



Teaching All Learners to Be Mathematically Skillful

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Marino Institute of Education, Dublin, Ireland • December 1, 2012

SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN

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Today's session

1. What does does it mean to be mathematically skillful?
2. What are high-leverage teaching practices?
3. High-leverage practices for teaching students to be mathematically skillful

Write your professional learning goals for this session.

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The 8's problem

Write as many 8's as you like. Insert plus signs (+) so that the value of the expression is equal to 1000.

Note: You can string 8's together, such as 888.

- How many different equations can you write following this rule?
- What is your approach?
- Can you find all of them? How do you know?

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Solutions to the 8's problem

number of 888's	number of 88's	number of 8's	number of terms
0	0	125	125
1	0	14	15
0	1	114	115
1	1	3	5
	2	103	105
	3	92	95
	4	81	85
	5	70	75
	6	59	65
	7	48	55
	8	37	45
	9	26	35
	10	15	25
	11	4	15

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Opportunities for learning maths

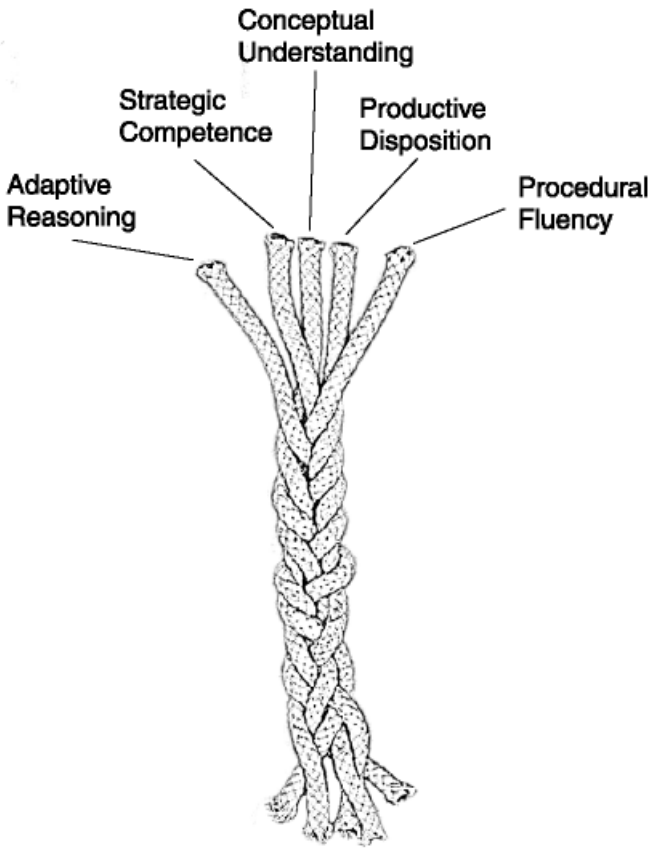
What opportunities for learning maths are there in the 8's problem?

How much of the learning opportunity was in:

- The problem itself
- The way the task was framed and set up
- How we used the task together

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Strands of mathematical proficiency



- ***Conceptual understanding*** - comprehension of mathematical concepts, operations, and relations
- ***Procedural fluency*** - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic competence*** - ability to formulate, represent, and solve mathematical problems
- ***Adaptive reasoning*** - capacity for logical thought, reflection, explanation, and justification
- ***Productive disposition*** - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

Kilpatrick, J., J. Swafford, and B. Findell. (2001). *Adding It Up: How Children Learn Mathematics*. Washington, DC: National Academy Press.

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Viewing focus

1. Are there important learning opportunities for students, or is this a waste of class time?
2. If our goal is to focus on increasing the power of teaching, what are some of the teaching practices to identify in this segment?

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Making reasoned decisions about the use of instructional time

NOT GOOD USE OF TIME

1. Students might get confused about even and odd numbers
2. Teacher should correct Sean's idea about 6 being both even and odd
3. Too much to cover to spend time on a confused discussion of a simple topic
4. Only some students participating

'TEACHABLE MOMENT'

1. Reconciling three different definitions of even and odd numbers
2. Learning to make mathematical arguments and counterarguments
3. Critiquing mathematical claims and arguments
4. Developing a definition of even and odd number

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What are some practices of teaching visible in the segment?

