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Using Mathematics in Teaching:

Seeing Teaching As Mathematical Problem Solving

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Michigan Council of Teachers of Mathematics
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Credits to my collaborators: Hyman Bass, Mark Hoover, Heather Hill
and others at University of Michigan

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Knowing Even and Odd Numbers

Is 8 even or odd?

Is 26 even or odd?

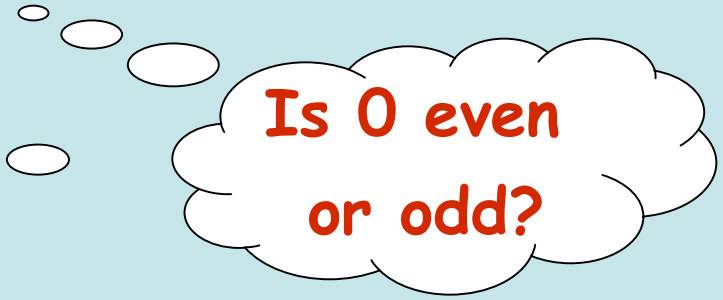
Is 1,376 even or odd?

Is 0 even or odd?

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Knowing Even and Odd Numbers for Teaching

What is a definition of even numbers that is both mathematically correct and usable by third graders?



Why does it work to only look at the units digit to determine whether a large number is even or odd?



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Overview

1. Clarifying the issue of teachers' mathematical knowledge
2. Seeing teaching as mathematical work
3. Developing a practice-based approach to learning mathematics for teaching

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1. Clarifying the Issue of Teachers' Mathematical Knowledge

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Shifting the Focus: From Knowing to Knowing and Using in Teaching

- Many criticisms of teachers for not knowing mathematics
- Evidence: teachers' scores, research on teacher knowledge, anecdotes
- Remedy: increase mathematics requirements for teachers, add mathematics content to professional development

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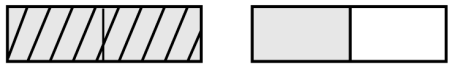
A Closer Look:
Knowing Multiplication of Fractions

$$1\frac{1}{2} \times \frac{2}{3}$$

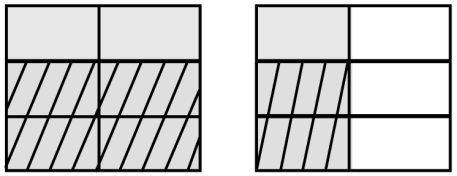
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Which of the following correctly represents the meaning of $1\frac{1}{2} \times \frac{2}{3}$?

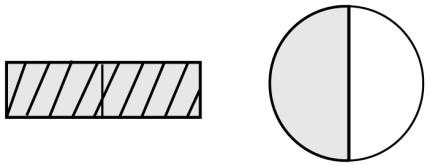
A)



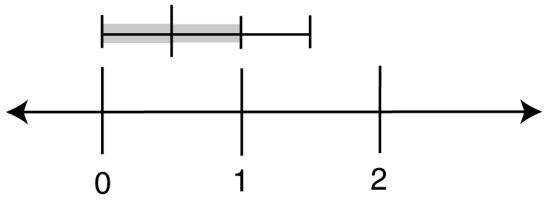
B)



C)



D)



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What is the actual issue that we are trying to address?

The quality of mathematics teaching and learning

Teachers' knowledge of mathematics and their ability to use it in their teaching

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What is Necessary Mathematics Knowledge for Teaching?

1. Examine teaching, and identify the mathematical work that teaching entails
2. Analyze what mathematical knowledge — topics and skills — is needed to do that work
3. Analyze also the qualities of that knowledge — how it must be understood and known to be serviceable for the work

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2. Seeing Teaching as Mathematical Problem Solving

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What Mathematical Problems Arise in Teaching?

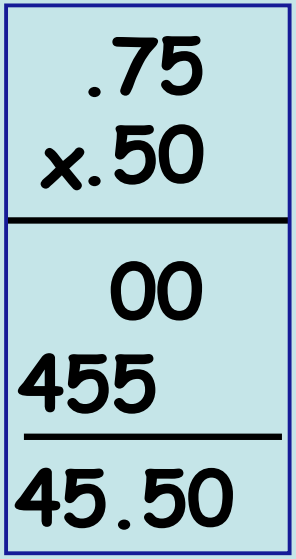
Examples

- Analyzing errors
- Giving and evaluating explanations
- Appraising unexpected claims, solutions, and methods
- Choosing and using representations
- Building and examining correspondences
- Choosing and using definitions
- Interpreting and responding to students' ideas

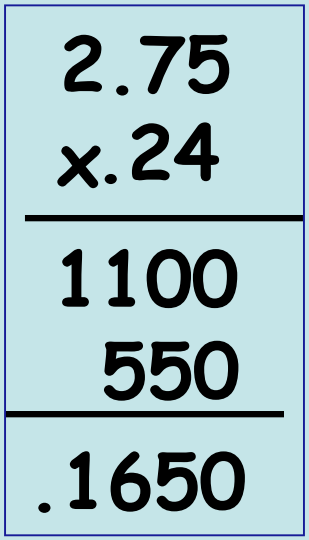
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Analyzing Errors

Student A



Student B



Are both students making the same types of errors? What are they doing? Why might they do these things?

