

Learning to Teach the Common Core

Deborah Loewenberg Ball*

November 28, 2011

*With grateful acknowledgements to Hyman Bass, Laurie Sleep, and Mark Thames.

SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

What is so important about the Common Core State Standards?

1. That they are common. This is the first step in one of the most important differences between the U.S. and other systems.
2. That they give such central content role to mathematical practices. This takes another step from the NCTM "process standards"* and the strands of mathematical proficiency in *Adding It Up*.

*Principles and Standards for School Mathematics, 2000; National Research Council, 2001.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

The mathematical practices (CCSS)

MP.1. Make sense of problems and persevere in solving them.

MP.2. Reason abstractly and quantitatively.

MP.3. Construct viable arguments and critique the reasoning of others.

MP.4. Model with mathematics.

MP.5. Use appropriate tools strategically.

MP.6. Attend to precision.

MP.7. Look for and make use of structure.

MP.8. Look for and express regularity in repeated reasoning.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

"Practice" in school mathematics

1. School mathematics has always been about "practice," but it has been more about *drill and practice*, not *the practice of mathematics*.
2. The Common Core State Standards offer the possibility of re-orienting school mathematics around a more robust conception of mathematical competence.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

What are some resources for implementing this more robust view of mathematics?

1. A history of efforts to incorporate some version of mathematical practices: Process standards of NCTM; Strands of mathematical proficiency in *Adding It Up*.
2. Extensive research literature on certain practices – problem solving; reasoning and proving; modeling.
3. English language arts (at the elementary level), particularly reading and writing, have long been dominated by a focus on practices. This presents a domain of school learning in which teachers have successfully made practices central to instruction.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

What are the obstacles to making the Common Core "common"?

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

What are obstacles to making the Common Core "common"?

1. Textbooks and assessments are not aligned with the CCSS.
2. Misperception that mathematical practices are processes, or the means to learning the "real content."
3. Some people may not think that all students are capable of engaging in mathematical practice.
4. Teachers' lack of opportunity to learn MKT with respect to the mathematical practices.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Learning to teach the CCSS mathematical practices

1. Developing appreciation that mathematical practices are **basic to the actual content**, not just part of the means, or something to do once the "basics" are mastered.
2. Developing conviction that **all students can**—and must—develop proficiency with mathematical practices.
3. Developing **mathematical knowledge for teaching (MKT)** with mathematical practices.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

① Mathematical practices are basic skills.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Learning opportunities

1. Explore mathematical problems and tasks and analyze the practices involved.
2. Examine mathematical situations in everyday life and analyze the practices involved.
3. Compare people's approaches and solutions and make visible the centrality of practices.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Analyzing problems

I have pennies, nickels, and dimes in my pocket.

If I pull out two coins, what amounts of money might I have?

Prove that you have all the solutions.

*Adapted from the NCTM (1989) *Curriculum and evaluation standards*.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

What mathematical core content is involved?

TOPICS

1. Combinations
2. Addition

PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Analyzing problems

Use only 8s and + signs to write equations for 1000. You can use as many 8s as you want.

Find all the solutions.

*Adapted from I. Gelfand, *Algebra* (1993).

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Solutions. . .

① 8 + 8 + 8 + . . . +8 + 8 = 1000 (125 8s)

② 888 + 88 + 8 + 8 + 8 = 1000 (1 888, 1 88, 3 8s)

③ 888 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8+ 8 + 8 + 8 + 8 + 8 + 8 = 1000 (1 888, 0 88s, 14 8s)

④ . . .

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

number of 888's	number of 88's	number of 8's	number of terms
	0	125	125
	1	114	115
	2	103	105
	3	92	95
	4	81	85
	5	70	75
	6	59	65
	7	48	55
	8	37	45
	9	26	35
	10	15	25
	11	4	15
1	0	14	15
1	1	3	5

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

What mathematical core content is involved?

TOPICS

1. Equations
2. Addition and multiplication
3. Place value

PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

What is the chance of rain?

Weather forecast:

Saturday: **50% chance of rain**

Sunday: **50% chance of rain**

What a bummer! I wanted to play golf, but there is a 100% chance of rain this weekend!

Is this correct? Explain.