1. Identifying appropriate mathematical learning goals
2. Managing time closely (in relation to mathematical learning goals)
3. Establishing and maintaining a safe and respectful learning environment
4. Interpreting and responding to student 'errors'
5. Leading a class discussion of a 'discussable' mathematical idea, question, or solution
   - Developing norms for mathematical and respectful talk
   - Identifying and 'steering' to the mathematical point
   - Allocating turns
   - Monitoring and ensuring learner' engagement
6. Posing strategic questions
7. Explicitly focusing on mathematical language
8. Signalling what it means to be 'good at maths'
9. Others?

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Responsible teaching is unnatural and intricate work

1. Teaching requires teachers to do things that are more than commonsense.
2. Teaching involves some moves or orientations that are unnatural in regular adult life.
3. Teaching practice is intricate, requiring a complex combination of knowledge, skill, timing, and relational work.

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High-leverage instructional practices

High-leverage practices (HLPs) are instructional tasks and activities that powerfully promote learning and are fundamental to skillful teaching.

Based on work done at the University of Michigan School of Education in the redesign of our teacher education program, and at TeachingWorks, a new organization whose mission it is to improve the quality of teaching and learning by transforming. teachers' education.

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Examples of high-leverage practices

1. Explaining, representing, and modelling core content
2. Setting up and managing small-group work
3. Recognizing and identifying common patterns of student thinking in a content domain
4. Posing questions
5. Eliciting and interpreting learners' thinking
6. Establishing norms and routines for a respectful and effective learning environment
7. Selecting and using specific methods to assess students' learning on an ongoing basis
8. Conducting a meeting with a parent or caregiver

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Choosing strategic examples

Which of the following lists would be best for assessing whether your learners understand decimal ordering?

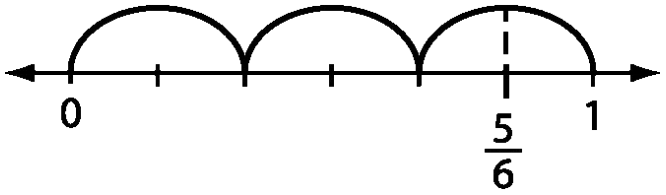
Justify your choice.

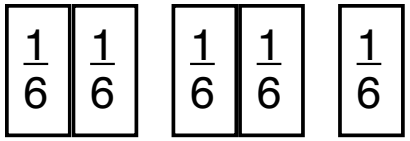
A.	.5	7	.01	11.4
B.	.60	2.53	3.12	.45
C.	.6	4.25	.565	2.5

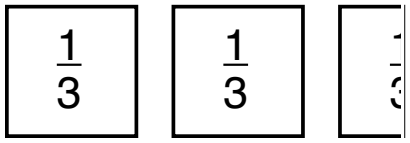
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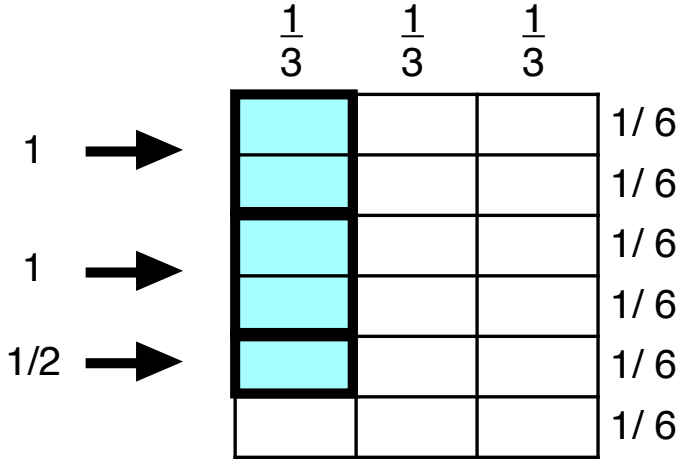
Analyzing and narrating representations

