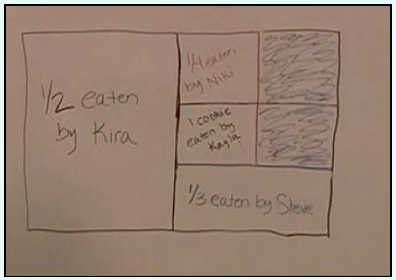
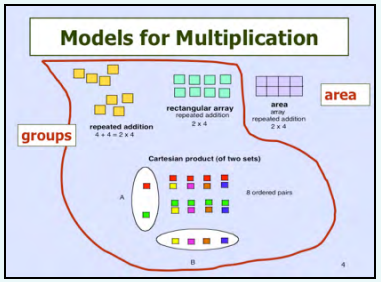
A Practice-Based Perspective on Mathematical Knowledge for Teaching

Deborah Loewenberg Ball and Hyman Bass
Imani Masters Goffney
University of Michigan
Center for Proficiency in Teaching Mathematics





IHE Network Conference
Milwaukee Mathematics Partnership
August 25 & 26, 2005

www-personal.umich.edu/~dball/

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From Yesterday….

- Many positive comments
- Liked watching videos
- Liked the focus on equity
- Small group discussions sometimes too long
- Small group discussions sometimes too short
- Want to talk about assessment today

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Overview of Workshop

1. Foundations: Major problems of mathematics education, and the need to focus on instruction
2. Teaching mathematics for proficiency and equity: Focus on reasoning and language
3. Knowing mathematics for teaching
4. Assessing mathematical knowledge for teaching
5. Developing teachers' mathematical knowledge

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What are Central Problems of Mathematics Education?

- Ineffective curriculum: Too many students not learning mathematics well enough for
  - Continued mathematics study
  - Use in everyday settings
  - Workplace
- Pervasive inequality: Unequal distribution of mathematical success by race, social class
- Lack of capacity for improvement
  - Public understanding, support, investment
  - Teacher shortages, teacher development and support
  - Teacher educator development and resources

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Focus on Teaching and the Resources Needed for Teaching

- Teaching is an important and often invisible contributor of learning.
- A major share of teaching is mathematical work.
- Mathematical knowledge for teaching is the knowledge needed for the work of teaching.That work entails specialized content knowledge distinct from that required by other mathematically-intensive professions.
- Knowing mathematics in special ways may be important to improving achievement of students, especially for those who are most "school-dependent."

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Common Framing of the Problem
⟶ Failure to Solve the Problem