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Building Correspondence Between a Mathematical Idea and a Model

$38 \div 4$

Write three different stories that correspond to this division expression and that reveal different interpretations of the meaning of division or its mapping to specific situations.

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Analyzing Mathematical Procedures

$$\begin{array}{r} 299\ 12 \\ \cancel{3002} \\ -\quad 783 \\ \hline 2219 \end{array}$$

How was this problem done?

Is this a method that would work to subtract any two whole numbers?

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Appraising an Unexpected Claim

A student claims she has discovered a new method for comparing fractions. She looks at the numerator and the denominator, and the closer they are to each other, the greater the fraction. An example:

$$\frac{4}{7} \qquad \frac{7}{8}$$

Is the general claim true? Will this work for any two fractions?

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3. Developing a Practice-Based Approach to Learning Mathematics for Teaching

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What Mathematical Problems Arise in Teaching?

Examples

- Analyzing errors
- Giving and evaluating explanations
- Appraising unexpected claims, solutions, and methods
- Choosing and using representations
- Building and examining correspondences
- Choosing and using definitions
- Interpreting and responding to students' ideas

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Example:

Using Records of Practice to Study Mathematics for Teaching

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Video Clip from 3rd Grade: A Student's Novel Idea About the Parity of 6

- Mid-January (ten days into number theory unit)
- Diverse classroom, many English language learners
- Goals in this classroom all year: taking mathematics seriously, attending carefully to other students' ideas, justifying claims, and working together on mathematics have been a goal all year

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Sticks of gum cost 2¢.

Pencils cost 7¢.

How many different combinations of gum and pencils can I buy and spend exactly 30¢?

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Students' Conjectures

An odd number plus an odd number equals an even number.

An even number plus an even number equals an even number.

An odd number plus an even number equals an odd number.

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Designing Opportunities to Learn Mathematics From the Study of a Video Clip

What **mathematical issues** do you notice in this video clip?

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Insert Shea video here

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Designing Opportunities for Teachers to Learn Mathematics From the Study of a Video Clip

Task #1

Lin used 10 to challenge Shea's claim. Why was this a good choice?

What other number would be strategic to ask Shea about, and why?

Task #2

Three different definitions of even number are being used in this lesson episode. What are they, and who is using which one?

Are the three definitions equivalent? Prove your answer.

Task #3

Your principal is concerned that you let this discussion go on. You were worried about this, too.

Can you explain what mathematics you were working on with the students by letting this discussion continue?

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Solving Problems in Learning Mathematics for Teaching

Task	What opportunities for teachers to learn mathematics for teaching does the task offer?
Task #1: Lin used 10 to challenge Shea's claim. Why was this a good choice? What other number would be strategic to ask Shea about, and why?	
Task #2: Three different definitions of even number are being used in this lesson episode. What are they, and who is using which one? Prove that the three definitions are equivalent.	
Task #3: Your principal is concerned that you let this discussion go on. You were worried about this, too. Can you explain what mathematics you were working on with the students by letting this discussion continue?	

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Solving Problems in Learning Mathematics for Teaching

Task	What opportunities for teachers to learn mathematics for teaching does the task offer?
Task #1: Lin used 10 to challenge Shea's claim. Why was this a good choice? What other number would be strategic to ask Shea about, and why?	Seeking counterexamples Factorization Analyzing structure Generalization
Task #2: Three different definitions of even number are being used in this lesson episode. What are they, and who is using which one? Prove that the three definitions are equivalent.	Knowledge of definitions Analyzing and reconciling multiple definitions Justifying claims
Task #3: Your principal is concerned that you let this discussion go on. You were worried about this, too. Can you explain what mathematics you were working on with the students by letting this discussion continue?	Seeing, identifying, and naming mathematical ideas, skills, and habits Communicating about mathematics to others Definitions of even and odd numbers Exploring patterns Importance of naming Evaluating conjectures Justifying claims Connections to modular arithmetic

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Conclusions

1. Knowledge needed for teaching is different from what is needed for other occupations or professions where mathematics is used (e.g., physics, mathematics, carpentry, tailoring, business).

2. Knowledge needed for teaching must be usable for the mathematical work that teachers do.

3. Mathematics teaching involves specialized mathematical problem solving.

4. Learning mathematics for teaching can be grounded in practice by designing opportunities for teachers to solve mathematical problems that arise recurrently in teaching.

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