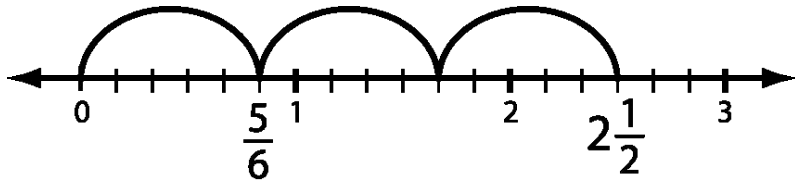
Which of these can be used to represent $\frac{5}{6} \div \frac{1}{3}$? Explain with reference to all parts of the expression.











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Analyzing non-standard (but correct) responses

Which person is using a method that could be used to multiply any two whole numbers?

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

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Ten high-leverage instructional practices that promote learners' mathematical skill

① Choosing and using mathematical tasks that support the development of mathematical skills
② Choosing examples
③ Teaching and using academic language
④ Leading a productive whole-class math discussion
⑤ Responding productively to students 'errors'
⑥ Using homework equitably
⑦ Using specific mathematically-focused positive reinforcement
⑧ Using public recording (posters, whiteboard)
⑨ Diagnosing common patterns of student thinking (and not-so-common)
⑩ Assessing students' mathematical skill and teaching responsively

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① Choosing and using mathematical tasks that support the development of mathematical skills

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Analyzing others' solutions

A fourth year learner did the calculations below. For each one, decide whether the answer is correct or incorrect and explain how you know. If the answer is incorrect, try to explain what the error is.

$$\begin{array}{r} 48 \\ +\ 27 \\ \hline 615 \end{array}$$

$$29 + 37 + 18 = 84$$

Is the answer correct or incorrect?

How do you know?

If the answer is incorrect, try to explain what the error is.

If the answer is incorrect, please do it correctly here:

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Reasoning with tools



Which Cuisenaire rod is three times as long as a red one?

Which rod is half as long as an orange one?

One rod is one-fourth as long as another rod. What colors might they be?

Which rod is one-third as long as the dark green one?

Which rod is five times as long as a red one?

The blue rod is how many times as big as the white Cuisenaire rod?

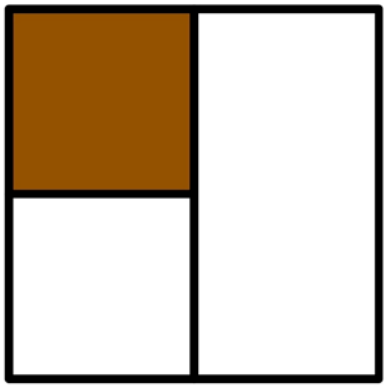
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Reasoning with representations

What fraction of the rectangle below is shaded brown?



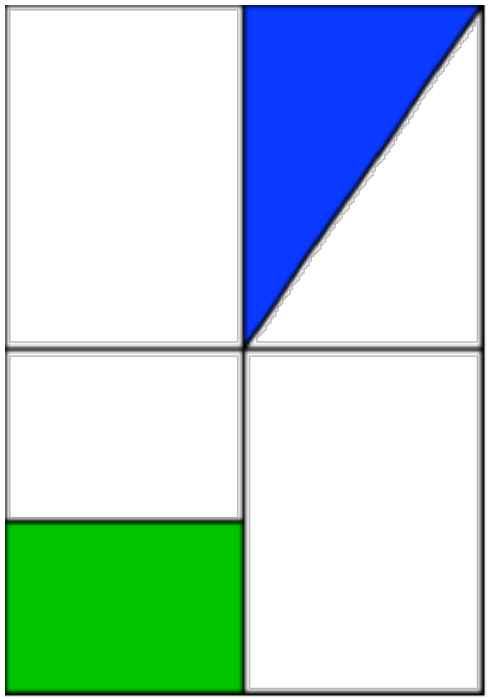
What fraction of the rectangle below is shaded brown?



What fraction of the big rectangle is shaded blue?