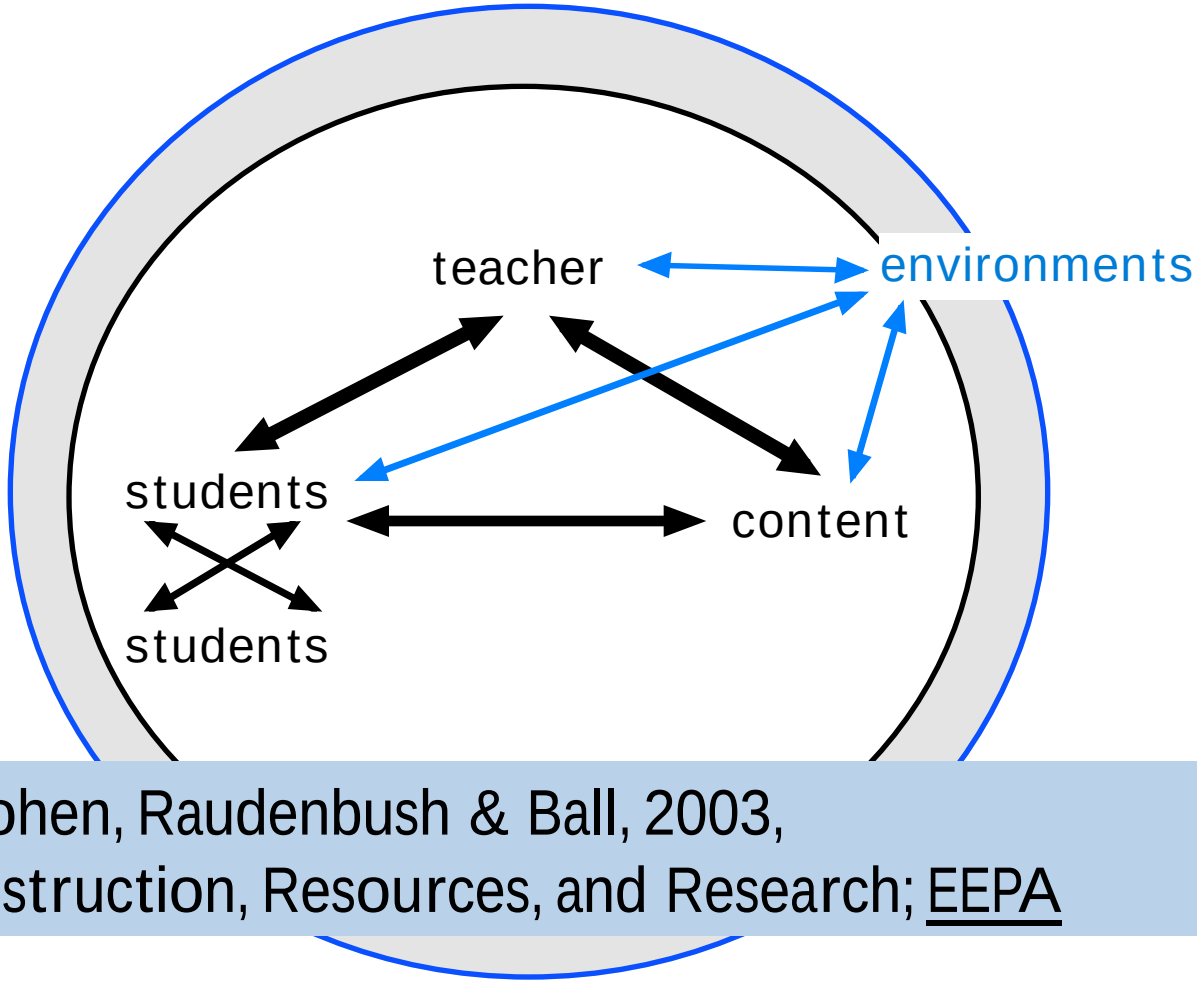
- Teachers do not know mathematics well enough (evidence and causes vary)
- Concentration of under-qualified teachers in urban and high poverty schools

- Increase number and rigor of mathematics requirements for teacher certification
- Recruit mathematically-trained people into teaching

but -- these may not improve equitable access to high quality learning of mathematics for every student

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Using Knowledge IN Instruction



