Generalized Vickrey Auction Mechanism

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Generalized Vickrey Auction (MacKie-Mason and Varian 1994)

- Problem:

- Vickrey: Award good to highest bidder, at second highest price

  1. truthtelling is a dominant strategy

  2. bid doesn't affect price paid; truthful bid wins whenever true value $>$ price

- Example

  1. Goal: give good to max$[v_1, v_2]$

  2. Suppose agent 1 announces truth: $b_1 = v_1$

  3. Then agent 1 wins when payoff positive, $v_1 - b_2 > 0$, never when negative

  4. So, dominant strategy

- Generalized Vickrey:

  1. multiple units

  2. multiple goods with dependent demands

  3. goods with externalities (agents care how much others consume)

Intuition for Generalized Vickrey Auction

- To induce truthtelling, give each participant her true utility plus all social surplus of the other agents measured using their reported utility functions

- i is getting max[r_i + payment], cares about u_i + payment, so, report $r_i = u_i$

- whatever others report, getting max of what you value, so truthtelling is dominant

- But, giving *each* agent *total surplus* is very costly

- Can require a payment if it depends only on reports made by *other* agents (the "second price" in Vickrey)

Formal Statement (Pure Exchange)

- Social objective function (for allocation x)

$$\max \sum_a u_a(x)$$

1. each user p reports utility fcn, $r_a(x)$

2. auction manager computes candidate social welfare max, $r(\cdot)$ as true,

$$x^* = \arg\max \sum_a r_a(x)$$

subject to any constraints

$$F(x) = 0$$

an assigns action x_a^* to agent $a = 1, \ldots, A$. Then compute

$$W_{-a}(x^*) = \sum_{b \neq a} r_b(x^*)$$

3. Agent a receives payoff

$$u_a(x^*) + [W_{-a}(x^*) - G_a(r_{-a})],$$

where $G_a(r_{-a})$ is an arbitrary function of other agents' reported utilities, independent of the effects of agent a's report

4. A convenient $G_a(\cdot)$ is

$$G_a(r_{-a}) = \max_a \sum_{b \neq a} r_b(0, x_{-a})$$

subject to

$$F(0, x_{-a}) = 0$$

Interpreting the Payment

1. $[W_{-a} - G_a]$ is the net increase (decrease) in welfare of all others created by user a's participation

   - a receives own direct utility plus her net effect on social welfare of others

   - e.g., if she creates incremental congestion, she pays precisely the congestion cost

   - since a gets all of the incremental surplus, wants to truthfully reveal her preferences

2. Note: Computationally intensive. The GVA mechanism requires solution of the social welfare optimization problem once, but the payment function requires another solution for each participant a

Example: Multiple Units

- 3 units
- Agent 1 wtp: (10,8,6)
- Agent 2 wtp: (9,7,5)
- Efficient allocation: 2 units to #1, 1 unit to #2
- GVA:
  1. agent #1 pays $7 + 5 = 12$ (surplus that would be generated if he didn't participate)
  2. agent #2 pays 6

- Slightly different than $M+1$st auction ($p = 7$)

- If #2 lies, says second unit worth $8+\epsilon$, then gets $9+7-(8+6) = 2$ instead of $9-6 = 3$