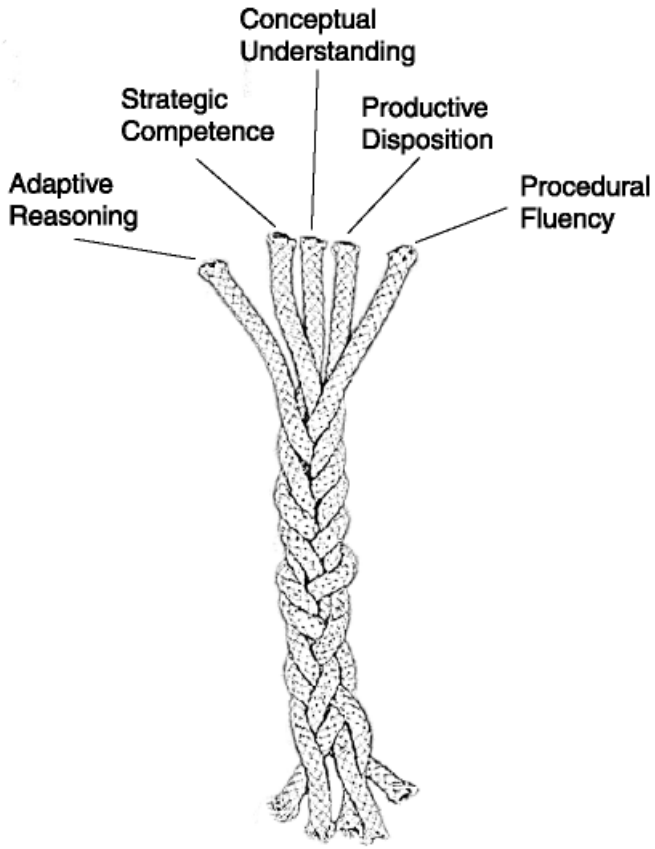
What fraction of the big rectangle is shaded green?

What fraction of the big rectangle is shaded altogether?



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Analyzing tasks



How do these tasks promote the development of mathematical skill?

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Attending closely to wording

POSSIBLE PROBLEM:

I have 1¢, 5¢, and 10¢ coins in my pocket. If I pull out two coins, what amounts of money might I have?

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Reasoning about different wording

1. I have 1¢, 5¢, and 10¢ coins in my pocket. If I pull out two coins, what amount of money might I have?
2. I have 1¢, 5¢, and 10¢ coins in my pocket. If I pull out two coins, how many combinations are possible?
3. I have 1¢, 5¢, and 10¢ coins in my pocket. If I pull out two coins, how many different amounts of money are possible? Prove that you have found all the amounts that are possible.

What are the differences among these three formulations?

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Necessary for challenging learners with complex tasks

- Help students to believe they can do hard work.
- Scaffold complex work appropriately.
- Make wise judgments about what to leave open and what needs to be made explicit.
- Comment on mathematics, and mathematical productions, not features of students.
- Support error as a fruitful site for mathematical work, and teach students to use error productively.

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② Responding productively to learners' 'errors'

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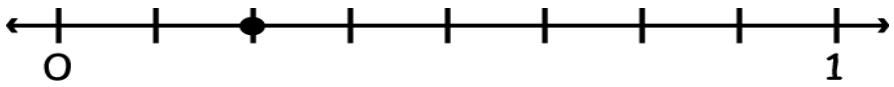
Learners' errors as a crucial site for teaching

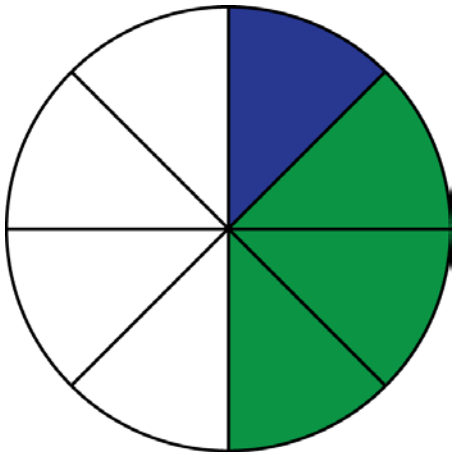
1. Often occur at the heart of the content; working on them can help make instruction more focused
2. Expose and can provide 'inoculation' for probable confusions
3. Develop learners' independent skills at analysis (mathematical self-regulation)
4. Can change children's conceptions of what 'being good at maths' means

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'Safe' tasks

What makes these tasks "safe"?

