Mathematical Content and Applications for the Teaching of Elementary School Mathematics

Math 399/Ed 310/Ed 640
June 2004

Course Instructor:

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Meeting Dates and Times:

One Credit: Sunday, June 6: 1:30pm - 3:30pm
Monday, June 7: 9:00am –11:00am
Tuesday, June 8: 9:00am –11:00am
Wednesday, June 9: 9:00am –11:00am
Thursday, June 10: 9:00am –11:00am
Friday, June 11: 9:00am –11:00am
and
Friday, June 18: 9:00am-12:00pm

Two Credits: All of the above plus:
Monday, June 14: 9:00am-12:00pm
Tuesday, June 15: 9:00am –12:00am
Wednesday, June 16: 9:00am –12:00am
Thursday, June 17: 9:00am –12:00am

Location: Week One: Eagle Crest Conference Center, Ypsilanti, MI
Week Two: Room 4212, School of Education, University of Michigan

Class email list: SummerMathClass@umich.edu

I will use email extensively to communicate with you; I encourage you to do the same with me, and with others in the class. Please check your email regularly.

Note: To make the management of email easier and more reliable, attachments should be titled with a conventional label, e.g.: <yournname_assign1.doc> or <yourname_midterm.doc>.

Course website: http://www-personal.umich.edu/~dball/summermath.html

I will use our course website to post assignments and documents from class. On it you will also find links to some interesting mathematics teaching resources. Be sure to let me know if internet access will be a problem for you during the course.

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Course Focus and Goals

This is a mathematics course designed specially for people who seek to become teachers of mathematics. Teaching mathematics requires a great deal of specialized mathematical knowledge—knowledge that is different from what it takes to do well in a math course as a student yourself or to be good at other jobs that require mathematics. Your own understanding, fluency, and comfort with mathematics will be important to your effectiveness as a teacher. In preparing to teach, you will need to determine the mathematical goals of activities, anticipate the varied ways students might respond, and prepare mathematically for what might happen as a lesson unfolds. You will need to prepare good questions to ask. You will need to generate easier as well as harder versions of problems, either as back-up plans or as ways to focus or extend students' work. You will need a keen sense of the complexity of particular mathematical ideas, and ways they can be scaffolded for students' learning. When your students have trouble, or get answers wrong, you will need to do more than know that they are confused, or that they have incorrect solutions: You will need to be able to figure out what they are doing mathematically, and whether it makes sense.

One thing to notice about all these examples in the previous paragraph is that they are all instances of mathematical thinking that teachers have to do in the course of their work. They are not all things that other adults have to be able to do with mathematics. This is what I mean when I say that teaching requires *specialized mathematical knowledge*, and the course is designed to help you revisit and extend your own mathematical knowledge and build it into this specialized kind of knowledge you will need as a teacher.

We will work in this course on developing mathematics knowledge that is useful for teaching. In particular, we will focus on *meanings and representations of fractions*. We will explore a variety of fraction representations/interpretations (e.g., fractions as parts of wholes, as portions of discrete sets, as division, as points on a number line), and use these representations to solve problems and to explain and justify solutions. Sometimes a fraction represents a number, but often fractions represent a relationship between two quantities. All of these meanings are useful and important, and we will make an explicit attempt to learn about them in ways that connect to and make them usable in teaching.

Attention to *mathematical practices* will cut across all of our work. Particular emphasis will be placed on *explanation, representation and recording,* and *language*:

> Explanation: *Explanation* refers to the forms of expression that justify mathematically and help others understand why a mathematical claim is true. We will explore central features of an adequate mathematical explanation; purposes of explanation; the social nature of explanation (difference between explaining to oneself and explaining to others); providing adequate explanations; listening to and appraising other's explanations.
>
> Representation and recording: Choosing and using representations (verbal, symbolic, visual, material, manipulative, technological); examining correspondences and equivalences among representations; attending to and making sense of representations different from one's own
>
> Language: Naming, defining, precision, ambiguity, shared terms

These practices were selected because of their centrality in learning and doing mathematics, as well as their crucial role in teaching mathematics. Teachers must explain mathematics, and must size up students' explanations. Classrooms are settings where mathematical ideas are constantly being represented and recorded, in public and in individual work. And since talk is the medium of teaching and learning in any view of teaching, teachers' care with language matters to their

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clarity, their capacity to help students understand, and their ability to consider seriously how to manage the connections and conflicts between everyday and mathematical language.

Course Requirements

Attendance and class participation: Your participation in our class activities and discussions is important not only for your own learning but also the learning of others. Sharing your solutions, undeveloped ideas, and puzzlements with the group, as well as responding to classmates' ideas, statements, and questions, are all critical to our work together. As a teacher, you need to do more than understand your own thinking, or your own solutions to math problems—you have to be able to track on others' thinking, figure out what others are saying, and determine whether and how they make sense mathematically. In our class, the "others" will be your classmates. But in the future, they will be your students. So listening to and interacting with others in our class is explicitly to help you develop mathematical dispositions and skills that matter for teaching.

I expect you to attend every class, to arrive on time for a prompt start, and to participate in and contribute to class. If circumstances prevent you from attending class, please send me an email or call in advance.

Notebook: Another form of participation in the course will be to keep mathematical and other drawings, notes, and reflections in a special mathematics notebook. The work you do in your notebook is a central part of the course. You will use the notebook to create a record of your own work and thinking, as well as our class' accumulating understandings, investigations, conjectures, arguments, and solutions. Your notebook will be a place for you to track and record your thinking about issues and ideas. It will also provide an additional avenue of communication between you and me. You will use your notebook to record the work we do in class including solving mathematics problems and analyzing the work of teaching.