Cohen, Raudenbush & Ball, 2003,
Instruction, Resources, and Research; EEPA

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Clarifying the Problem

The quality of mathematics teaching and learning

Teachers' knowledge of mathematics and their ability to use it in teaching

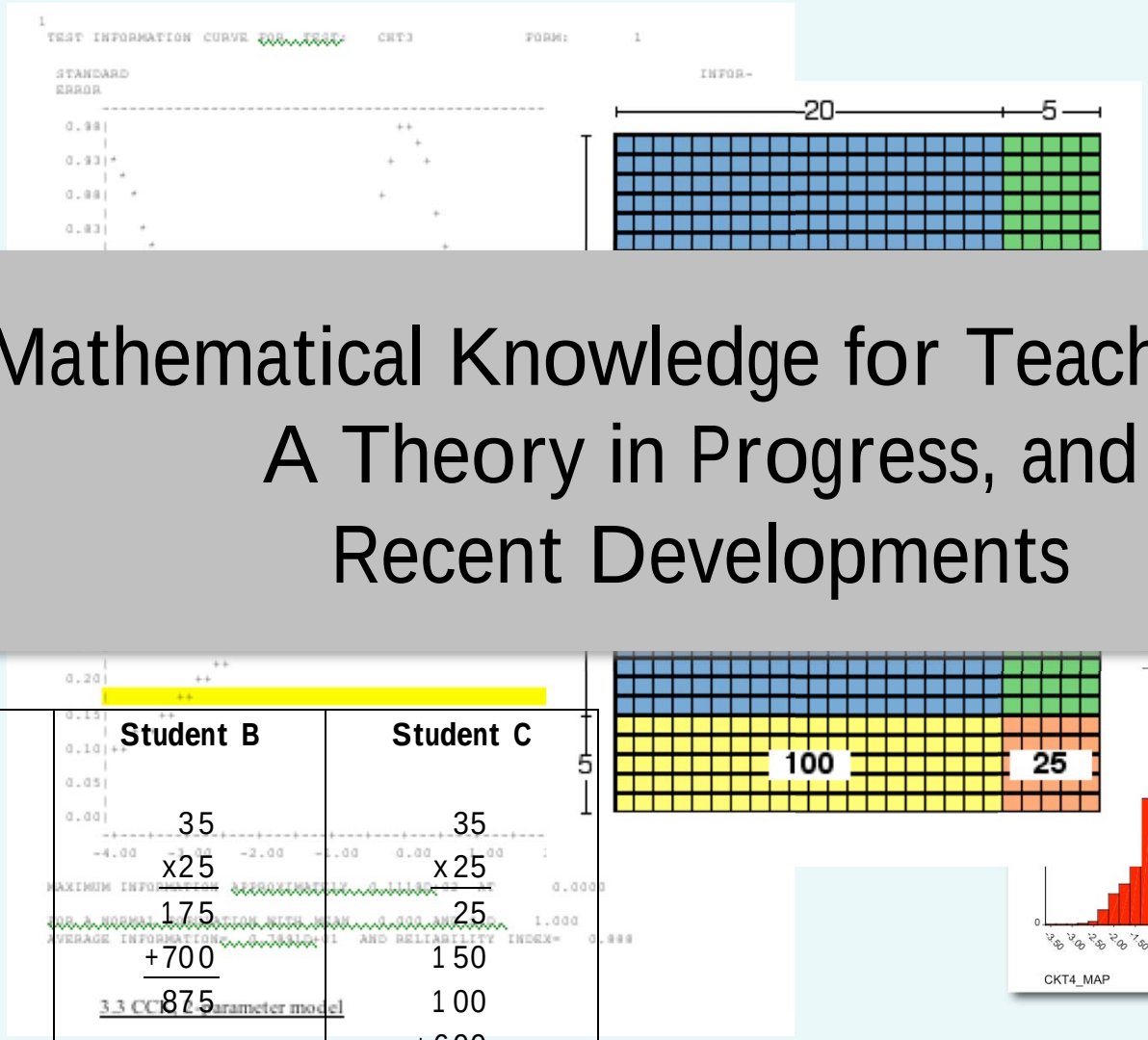
What mathematical resources do teachers need in order to teach mathematics to all students effectively?

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A Practice-Based Approach to Answering the Question

1. **Study instruction**, and identify the mathematical work of teaching
2. Analyze **what mathematical knowledge** is needed to do that work effectively, and **how** it must be understood in order to be useful for the work
3. Develop, test, and refine **approaches to helping teachers** develop and use mathematical knowledge for teaching, study results

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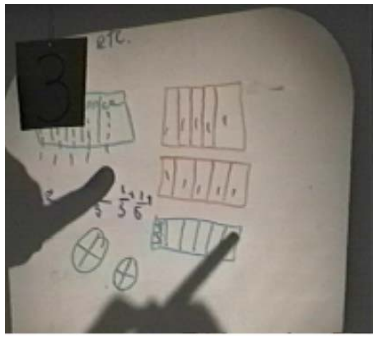
Mathematical Knowledge for Teaching: A Theory in Progress, and Recent Developments

Student A	Student B	Student C
35 x 25 1 25 +75 875	35 x25 175 +700 875	35 x 25 25 1 50 1 00 +600 875

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What Do we Mean By "Mathematical Knowledge for Teaching"?





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Mathematical Knowledge for Teaching

What do we mean when we use this term, "**mathematical knowledge for teaching**"?

- Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching

What do we mean by the "**work of teaching**"?

- The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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What *is* "Mathematical Knowledge for Teaching"? An Example from Subtraction

Subtract:

$$\begin{array}{r} 307 \\ -\ 168 \\ \hline \end{array}$$

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Analyzing Incorrect Answers for

$$\begin{array}{r} 307 \\ -\ 168 \\ \hline \end{array}$$

(a)

$$\begin{array}{r} 307 \\ -\ 168 \\ \hline 261 \end{array}$$

(b)

$$\begin{array}{r} 307 \\ -\ 168 \\ \hline 169 \end{array}$$

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Analyzing Correct Answers for

$$\begin{array}{r} 307 \\ -\ 168 \\ \hline \end{array}$$

(a)

$$\begin{array}{r} 307 \\ -\ 168 \\ \hline -1 \\ -60 \\ 200 \\ \hline 139 \end{array}$$

(b)

$$\begin{array}{r} 307 \\ -\ 168 \\ \hline 2 \\ 30 \\ -107 \\ \hline 139 \end{array}$$

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Other Work of Teaching

- Developing and using definitions
- Choosing and using representations
- Choosing numerical examples
- Using and defining terms
- Using notation
- Explaining

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Hypotheses About Knowledge of Mathematics for Teaching

PREVALENT HYPOTHESES

1. Teachers need knowledge of the curriculum, plus N levels more knowledge
2. "Pedagogical content knowledge" (— but what is it?)