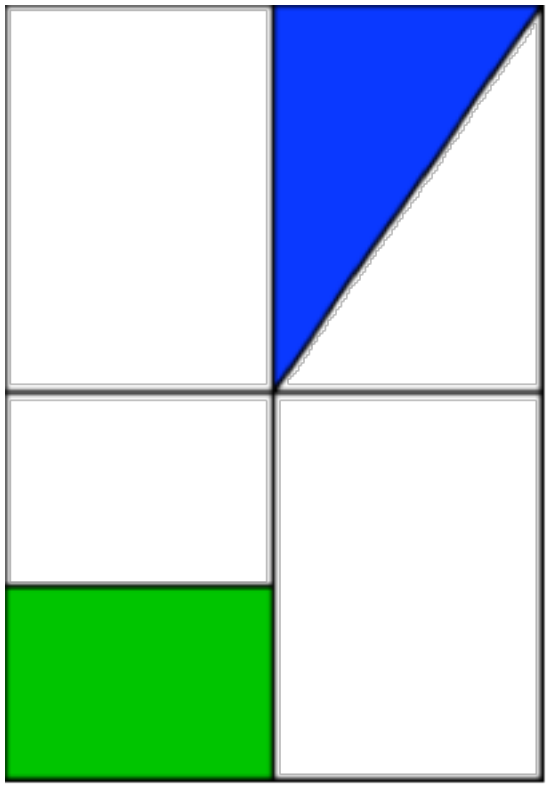




1. Much of the thinking is done <u>for</u> the children (e.g., equal partitioning, shading, labeling)
2. Protect from error — supports getting "correct" answer

THUS, THESE TASKS ARE ACTUALLY POTENTIALLY <u>UN</u>SAFE.

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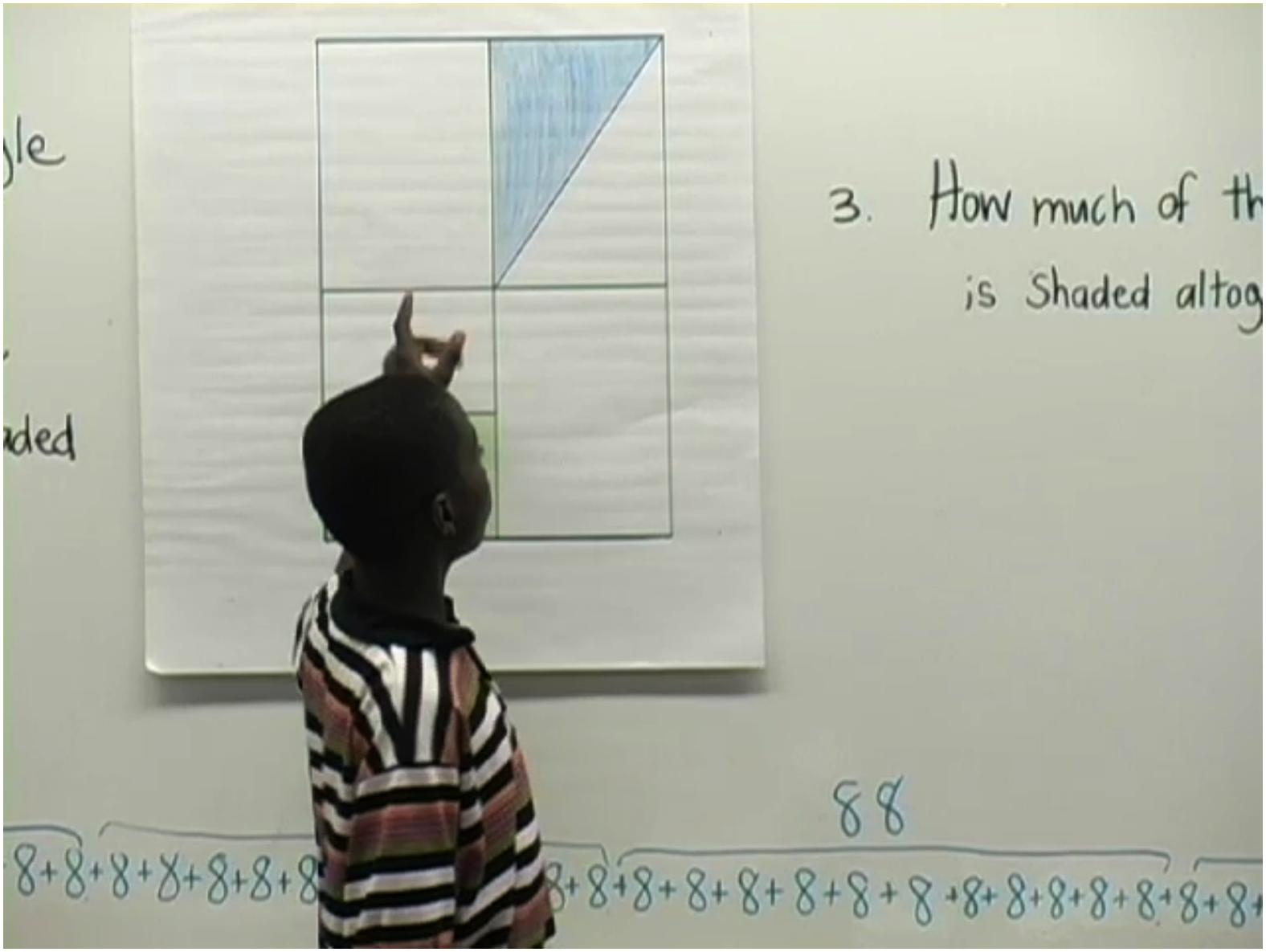


