematical knowledge"?

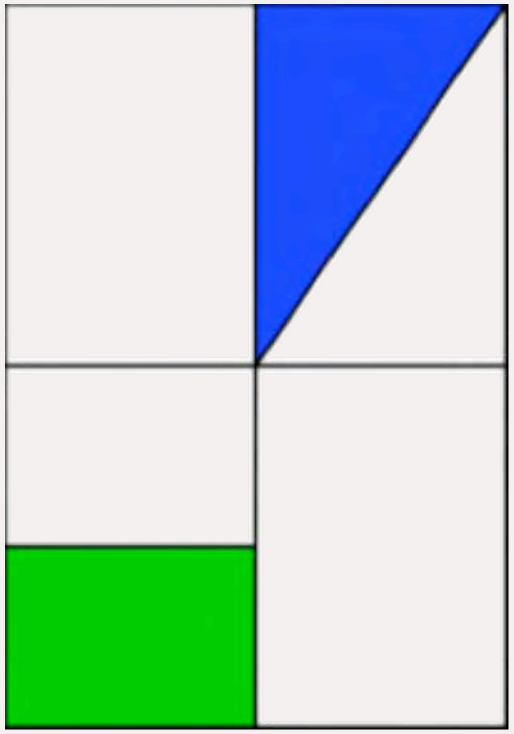
- Mathematical ideas, concepts, procedures (e.g., number systems, fraction, addition)
- Mathematical practices (e.g., representing, proving, explaining, generalizing . . .)
- Mathematical language
- Conceptions of the domain of mathematics

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1. Taking a step back: What is involved in teaching mathematics?

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Mathematical task, grade 4-5



- What fraction of the big rectangle is shaded blue?
- What fraction of the big rectangle is shaded green?
- What fraction of the big rectangle is shaded altogether?

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Video clip

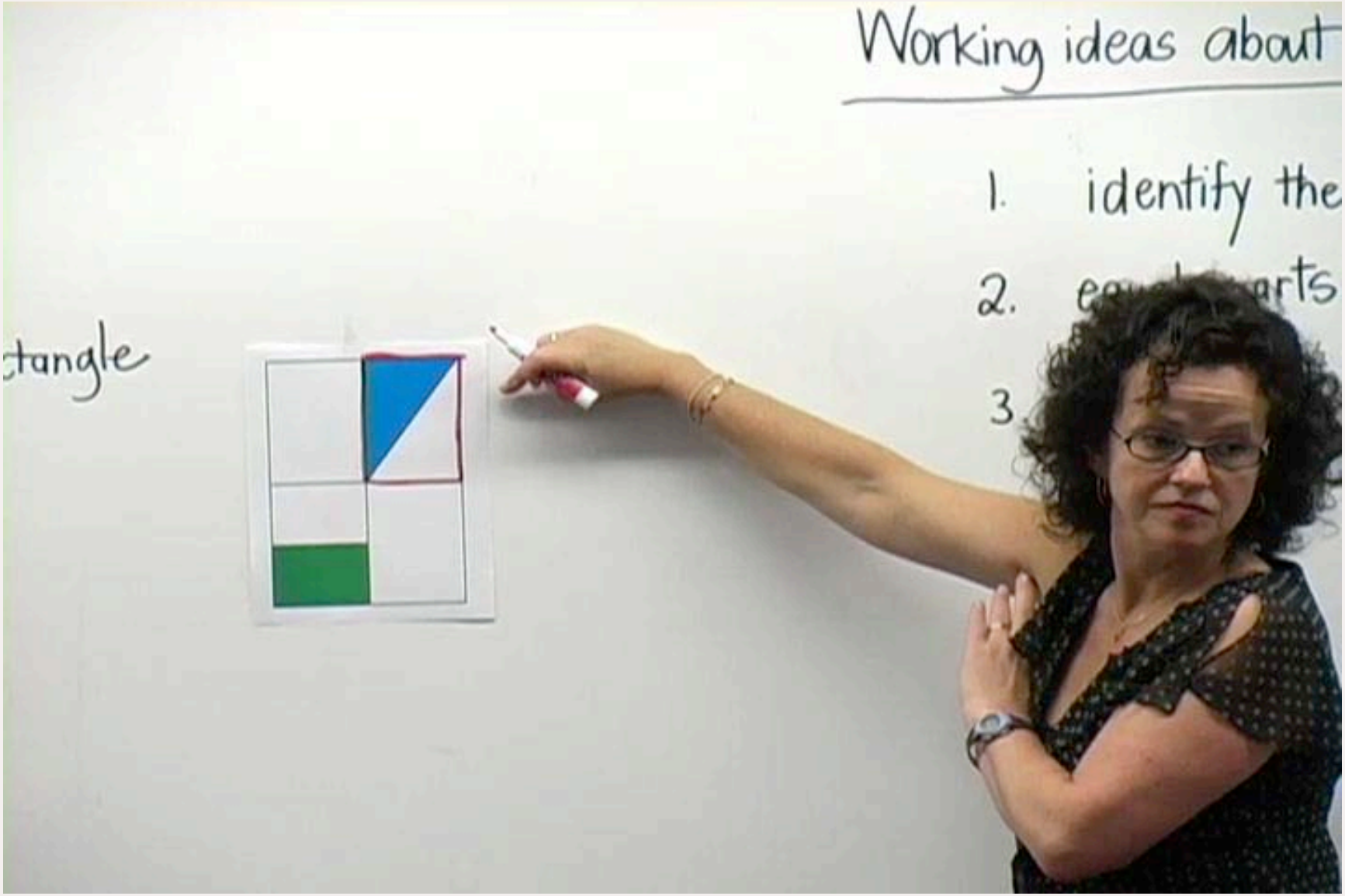
- Discussion of warm up problem
- Focused attention on equal parts
- Developing working ideas about fractions
    1. Identify the whole
    2. Make equal parts
    3. Count how many equal parts out of the whole

