

Expanding Who is "Good at Math" From the Beginning

Deborah Loewenberg Ball

SEYHAN N. EĞE MEMORIAL SYMPOSIUM • OCTOBER 27, 2009

SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN

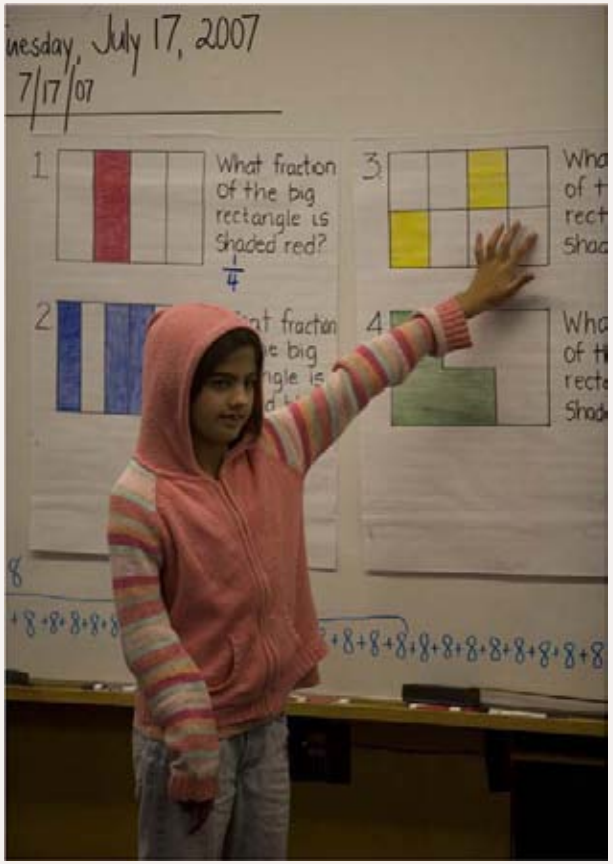
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An urgent problem, and a new risk

1. Enormous gaps in learning opportunities and disparities in achievement (within U.S. and in international comparisons)
2. Rapidly changing school population
3. Higher, more complex academic goals
4. High expectations for <u>all</u> students



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In the spirit of Seyhan Eğe: The main argument of my talk

1. The foundations of mathematical engagement and capability must be built from the earliest grades.
2. This requires redefining what it means to be "good at math" in school, and broadening interest in and participation in complex mathematical work.
3. Achieving these goals is key professional work of the teacher.

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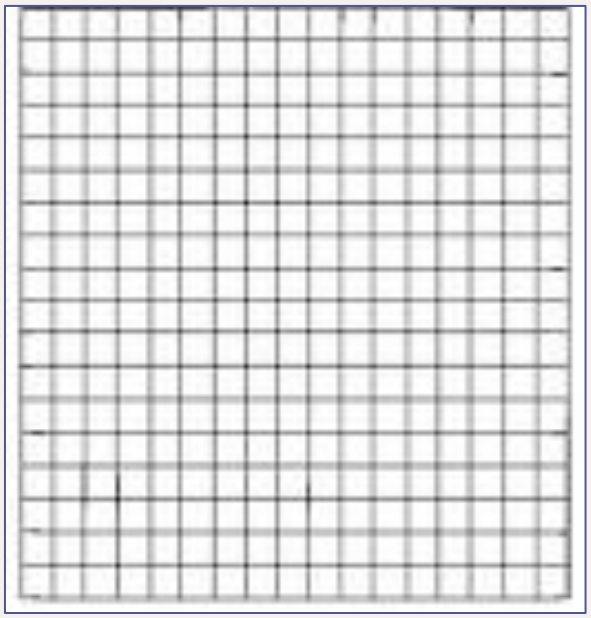
1. Young children have profound mathematical insights:

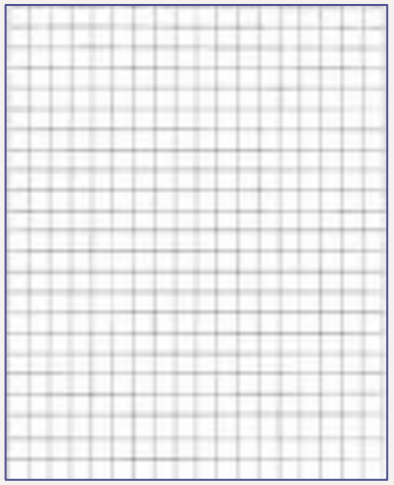
Three brief snapshots of mathematics learning

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Measuring area in first grade



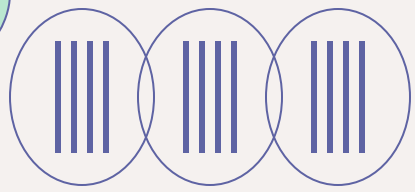




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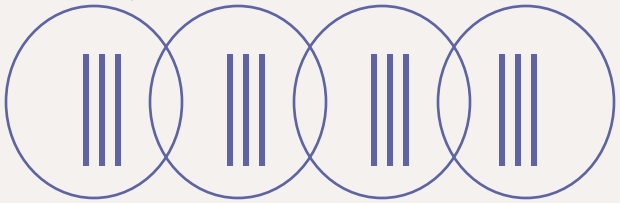
How much is three-fourths of a dozen?

