Pricing Congestible Network Resources

by

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1. Introduction

Motivation

Questions Addressed

Relation to club literature

2. Model Features

x_i is i's usage, $X = \sum_i x_i$ is total usage

Users care about usage and delay (Y):

$$U_i = u_i(x_i, Y) = m_i$$

Delay depends on utilization: $Y = X/K$

Only cost of service is cK

3. Efficient Use and Capacity

Review basic welfare economics of congestion externality

Given K, what is efficient usage pattern?

$$W(K) = \max_{x_j} \sum_{j=1}^{n} u_j(x_j, Y) - c(K)$$

Optimal solution:

$$\frac{\partial u_i(x_i, Y)}{\partial x_i} - \frac{1}{K} \sum_{j=1}^{n} \frac{\partial u_j(x_j, Y)}{\partial x_j} \qquad (1)$$

Decentralize with "shadow price":

$$p_e = \frac{1}{K} \sum_{j=1}^{n} \frac{\partial u_j(x_j, Y)}{\partial Y}$$

p_e is the marginal congestion cost of x_i

With many users, consumer optimization against p_e yields social optimum

Capacity

$W'(K) > 0$ if and only if $p_e X - c'(K) K > 0$

Hence, p_e efficiently allocates and sends right signal for capacity expansion

4. Competitive Pricing: Two-Part Tariffs

Two-part tariff: many possible suppliers (Y)

connection fee: $q\ Y$

usage fee: $p\ Y$

special cases: pure connection and pure usage pricing

Consumers

$$\max_{x_i, Y} u_i\ x_i, Y \quad p\ Y\ x_i \quad q\ Y$$

Producers

$$\max_{X, K} p\ Y\ X \quad nq\ Y \quad c\ K$$

First-order conditions (using consumer FOC):

$$\frac{@u_i\ x_i, Y}{@x_i} \qquad \frac{1}{K} \sum_{j\ 1}^{n} \frac{@u_j\ x_j, Y}{@Y} \qquad 2$$

$$Y \sum_{j=1}^{n} \frac{\partial u_j(x_j, Y)}{\partial Y} \quad c'(K) \quad K \qquad (3)$$

Compare (2) to (1): social optimum for usage

Combining (2) and (3): $p \quad Y \quad X \quad c'(K) \quad K$ (efficient investment)

Competitive market price is equal to sum of congestion costs: rhs of (2) is aggregate wtp for producer to sell less

Entry

Zero profit condition:

$$\frac{n q(Y)}{c(K)} \quad 1 \quad \frac{c'(K)}{c(K)/K} \quad 1 \quad \frac{1}{e},$$

With increasing returns: elasticity < 1

When MC small, most cost covered by subscription fee

How many qualities offered?

some users might prefer lower quality (more delay) in exchange for lower price

cost conditions and network externalities determine equilibrium variety

trade-off between consumer gains and costs of provision

if $c_1 K_1 \quad K_2 \quad c_1 K_1 \quad c_2 K_2$ then see as many varieties as customer types

Customer heterogeneity

Different preferences for delay at different times (or for different uses)

If consumer set-up costs small: "restaurant equilibrium"

Otherwise, demand for multiple service qualities from single producer

Increases complexity of pricing problem: multiple dimensions of congestion and resource cost

Capacity expansion

Case 1: Keep X fixed, charge more for reduced delay

Change in profits:

$$\left[q'(Y) - p'(Y)\, x_j \frac{dY}{dK} \right] \tilde{N}K - c'(K)\, \tilde{N}K$$

After manipulation:

$$p\left(\frac{X}{K}\right) - c'(K) - \tilde{N}K > 0$$

which is social optimum condition

Likewise if price fixed, X increases due to lower delay

5. Competitive Pricing: No Usage Fees

Why?

usage pricing expensive to implement

politics

Equilibrium

Special case:

$$u_i(x_i, Y) = v_i(x_i) - D(Y)$$

where $D(Y)$ are convex "delay costs"