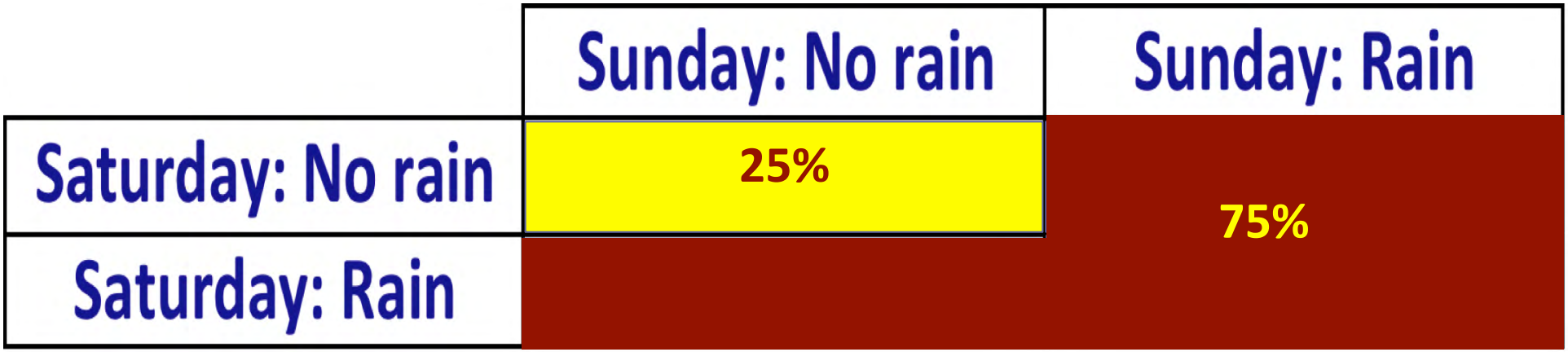
This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

What is the chance of rain on the weekend?

	Sunday: No rain	Sunday: Rain
Saturday: No rain	25%	25%
Saturday: Rain	25%	25%

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

What is the chance of rain on the weekend?



This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

What mathematical core content is involved?

TOPICS

1. Probability

PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Learning opportunities

1. Explore mathematical problems and tasks and analyze the practices involved.
2. Examine mathematical situations in everyday life and analyze the practices involved.
3. Compare people's approaches and solutions and make visible the centrality of practices.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

② All students can master mathematical practices.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Learning opportunities

1. View classroom examples of diverse classrooms where students are expected to use mathematical practices.
2. View examples of diverse students' mathematical work and thinking.
3. Engage students in problems that expose the importance of mathematical practices and study their work.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

View video clip

Diverse class, over half are ELLs

Mid-year, in a unit on even and odd numbers

Focal question for viewing:

What mathematical practices do you see the students using or learning to use?

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Link for video

http://hdl.handle.net/2027.42/65013

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Focal question

What mathematical practices do you see the students using or learning to use?

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Mathematical practices in use

- Understand the approaches of others (3)
- Continually ask oneself, "Does this make sense?" (1)
- Understand and use stated assumptions, definitions, and previously established results in constructing arguments. (5)
- Make conjectures and build a logical progression to explore their validity, (2, 3)
- Consider the available tools (representations) when solving a mathematical problem. (5)
- Abstract a given situation, represent it, manipulate the representation. (2)
- Construct viable arguments and critique the reasoning of others. (3)
- Examine claims and make explicit use of definitions. (3, 6)
- Determine (generalize) domains to which an argument applies. (2, 8)
- Attend to precision (use clear definitions). (6)
- Give carefully formulated explanations to others. (3)
- Look for and make use of structure. (7)
- Evaluate the reasonableness of results. (3)

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Learning opportunities

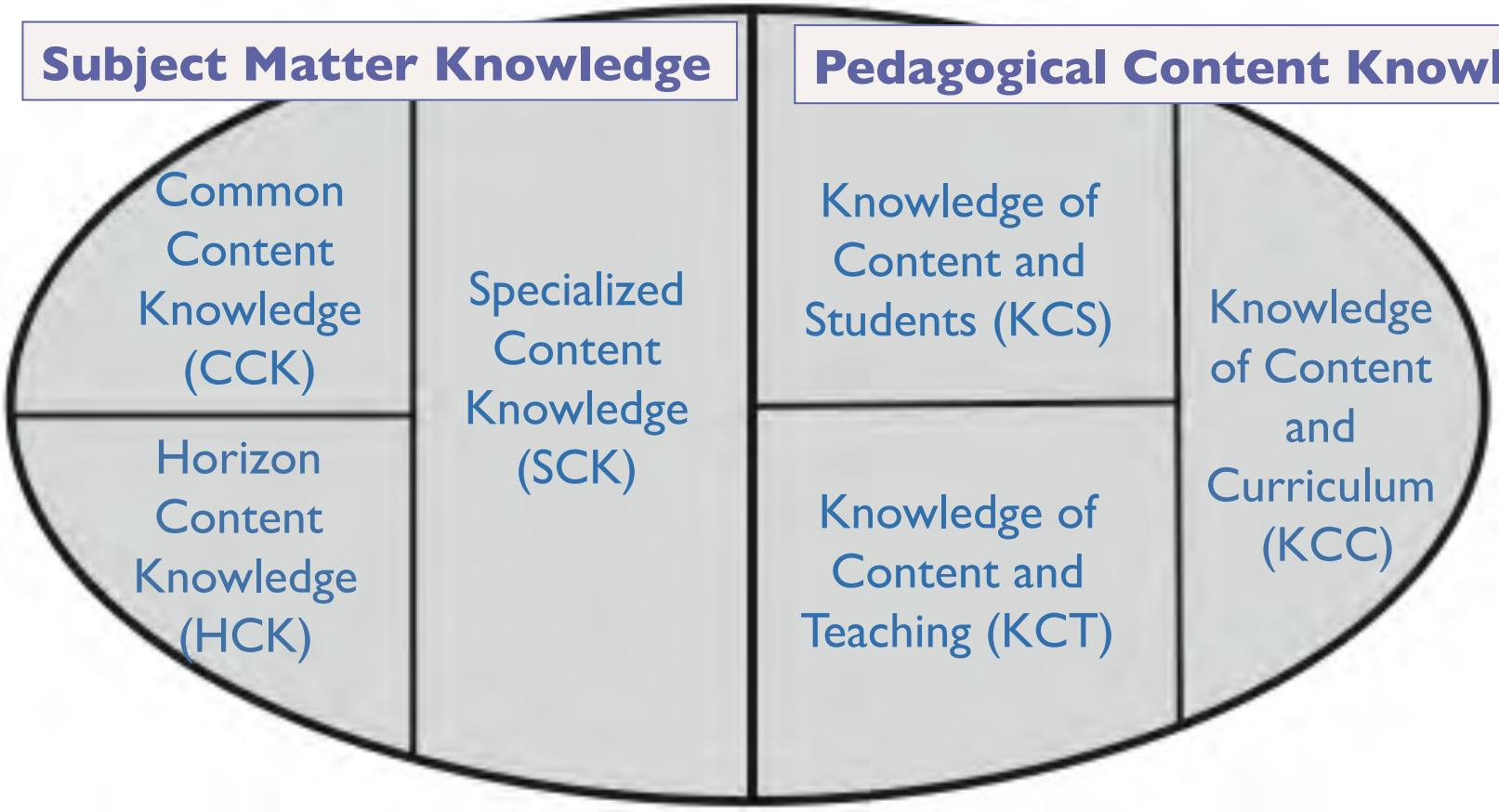
1. View classroom examples of diverse classrooms where students are expected to use mathematical practices.
2. View examples of diverse students' mathematical work and thinking.
3. Engage students in problems that expose the importance of mathematical practices and study their work.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