Shouldn't one fourth have **four** in it?



$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$

Here's **four** groups, so one of these is **one-fourth**.



$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$

There's only three groups, so one of these makes **one third.**

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Rania:

I still disagree. . . I want to say something. This is what I think three-fourths is— I'm saying that three-fourths is like three groups of four.



Kevin:

I think she disagrees because she think like—she thinks one—the reason she thinks one fourth should have four is because of the four in the bottom number.

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Shea:
Can we vote?

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Making and proving a conjecture in third grade

An odd number plus an odd number equals an even number.

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2. Nurturing young children's mathematical instincts

"Sowing", using, and extending students' mathematical ideas

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Expecting and enabling complex mathematical work from the beginning

1. Broadening what it means to be successful in math class
2. Using appropriate mathematical problems and questions
3. Making mathematical practices explicit
4. Developing specialized mathematical language
5. Constructing a social and intellectual culture of mathematical seriousness and respect for others' ideas
6. Supporting students' work both publicly and privately

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Video clip

- Grade 3
- Work on even and odd numbers
- Discussion of unexpected student idea
- Practices of mathematical argument and analysis
- Norms of respect

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Who is participating in mathematics, and what counts as "participation"?

1. What are the mathematical tasks?
2. How are mathematical practices featured?
3. How is specialized mathematical language being developed?
4. What is the social and intellectual culture?
5. How is students' work being supported?

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Broadening what it means to be – and who is — "good at math"

WHAT IS BEING "GOOD AT MATH"?

- Proposing an idea in class
- Reasoning, and critical listening; expecting mathematical justification
- Critiquing claims using evidence and logic
- Attending to definitions of even and odd numbers (three unreconciled definitions of "even number" in play)
- Making public presentations and representations of mathematics
- Treating others' mathematical ideas with interest and respect

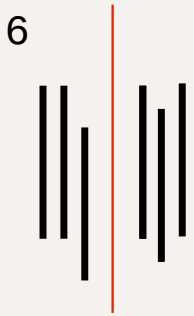
WHO IS PARTICIPATING IN MATHEMATICS, AND HOW?

- Shea has a mathematical idea about 6; he appropriates (inappropriately) the name "even and odd"
- Tina questions Shea's claim, using definition
- Kip (and others) request a proof
- Shea represents on board
- Lin clearly articulates Shea's idea, then generalizes it in order to convince him it is unreasonable
- Ogechi makes more precise the definition of odd numbers

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Students' definitions of even number

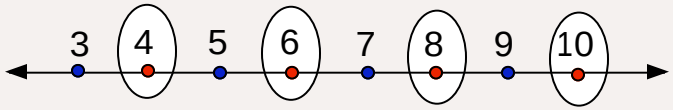
1. An even number is a number you can split in two equal parts without having to break anything in half.



2. An even number is a number that when you group it in twos, there are none left.



3. The even numbers are every other number on the number line, like 2, 4, 6, and so on.



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3. More students could be very good at much more mathematics:

The role of the teacher in cultivating broad mathematical engagement and achievement

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Teaching that is both responsible and responsive

- Hears the mathematics in students' representations and talk
- Notices misconceptions in students' representations and talk
- Balances mathematical rigor with generosity toward emerging ideas
- Manages mathematical opportunities wisely

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What does this "responsible and responsive" teaching take?

- Knowing and loving math
- Holding high expectations of students

A reasonable, but surprisingly insufficient answer.

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Knowing and loving mathematics is not enough

. . . because teaching mathematics involves getting other people to know, do, and love mathematics

Is it necessary, but insufficient?

A paradox.

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A fundamental paradox

- Good teachers must know and love the domains they teach.
- But to be good teachers, they must be fascinated by other people's thinking, learning, and work in the domain, not only their own.
- But being fascinated depends on knowing the domain really well.

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Imagine . . .

- A piano teacher whose students mostly watch her play
- A Chinese teacher whose students mostly listen to her speak
- A mathematics teacher who stands at the board talking and doing mathematics for most of each class

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What are the tacit assumptions underlying "performance pedagogy"?

1. That observing expert performance can enable learners to perform a practice
2. That experts can explicate what they are doing that must be learned
3. That learners' learning trajectories mirror experts' accomplished practice

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What is the work of getting others to learn and do?

Building bridges between

learners' experience and ways of thinking

and

domain-specific skills, knowledge, dispositions, and capacities

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What does this "bridge building" entail?

- Understanding the domain in multiple ways
- Explicit knowledge of tacit aspects of the domain
- Seeing the domain from others' viewpoints

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"High expectations" are also not enough

- Helping students to do hard work
- Scaffolding complex work appropriately
- Making wise judgments about what to leave open and what needs to be made explicit
- Commenting on mathematics, and mathematical productions, not features of students
- Supporting error as a fruitful site for mathematical work, and teaching students to use error productively

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High quality teaching, and how to develop it

- None of this is possible without highly skilled teaching
- This requires
  - ① special mathematical knowledge that is both deeply rooted in disciplinary practice and ideas and flexible enough to connect with learners' thinking
  - ② instructional judgment and practice
  - ③ cultural sensitivity and relational skills
- Not simply "natural" — requires a system of professional preparation and paraprofessional support, at scale (a topic for another talk!)

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THANK YOU!

Slides will be available at my website:

(Google “Deborah Ball”)

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