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Viewing focus

- What is the work of teaching in this clip?
- What mathematical "knowledge" is entailed by that work?

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Examples of work of teaching

1. Selecting/designing tasks
2. Identifying and working toward the mathematical goal of the lesson
3. Listening to and interpreting students' responses
4. Teaching students what counts as "mathematics" and mathematical practice
5. Making error a fruitful site for mathematical work
6. Attending to ambiguity of "big rectangle"
7. Deciding what to clarify, what to make more precise, what to leave in student's own language

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2. Knowing mathematics for teaching

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From "teacher knowledge" to "knowledge for teaching"

- Dewey, Schwab, Bruner
  - "Psychologizing" subject matter
  - Disciplinary structures (nature of knowing)
  - Basic integrity of the subject
- Shulman and colleagues: pedagogical content knowledge

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A practice-based theory about knowing mathematics in and for teaching

1. Study instruction and identify the mathematical work of teaching
2. Analyze what mathematical knowledge is needed to do that work effectively, and how it must be understood to be useful for the work
3. Develop, test, and refine measures of MKT using multiple methods as a means to evaluate professional education, investigate effects on students' learning, and improve theory
4. Develop and evaluate approaches to helping teachers learn mathematical knowledge for teaching

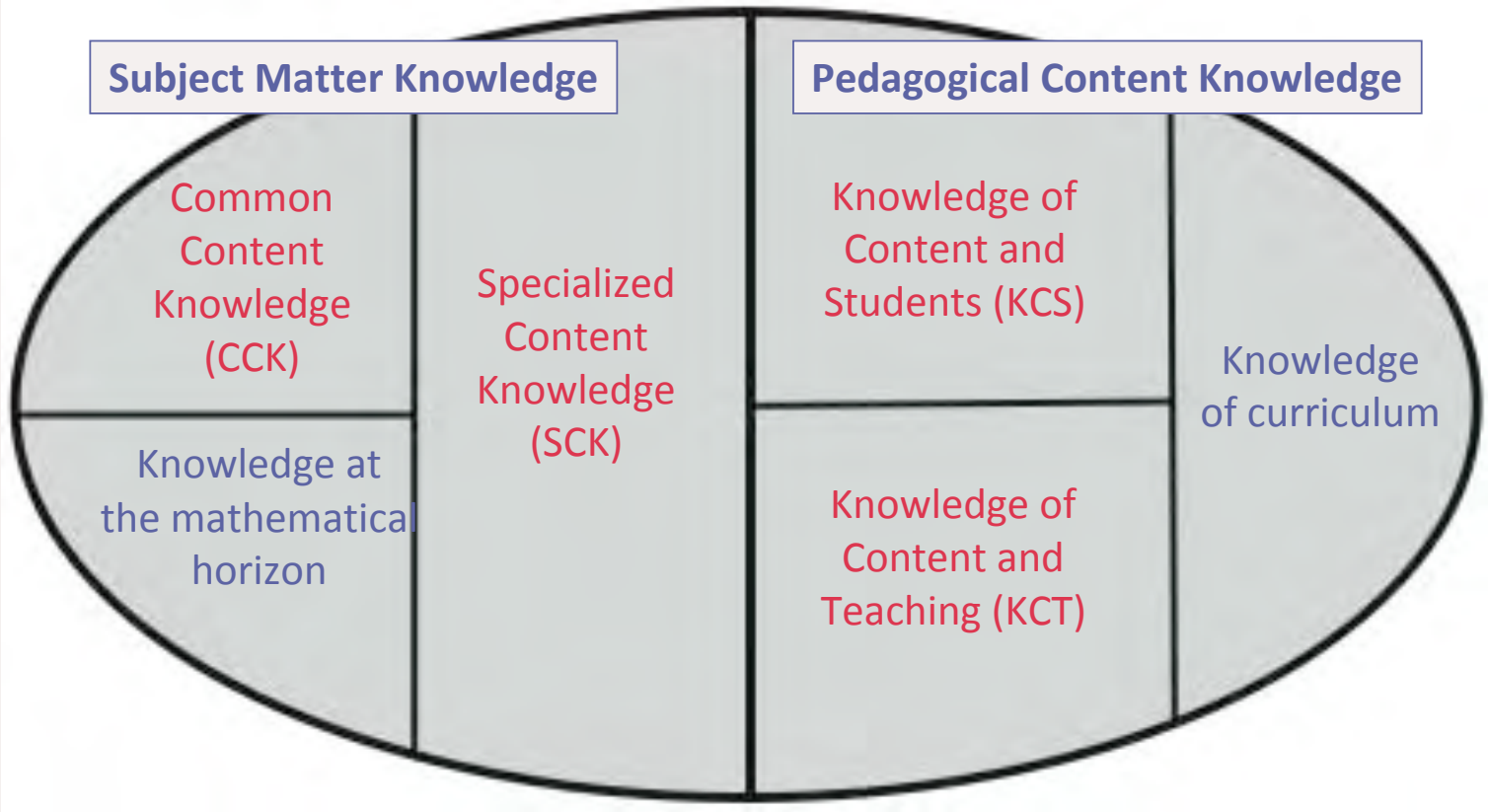
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Mathematical knowledge for teaching (MKT)

- Frame: knowledge used in practice
  - "knowledge *entailed by* the work of teaching"
- What do we mean by "knowledge"?
  - Mathematical knowledge, skill, habits of mind
- What do we mean by the "work of teaching"?
  - The activities in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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Mathematical knowledge for teaching



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Common content knowledge (CCK)

Calculate:

$$\frac{5}{6} \div \frac{1}{3}$$

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Specialized content knowledge (SCK)

$$\frac{5}{6} \div \frac{1}{3} = \frac{10}{12} \div \frac{4}{12} = 10 \div 4 = 2\frac{1}{2}$$

Is this a fluke?

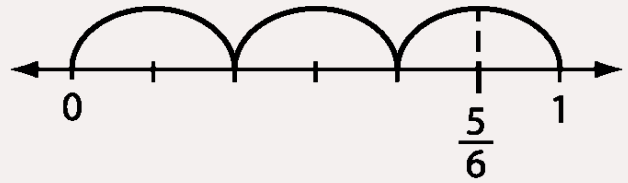
Does it work in general?