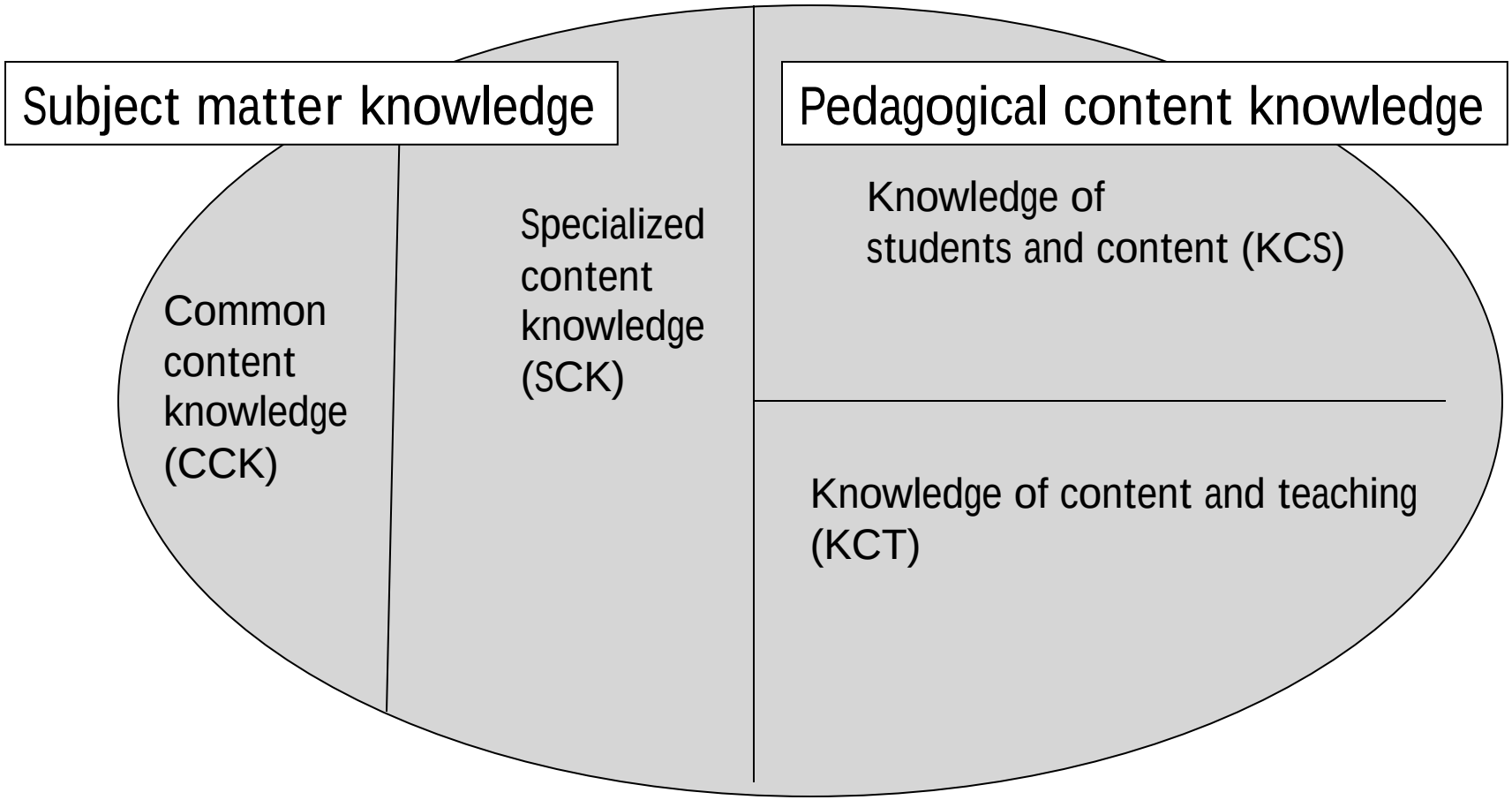
OUR CURRENT HYPOTHESES

- **Common** content knowledge
- **Specialized** content knowledge
- Knowledge of content and **students**
- Knowledge of content and **teaching and curriculum**

What does this have to do with "pedagogical content knowledge"?