③ Develop MKT for mathematical practices.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Mathematical Knowledge for Teaching (MKT)



This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Learning opportunities

1. Engage in solving problems and discussing the practices used in solving them.
2. Unpack practices in detail: explain, represent, consider what might be involved in making them learnable.
3. Study cases of students and develop explicit knowledge of how they approach mathematical practices.
4. Develop and compare ways to teach specific practices.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Unpacking mathematical explanation (MP 2 & 3) (SCK)

Has a clear purpose

- Makes clear at the outset what is being explained and why you start there; and carefully connects the explanation to the question or idea being explained
- States what is known and what needs to be determined

Has a logical structure

- Starts at the beginning, and goes step-by-step in a logical fashion
- States what you are doing and why you are doing it throughout
- Summarizes at the end what has been explained, and links that back to the original question, claim, or problem

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Unpacking mathematical explanation (continued)

Uses representations and language clearly and carefully

- Strives to be as simple and clear as possible
- Uses mathematical language accurately and consistently
- Defines terms as needed
- Uses representation(s) accurately
- Carefully labels any diagrams
- Uses units (when applicable)
- Makes explicit correspondences between the problem, the verbal/written explanation, and any representations

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Unpacking mathematical explanation (continued)

Focuses on meaning and is geared to the learner

- Shows what something means or why is true, and is convincing to the person to whom you are explaining
- Takes into account the background knowledge of the listener/reader
- Uses words that will be understood by the listener/reader
- Breaks things down – does not assumes the listener/readers knows what you are thinking
- Elaborates and emphasizes the part of the explanation that is most complex or confusing
- Uses representations that highlight meaning

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Studying students' development, use, and learning of practices

- What do they find difficult?
- What is typical at different ages?
- What do they do "naturally"?
- How do they talk or use language or represent mathematically?

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Examples from video

Sean, Mei, Ofala, Betsy:

Use of pictorial diagram to represent general case of even and odd numbers (5)

Mei and Betsy:

Generalizing (reasoning abstractly) (2)

Sean, Ofala, and Betsy:

Looking for and using mathematical structure (7); constructing viable arguments (3)

Cassandra, Keith, Tembe, Mei, Ofala:

Critiquing the arguments of others (3)

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Developing approaches to teaching mathematical practices

Examples

Being explicit by:

- Helping students learn to make charts (coin problem, 8s problem, weather probability) (5)— instead of just letting them work
- Supporting the development of explanation: making poster with features of good mathematical explanation, scaffolded questioning (2)— instead of just asking for explaining

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu

Learning to teach the CCSS mathematical practices

1. Developing appreciation that mathematical practices are **basic to the actual content**, not just part of the means, or something to do once the "basics" are mastered.
2. Developing conviction that **all students can**—and must—develop proficiency with mathematical practices.
3. Developing **mathematical knowledge for teaching (MKT)** with mathematical practices.

This work is licensed under the Creative Commons Attribution-Noncommercial-No Derivative Works Version 3.0 United States License: http://creativecommons.org/licenses/by-nc-nd/3.0/us/
© 2011 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109 • mtlt@umich.edu