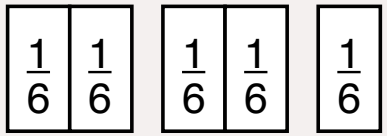
If so, why does it work?

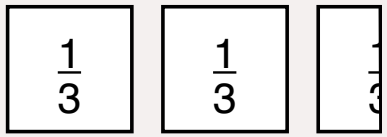
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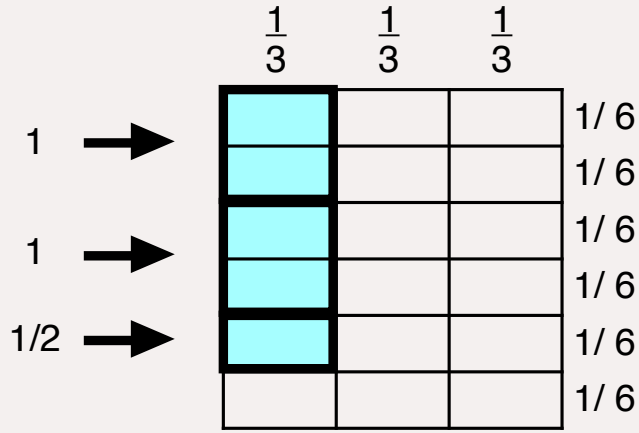
Specialized content knowledge (SCK)

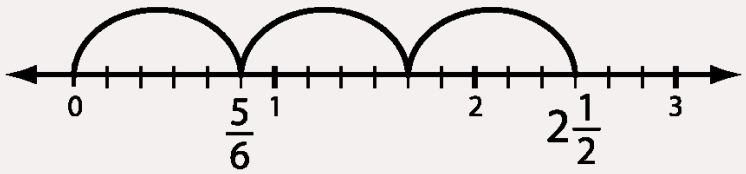
Which of these can be used to represent $\frac{5}{6} \div \frac{1}{3}$?











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$$\frac{5}{6} \div \frac{1}{3} = 2\frac{1}{2}$$

Knowledge of students and content (KCS)

- What are common errors students make when dividing fractions?
- How do students' experiences with division of whole numbers support their understanding of division of fractions? How does it confuse them?
- What difficulties do students typically have interpreting the answer to a division of fractions problem?

Knowledge of teaching and content (KCT)

- Which representation would you use to introduce the meaning of division of fractions? Or to explain the invert and multiply algorithm?
- What sequence of problems would you use to begin work on division of fractions?
- In a whole-class discussion, what solution methods would you want presented, and in what order?

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Other tasks of teaching mathematics

- Responding to students' "why" questions
- Unpacking and decomposing mathematical ideas
- Explaining and guiding explanation
- Using mathematical language and notation
- Generating examples
- Sequencing ideas
- Choosing and using representations
- Analyzing errors
- Interpreting and evaluating alternative solutions and thinking
- Analyzing mathematical treatments in textbooks
- Making mathematical practices explicit
- Attending to issues of equity (e.g., language, contexts, mathematical practices)

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3. Measuring the use of content knowledge as teacher quality

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Using MKT to develop measures of teacher knowledge

- Focused on the use of mathematical knowledge in teaching:
  - Started with number concepts, operations, algebra (K-5); later through 8, geometry, probability, proportional reasoning
  - Focused on recurrent tasks of teaching where content knowledge is used (e.g., analyzing = solutions, choosing examples, representing content, explaining, evaluating student work
- Can be used at scale
- Many stages of work to develop items

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Sample item A:
Choosing examples: Ordering decimals

Which of the following lists would be best for assessing whether your students understand decimal ordering?

A. .5 7 .01 11.4

B. .60 2.53 3.12 .45

C. .6 4.25 .565 2.5

D. These lists are all equally good for assessing whether students understand how to order decimal numbers.

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Sample item B:

Evaluating unconventional solutions: Multiplication

Which student is using a method that could be used to multiply any two whole numbers?

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

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Try to write a specialized content knowledge item.