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Shulman's Original Category Scheme (I985) Compared with Ours



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Ball, D., Hill, H., Bass, H. (2005). Knowing mathematics for teaching: Who knows enough to teach third grade, and how can we decide? *American Educator*, Fall 2005, 14 - 22, 43 - 46.

Test of Mathematical Knowledge for Teaching

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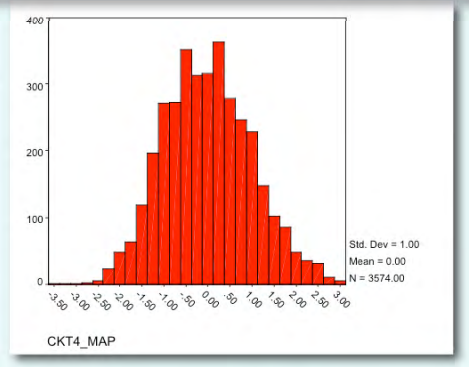
Lunch Break

- During lunch we would like you to meet with colleagues to discuss how you would like to continue dialogue and work across campuses-- for example, continued work on mathematical definition, language or issues of equity by level of mathematics (remedial algebra, calculus, etc.).

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DEVELOPING AND STUDYING TEACHERS' MATHEMATICAL KNOWLEDGE FOR TEACHING

Student A	Student B	Student C
35 x 25 1 25 +75 875	35 x25 175 +700 875	35 x 25 25 1 50 1 00 +600 875



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Can These Ideas Improve Students' Mathematics Learning?

DEVELOPING APPROACHES FOR HELPING TEACHERS LEARN MKT

STUDYING THE RELATIONSHIPS OF TEACHERS' MKT AND STUDENT ACHIEVEMENT

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Why Would We Want to "Measure" Teachers' Content Knowledge for Teaching?

- To understand role of teachers' content knowledge in students' performance
- To study and compare outcomes of professional development and teacher education
- To inform design of teachers' opportunities to learn content knowledge

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Study of Instructional Improvement

- Study of three Comprehensive School Reforms; teacher knowledge a key variable
- Instrument development goals:
  - Develop measures of content knowledge teachers *use* in teaching – not just *what* they teach
  - Develop measures that discriminate among teachers (not criterion referenced)
  - Non-partisan

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Problems As We Began This Work

- No way to measure teachers' content knowledge for teaching on a large scale
  - Small number of items, many written by Post, Ball, others appeared on every instrument
  - Nothing known about the statistical qualities of those items (difficulty, reliability)
  - Studies relying on single items -- single items unlikely valid or reliable measures of teacher knowledge

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Measures of Teachers' Mathematical Knowledge for Teaching

OUR GOALS:

- Measure mathematical knowledge used in teaching
- Contextualize the items in tasks (or situations) that all teachers face
- Usable measures:
  - Large sample of teachers (N=5500) = multiple choice
  - Not grade-specific (grade 1 - 6 teachers)
  - Not represent any single view of teaching (e.g., "reform")
  - Items that vary in difficulty

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Early Decisions and Activity

- Survey-based measure of content knowledge for teaching mathematics
  - 3000 teachers participating in SII
  - Multiple choice
- Specified domain map
- 5 people + 5 lbs cheese + 5 weeks = 150 items (May 2001)
- Large-scale piloting, summer 2001

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Original Sampling Frame

	Types of knowledge		
Mathematical content		Common and specialized content knowledge	Using knowledge of students and content
	Number		
	Operations		
	Patterns, functions, and algebra		

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Sample Items
K-5 Number and Operations

- **Common knowledge**

  What number is halfway between 1.1 and 1.11?