What fraction of the big rectangle is shaded blue?

What fraction of the big rectangle is shaded green?

What fraction of the big rectangle is shaded altogether?

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Interpreting and responding to learners' 'errors'

1. Is it really an error or is it in the way the student is expressing or writing it? (e.g., Sean)
2. Is the student actually correctly answering a different question? (e.g., Mamadou)
3. Is the 'error' an important conceptual issue that could be exploited productively for everyone's learning? (e.g., Sean, Mamadou)

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③ Teaching and using academic language

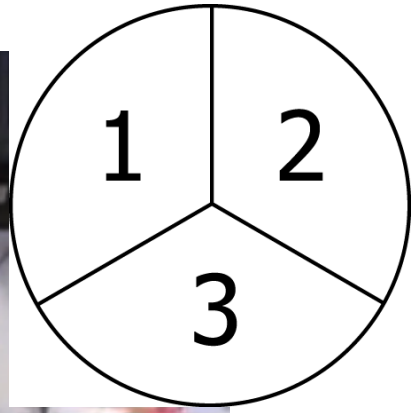
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Viewing focus

What are the components of the teacher's approach to introducing a new mathematical term?

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Introducing a mathematical term

1. 'What numbers can you get?' (*concept*)
2. 'We call those outcomes.' (*new term*)
3. 'Possible results of an experiment.' (*definition*)
4. Puts up poster. (*signals importance, supports remembering and using term*)
5. 'How many outcomes are there?' (*practice*)

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Why is language so important?

- Mathematical knowledge and reasoning depends on and is supported by mathematical language.
- Teaching and learning mathematics depends on and is supported by language.
- Mathematical language is both mathematical content to be learned and medium for learning mathematical content.