FOC with usage prices:

$$v_i'(x_i^e) = \frac{n}{K^e} D'(Y^e)$$

$$nD'(Y^e)Y^e = c'(K^e)K^e$$

FOC with attachment-only prices (satiation at $x^a; X^a$)

$$nD'(Y^a)Y^a = c'(K^a)K^a$$

Same form but different equilibrium n and/or Y

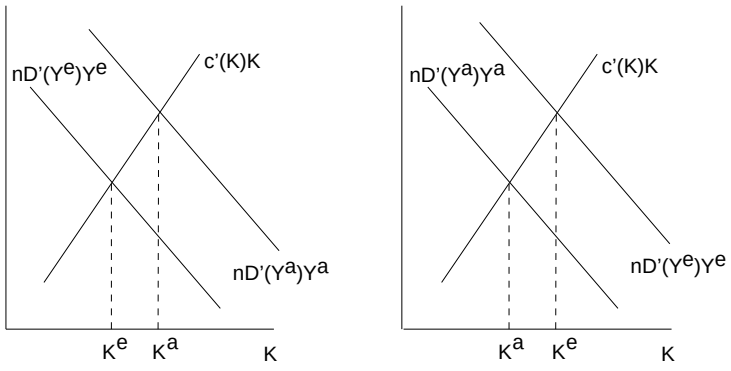


Figure 1. Determination of equilibrium capacity.

Same number of users (Figure 1)

lhs decreasing in K by convexity of D Y

rhs increasing if c^{00} K not too negative

With zero p, each consumer uses more: shifts D^{0} Y up (higher marginal congestion costs)

So, get higher capacity

But also get higher congestion (Figure 2)

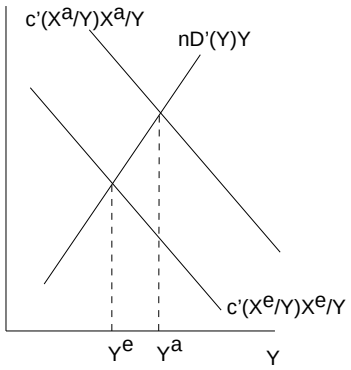


Figure 2. Determination of equilibrium congestion.

Number of users changes

given an alternative, user leaves network if

$$v_i x_i^a \quad u_i \quad D\ X^a=K^a \quad q\ Y^a$$

i.e., if net benefit from high usage is less than congestion cost (including access fee)

reducing n yields equilibrium shift down in $nD^0 Y\ Y$ curve (figure 1)

get lower capacity

"YogiBerra" equilibrium

small number of heavy users with high delay tolerance (low wtp for capacity)

high value users exit to alternative service

6. Monopoly

With usage pricing only: lower output, higher price, lower capacity

Two-part tariff: Generalizes Oi's model

Identical tastes: first best

set q to surplus

set p to efficient congestion price

Heterogeneity

Index users by t, u x; Y; t decreasing in t

T is marginal user

FOC:

$$\frac{p \quad c'(K=Y)}{p} \quad 1 \quad \frac{x(p;T)}{X(p;T)=F(T)}$$

Last term: marginal to average demand

price can be above or below marginal congestion cost

If consumer with lower total value (T) consumes less than consumer with higher total value: $p > c^0 K$

But, if T consumes more, $p < c^0 K$

"auto salesman equilibrium": lose money on every sale but make up for it on volume!

Figure 3: email vs. MTV

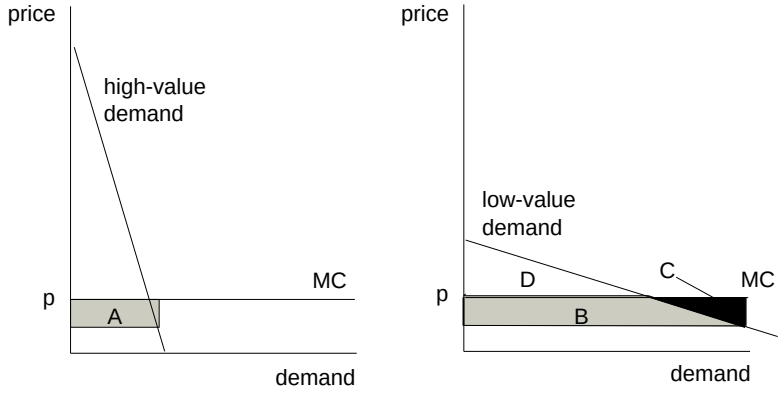


Figure 3. Pricing less than marginal cost may be optimal.

quite plausible in network applications: relatively low-value services can require huge amount of bandwidth

monopolist may underprice their congestion to get revenues – imposes congestion costs on high-value users

7. Concluding Remarks

A competitive market with two-part pricing can maximize social welfare

This assumes entry and sufficient variety: costs subadditive

Monopolist may price usage above or below efficient level

Attachment-only pricing: lower usage, higher congestion (Yogi Berra)