- **Specialized knowledge**

  1. Representing mathematical ideas and operations
  2. Providing explanations for mathematical ideas and procedures
  3. Evaluating mathematical methods, claims, or solutions

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Representing Number Concepts

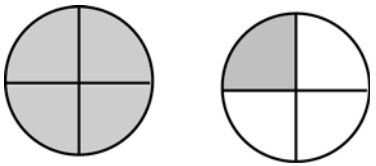
Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)

a) 5/4

b) 5/3

c) 5/8

d) 1/4



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Providing Mathematical Explanations

Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

a) Four is an even number, and odd numbers are not divisible by even numbers.

b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).

c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.

d) It only works when the sum of the last two digits is an even number.

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Evaluating Alternative Solution Approaches

Student A	Student B	Student C
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Which of these students is using a method that could be used to multiply any two whole numbers?

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Using Data to Test and Improve Theory

- Factor analyses
- Analyses of validity
- Uses of measures
  - To predict student achievement
  - To evaluate professional development

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Factor Analysis

- Enables analyses of the number of underlying factors in a data set
- Can link items to constructs
- Ask similar question of our data:
  - How many factors? What are they named?
  - How do items group?

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Overarching Findings: Factor Analyses

- Multidimensionality of mathematical knowledge for teaching
  - Knowledge of students and content different from content knowledge
  - Patterns, functions & algebra content knowledge for teaching
  - Geometry mathematical knowledge for teaching
  - Number and and operations mathematical knowledge for teaching
    - Specialized
    - Common
- Tentative conclusion: existence of professional mathematical knowledge for teaching

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Validating Our Measures

How do we interpret teachers' performance on our questions?

1. Their score reflects their mathematical thinking
   - **Cognitive interviews**
2. Higher scores mean higher-quality mathematics instruction
   - **Videotape validation study**
3. Scores reflect common and specialized knowledge of content
   - **Mathematician and non-teacher interviews**
4. Higher scores related to improved student learning
   - **Study of Instructional Improvement student gains analysis**

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Studying Links Between Teachers' Knowledge of Mathematics for Teaching and Student Achievement

Question: Do students learn more when teachers do better on our measures?

- SII data and analysis – measured CKT on questionnaire
- SII measures highly reliable, from test construction perspective

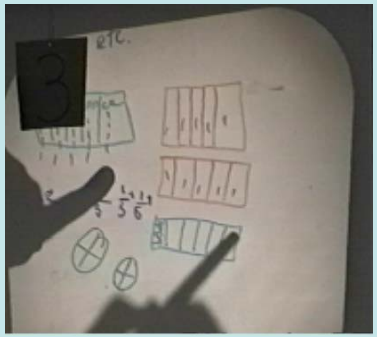
- Model: Student Terra Nova gains predicted by:
  - Student descriptors (family SES, absence rate)
  - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significantly related to achievement gains
  - Small effect (< 1/10 standard deviation)
  - But student SES is also on same order of magnitude

Hill, Rowan, & Ball, Effects of teachers' mathematical knowledge for teaching on student achievement; *American Educational Research Journal*, Summer 2005

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A RETURN TO THE WORK OF TEACHING





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Setting the Background

- Third grade (ages 8 - 10)
- Diverse class (language, race, culture)
- Mathematics: unit on fractions
  - Area representations
  - Number lines
  - Ordering fractions
- Teaching: Scaffolding students' mathematical work
  - Use this to develop a frame for language use in math class that addresses mathematical proficiency with attention to equity

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Mathematics Problem

Which is more: 4/4 or 4/8?

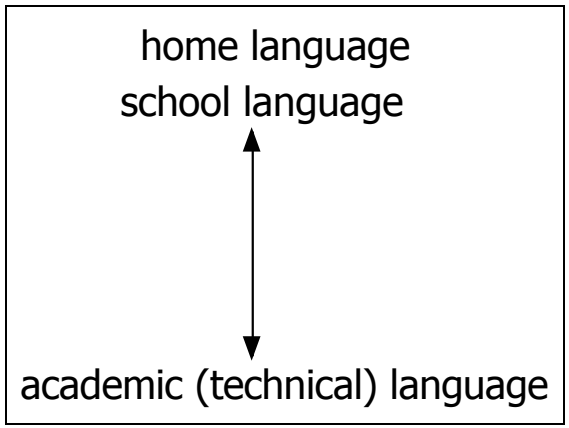
What might be some of the language challenges for third graders in a multi-lingual class?