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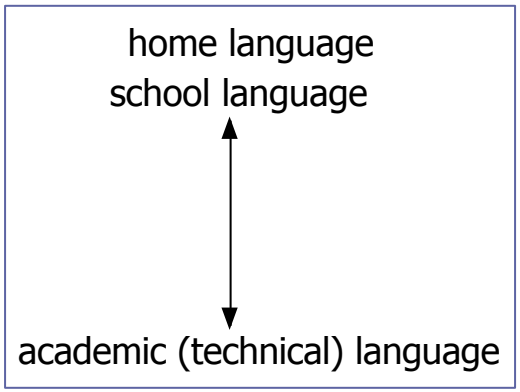
Academic and intellectual vocabulary

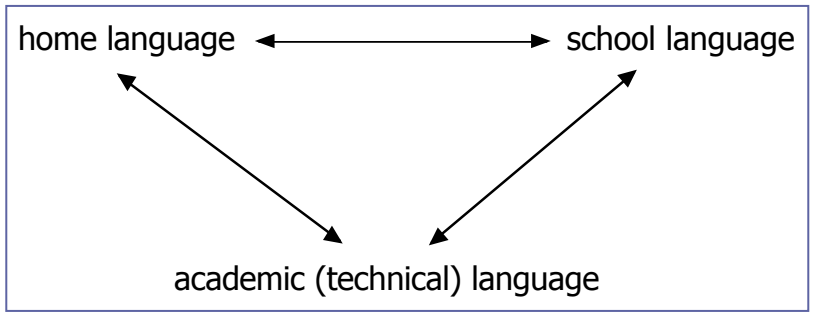
1. Specific mathematical concepts (e.g., outcome, even number, rectangle, equal)
2. Terms for knowledge or knowing (e.g., prove, conjecture)
3. Terms for intellectual work (e.g., confer, propose, argue)

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Issues with mathematical language: Mathematical vs. everyday language

- Mathematics often uses and specializes everyday language rather than coining a separate technical vocabulary -- both enabling and complicating entry to its register.
- Equity issue: Additional dimension for students who must navigate between home, school, and mathematical languages





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Challenges of using language

You ask your learners to explain what a rectangle is. One child offers a definition:

A rectangle is a flat shape. It has four square corners, and it is closed all the way around.

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What is the work for the teacher?

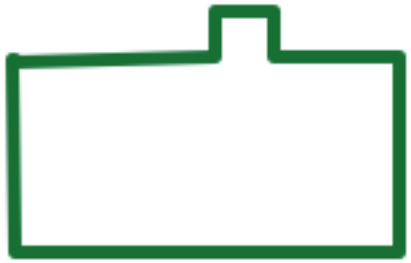
A rectangle is a flat shape. It has four square corners, and it is closed all the way around.

1. To see that something is missing
2. Decide what to do or say
3. Offer a counterexample

'Is this a rectangle?'

(straight sides)





(exactly four square corners)

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A rectangle is a flat shape with **straight sides that are connected at exactly** four square corners. It is closed all the way around.

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Dilemmas of using language

1. Learners necessarily use imprecise language.
2. We want to encourage students to express themselves.
3. Informal language can serve as a bridge or a support.
4. Attending to the needs and resources of second language learners.

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What makes a 'good' definition?

- Mathematically precise — correctly identifies the kind of object, process, property
- Usable by user community — based on already-defined and understood term

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What is a mathematically precise and usable definition of "even number" for third graders?

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Choosing a definition of 'even number'

a) An even number is a number that can be divided into two equal parts.

b) An even number is any multiple of 2.

c) An even number is any integer multiple of 2.

d) An even number is any number whose unit digit is 0, 2, 4, 6, or 8.

e) A whole number is even if it is the sum of a whole number with itself.

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a) An even number is a number that can be divided into two equal parts.

b) An even number is any multiple of 2.

All numbers, for example 7, 3/5, √2, p, are even!

c) An even number is any integer multiple of 2.

This is a correct definition of even number.

d) An even number is any number whose unit digit is 0, 2, 4, 6, or 8.

In this case, 36.7 is an even number!

e) A whole number is even if it is the sum of a whole number with itself.

This is a correct definition of evenness for whole numbers, and is consistent with the general definition for integers that will arrive later.

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Conclusions

Teaching all students to become mathematically skillful entails:

- Developing teaching skills with 'high-leverage' practices
- Developing specialized mathematical knowledge for teaching
- Practicing with the mathematical work of teaching (like we did today)

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