This notebook may not be like other journals you have kept in the past. Think of it like a sort of laboratory notebook where one keeps systematic records of experiments, or the notebooks that photographers keep about lighting, lenses, and settings. Bird-watchers keep notebooks; so do cooks and architects. These records all help these professionals keep track of, study, and improve their practice. You are developing a notebook to record and support your learning as a teacher of mathematics.

I will look at and respond to the work in your notebook, and will provide suggestions for amplifying your use of this medium for your own learning so that you get as much as possible out of the experience. My overall evaluation of your work in your notebook will take into account your progress over the course in learning to make a professional record.

Below are some specific qualities that distinguish work in notebooks. A notebook that is consistently thin or missing more than a very small amount of what is expected will be considered unsatisfactory.

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	Satisfactory	Exceptional	Needs to improve
Records of in-class work	• Organized to keep track of your own reactions and thoughts, solutions and experiments • Includes all entries • Records of representations and examples	• Clear records of others' thinking and work, solutions, and experiments • Annotations and reflections on specific aspects of the teaching and learning that goes on in the course itself • Organized for easy reference across topics or time (e.g., includes an index, headings, or color-coding of types of entries)	• Insufficiently organized • Difficult to read or understand • Missing some aspect of any entry
Post-activity and post-class reflections	• Comments thoughtfully on class activities, or ideas or experiences	• Comments connect to other experiences • Awareness of own and others' progress • Comparing own thoughts over time • Reflection both on learning as a student, and insights as a beginning teacher • Pursues a new insight or puzzles about a question or an idea in a new way	• Missing some elements of reflections Thin reflections

Daily Assignments: You will have assignments for each class. These will include mathematical problems that help you develop mathematical knowledge for teaching, as well as reflections on or extensions of the work we have done in class. You are encouraged to work together on assignments; however each person should turn in an individual write-up. Assignments will not be completed in your notebook, but you are encouraged to paste your assignments in your notebook after you have gotten them back so that your notebook is a good record of your thinking and your work.

I will give you regular feedback on your assignments and on the quality of your work. My goal is to help you make good use of this course's opportunities to learn mathematics for teaching.

Examination: There will be an in-class examination on Friday, June 11. Consistent with the course goals of developing your mathematical knowledge for teaching, this exam will ask you to demonstrate your performance in the main areas of the course. The tasks and problems (written and oral) you will be asked to do will be consistent with those you encountered during the week. You will be able to prepare in advance for the performances that the exam will ask of you, working with others if you choose. You may use your notebook, and other preparation, during the exam.

Application Project: Between Friday, June 11, and Friday, June 18, you will do a project involving some common problems and typical tasks of teaching mathematics. This activity will provide an important opportunity to apply the mathematics of this course to the work you will be learning to do as teachers. You will be able to work together or alone. On Friday, June 18,

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you will share the results of this professional work. Details will be provided at the end of the first week of class.

Course Materials

All materials needed for the course, including your notebook, will be provided. I will also have colored pencils, fine-line markers, scissors, glue sticks, and rulers available for work in your notebook; however, you may wish to bring anything you especially like to use.

Grading and Evaluation

The members of our class are at various stages in their professional careers. Some of you are undergraduates, currently enrolled in a teacher education program; some are new graduate students, about to begin an intensive one-year course of study in teacher preparation; others have recently graduated, soon to become first-year teachers. As a result, there are different circumstances under which you are all enrolled (e.g., course numbers, credit hours, etc.). Your enrollment status, however, has no bearing on the requirements and expectations for this course.

Everyone is expected to complete all course requirements. The effort, detail, and thoughtfulness you put into your work should reflect the standards of performance you will be expected to meet as a teacher:

- meticulous preparation,
- use and application of mathematical knowledge,
- careful consideration of alternatives,
- genuine curiosity about ideas (including those of others),
- collegial work,
- analysis and reflectiveness,
- clear expression, with respect for the place and value of precision;
- organization,
- timeliness.

Participating in this course is a unique professional opportunity. I will be happy to write you a professional letter for your teaching portfolio upon satisfactory completion of the course.

Your final grade will be composed as follows based on your performance of each of the course requirements. Specific details about grading criteria will accompany individual assignments and the exam.

Attendance and participation in class	20%
Notebook	20%
Daily assignments	20%
Examination	20%
Application project	20%

The Summer Institute

This course is the central component of the Summer Institute, an eight-day professional development program for teacher developers. These professionals work with preservice or

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practicing teachers in a variety of settings—from university mathematics departments to school districts. The focus of their work is mathematics.

The Summer Institute is designed to help teacher developers investigate ways to help teachers learn mathematical knowledge for teaching. Most of the institute participants already have extensive experience working with teachers. This institute offers them opportunities to learn new ideas about the mathematics that teachers need to know and ways to help teachers learn it. In addition, because professional development for teacher developers is not typical, for many of them this may be a new experience. They are most often the people responsible for providing professional development, not the ones participating in it.

Institute participants will be observing our class, and the class will also be videotaped. To show our appreciation for your participation, you will receive a $150 stipend at the end of the course. We will have a chance to meet without the observers for the first portion of our class on Sunday (from 1:30 – 2:00), and I will provide more information about the institute and answer any questions you may have.

The Summer Institute is an activity of the Center for Proficiency in Teaching Mathematics, jointly conducted by the University of Georgia and the University of Michigan. Faculty and graduate students from both universities are working together to design, conduct, and learn from the Summer Institute.

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