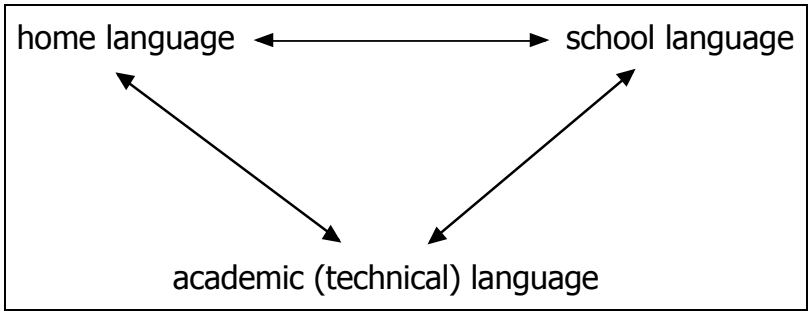
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What Do We Mean By “Multi-Lingual”?

Language

- Vocabulary, grammar, syntax, talk (how language is “delivered”)
- Discourse





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Viewing Focus

- What do you notice about mathematical language or representation?
- In what ways is mathematical representation being developed?
- How is equity being attended to?
- What is the teacher doing to structure students' work?

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Five Specific Foci for Attending to Equity

1. Attending to the relationship between everyday and mathematical language
   a) When everyday language assists, when it interferes
   b) Concern for definitions
2. Care with "contexts"
   a) Critical eye to when contexts create barriers or confusion and when they use children's experience to create connections and competence
3. Broadening what it means to be successful in mathematics and the space to do so
   a) Multiple forms of participation and kinds of contributions
4. Assigning competence: whose mathematical is made public
   a) Labeling work as important, bringing it into public, endorsed view
5. Explaining as central, and making explicit what it means to explain
   a) Requesting justification
   b) Creating culture of explanation and justification to classmates

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Looking Back: Language and Representations

- Mathematics, like any discipline, requires language to express and communicate concepts and reasoning. But mathematics requires more care and precision with language and representation than most domains.

- The accumulation of knowledge and techniques in mathematics demands a "compression" of language and notation (symbols), to keep things at human scale as the domain of knowledge expands. By the same token, teaching mathematics at the early levels requires an "unpacking" of these compressed versions, into more elementary forms.

- Mathematical constructs can be represented in multiple ways. It is wrong to think that there is a "best" way. A representation is a tool, and, like any tool, it is "best" for certain kinds of tasks, but not for others. For the mathematics learner and practitioner, what is important to know is a wide repertoire of representations, how to show correspondences that establish their mathematical equivalence, and what are the pros and cons of each one, so that one can make discriminating choices of them in a variety of situations.

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Explaining Mathematical Ideas

- Convincing yourself, convincing others
- Use of language (definitions)
- Use of drawings (and other representations) and making correspondences to ideas
- Making clear what things mean and why they make sense; showing why something is correct or "true"

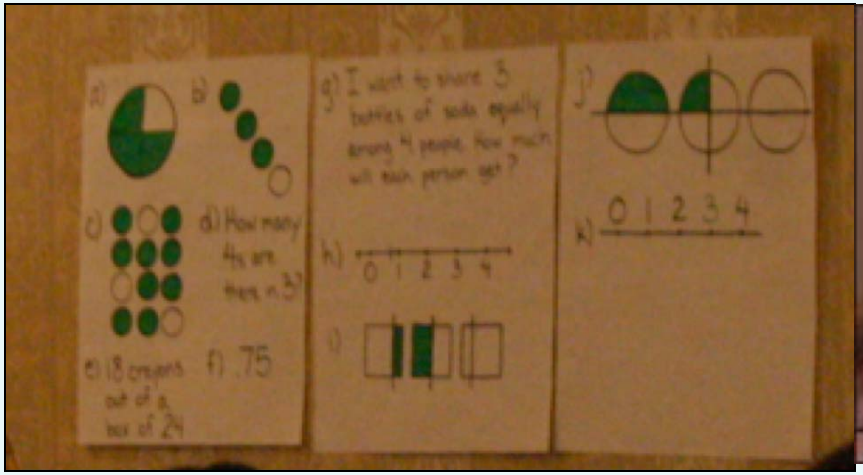
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Looking Back: Equity

- Attending to the relationships between everyday and mathematical language
- Care with “contexts” (everyday and mathematical)
- Broadening what it means to be successful in mathematics and the space to do so
- Assigning competence: whose mathematical work is made public
- Explaining as central, and making explicit what it means to explain