Ms. Lin looks up the definition of "even number" in several textbooks and reference books. Which of the following definitions is both mathematically correct and accessible below the middle school level?

a)

b)

c)

d)

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Validating measures of MKT

1. Teachers' scores reflect their mathematical thinking
   - Cognitive interviews
2. Scores reflect common and specialized knowledge of content
   - Mathematician and non-teacher interviews
3. Higher scores related to improved student learning
   - Student gains analysis
4. Higher scores mean higher-quality mathematics instruction
   - Videotape validation study

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Linking teacher knowledge and student achievement

- Questionnaire consisting of 30 items (scale reliability .88)
- Model: Student Terra Nova gains predicted by:
  - Student descriptors (family SES, absence rate)
  - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significant
  - Small effect (< 1/10 standard deviation): 2-3 weeks of instruction
  - But student SES is also about the same size effect on achievement

(Hill,Rowan, and Ball, AERJ, 2005)

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Using multiple sources of evidence to evaluate our claims

1) Scores capture teachers' mathematical knowledge

Cognitive interviews

2) Higher scores are related to improved student learning

Study of Instructional Improvement student gains analysis

3) Scores reflect different dimensions of MKT

Mathematician and non-teacher interviews

Item response theory and factor analysis

4) Higher scores are related to higher-quality mathematics instruction

Videotape validation study

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The challenge

- How can MKT, in particular SCK, be developed? How can opportunities for learning SCK be provided?
- Easy to work on mathematics, or on students; less easy to create opportunities to develop the specialized knowledge of mathematics needed for teaching

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3. Teaching practice

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Domains of teaching work

A) "Off camera": Before this episode

1. Learn about individual children and what they know, care about, are worried about, can do, etc.
2. Establish the environment to manage behavior
3. Teach intellectual habits (e.g., drawing, speaking to peers, knowing and being able to choose and make different kinds of mathematical moves)
4. Choose the specific problem: Which is more 4/4 or 4/8? Why that question? Why those numbers? What's a similar or better choice?

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B) During these 6 minutes

12:58:35—Open the discussion: use specific tone, body movement around the room, choose whom to call on, and call on that child

12:58:38—Watch students while walking around; figure out who is drifting and encourage students' attention; maintain tenor of class while Lin draws

1:00:58—Lin completes drawing. Decide what to do about "I took four out of it"; direct her to repeat, "more loudly"; ask others to comment; work to get other students to comment besides Bernadette

1:01:28—David comments. Work to understand; manage risk of losing class; decide not to take up; close interchange with David kindly.

1:02:40—Bernadette suggests the number line. Decide to have her work on the side; make her a number line to work on.

1:02:57—Kevin agrees and says first he did something else that was wrong. Decide to probe and to take this up; highlight for others; amplify by drawing incorrect picture on board

1:24:57—Pose question to assess students' understanding; make up specific question; decide how to take up answers

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Teaching as mathematically "natural" work, and the limits of this perspective

- Some aspects of teaching depend on mathematical instincts, habits of mind, practices
- So an additive view of learning to teach may make sense — add other knowledge to mathematical knowledge and habits

but —

- Teaching mathematics also involves doing things that are mathematically *unnatural*

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Examples of mathematics teaching as mathematically unnatural work

1. Unpacking mathematical ideas
2. Listening to mathematically imprecise language
3. Not automatically affirming correct statements
4. Hearing what others say, not what you think
5. Surfacing "error"

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4. Developing and refining expertise in mathematics teaching practice

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Create opportunities to learn —

- The practice of mathematics in and for teaching
- Teaching practice itself
- Explicitness about both, including ability to be fluent in narrating practice

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Challenges of centering program on teaching and mathematical practice

1. Fascination with mathematics itself
2. Fascination with children's thinking
3. Problem of expertise and tacit knowledge
4. Widely held view of teaching as uncertain, artistic, and unable to be specified
5. Underdeveloped pedagogy of practice

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Resources for centering program on teaching and mathematical practice

1. Clinical education in other professions (Grossman)
2. The growing archive of different kinds of cases and artifacts
3. A community of practice for developing a pedagogy centered in and on practice

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