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Mathematical Knowledge for Teaching: Exploring the Design and Teaching of Explicit Opportunities for Teachers' Learning





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Three Kinds of Learning of Mathematics

- Learning mathematics as a student
- Learning mathematics as a teacher
- Learning to teach mathematics

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Mathematical Content and Applications for the Teaching of Elementary School Mathematics

- A mathematics content course for prospective elementary school teachers
- Focused on **knowing and using mathematics for teaching**
  - ❑ Content
    - Mathematical topic: fractions
    - Mathematical practices: Using representations, using mathematical language, **making and evaluating mathematical explanations**
  - ❑ Applications
    - Using mathematics for tasks of teaching: Interpret and evaluate student work, discuss the mathematics in play in classroom lessons, analyze curriculum materials

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Making and Evaluating Mathematical Explanations

- The forms of expression that mathematically justify and help others understand why a mathematical claim is true
- Purposes and central features of mathematical explanation
- The social nature of explanation
- Providing adequate explanations
- Listening to and appraising others' explanations

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Cookie Jar Problem

There was a jar of cookies on the table. Kira was hungry because she hadn't had breakfast, so she ate half the cookies. Then Steve came along and noticed the cookies. He thought they looked good, so he ate a third of what was left in the jar. Niki came by and decided to take a fourth of the remaining cookies with her to her next class. Then Kayla came dashing up and took a cookie to munch on. When Pam looked at the cookie jar, she saw that there were two cookies left. "How many cookies were there in the jar to begin with?" she asked Kira.

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Anna asks: "How did you decide that the one cookie eaten by Kayla got its own square?"

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Launching Explicit Work on Mathematical Explanation

STUDENTS' WORK IN CLASS

- Are you convinced that the only solution to the Cookie Jar Problem is 12? If so, what has convinced you? If not, what is leaving you skeptical?

- Are there things you have noticed that you would call "features of good mathematical explanations"?

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Features of "Good" Mathematical Explanations for the Cookie Jar Problem

- Makes clear at the outset what is being explained, and why you start there, and carefully connects the explanation to the question or idea being explained
- Starts from the beginning, and traces the logical flow of the reasoning
- Should be logical and complete, makes conclusion clear and links back to original question or claim or problem
- Might number the steps if appropriate, or label parts of a diagram
- Strives to be as simple and clear as possible
- Defines terms as needed, uses available definitions as needed
- Uses representation(s) accurately (algebraic, geometric, etc.), and combining representations
- Links the language and diagrams clearly to the steps of the argument
- Shows what something means or why is true, and is convincing to the person to whom you are explaining
- Is calibrated to the context (considers the person to whom you are explaining, and what is already established as true and does not need more explanation)

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Exam: Question #1

- DIRECTIONS: This question will be completed in pairs. You and your partner will take turns being the presenter and the evaluator. After the presenter completes the explanation, the evaluator will appraise the explanation using the criteria we developed in class, and will provide feedback. Then you will switch roles.

- You are encouraged to prepare your explanations in advance, however you may not use any notes for this question. Although you will only explain one problem, you should do the other as well, so that you are prepared to listen closely to your partner.

- Paper and pens will be available for you to use during your explanation, as will copies of the problems and the list of criteria for good mathematical explanation. You should also refer to our list of criteria for good mathematical explanations, but do not use it rigidly (i.e., you need not comment on every item).

- For this question, you will be assessed on both your ability to provide and to evaluate a mathematical explanation. When you are listening to your partner's explanation, your role is to appraise the quality of the explanation. You should not be coaching your partner, and should listen neutrally so you do not distract or otherwise intervene in your partner's reasoning.

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Conclusion: Questions and Challenges

- Tackling issues of coverage
  - Choosing and justifying mathematical content (topics and practices) for **in-depth work**
  - Choosing and justifying content for **brief treatment**
  - Choosing and justifying content **to omit**
- Understanding and developing mathematical fluency for teaching
- Developing tasks and pedagogy appropriate for MKT: Do we really think that our courses/workshops should be taught as teachers would teach their own students?
- Developing performance assessments: knowing and using mathematics (e.g., explaining, representing)
- Rethinking the intersection of content and methods
- The professional development of professional developers

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Slides will be available at

http://www-personal.umich.edu/~dball/

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