Auction Protocols for Decentralized Scheduling

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“Markets ain’t what they used to be”

- Economics
  - existing instantiations perform well
  - failures:
    - incompleteness
    - externalities
  - solution:
    - marketize everything!
- Engineering
  - initial instances performed well
  - Generalized Vickrey Auctions: efficient outcomes even with externalities

But...

- Design space is huge
- The set of existing instantiations -- real and artificial -- has measure zero
- Failures
  - indivisibilities
  - complementarities
  - transaction costs
  - asymmetric information and strategic behavior

Creating new market spaces for advanced commerce

- Need to systematically map more of the design space
- Standard design loop:
  - construct market mechanisms for classes of allocation problems
  - analyze their performance
  - design better mechanisms
- MECHANISMS FOR ADVANCED COMMERCE ARE IN THEIR INFANCY

Mechanism Design is "Hard"

- Typical design goals:
  - Pareto efficiency: No feasible allocation benefits at least one agent without harming at least one other agent
  - Participatory efficiency: willingness; budget balance
  - Agent strategies are plausible
- Generally cannot satisfy all three of these for large regions of design space

Dominant agent strategies

- Gibbard-Satterthwaite (1973)
  - If agent preferences are ordinal but otherwise unrestricted, and
  - If there are at least three outcomes that are each sometimes socially desirable
  - THEN only dictatorial mechanisms can implement Pareto efficiency in dominant strategies

Bayesian-Nash Strategies

- Defn: Agents play strategies that provide best expected payoff if other agents also play Bayesian-Nash strategies
- Myerson-Satterthwaite (1980):
  - If agent valuations drawn from overlapping intervals
  - And trade is bilateral
  - THEN NO mechanism can implement both Pareto and participatory efficiency

Mechanism Design Search

- Which subsets of goals can we satisfy under what conditions?
- E.g.: bilateral trade can be implemented with ex post voluntary participation if the market maker tolerates a deficit (GVA)
- E.g., Commit-in-advance participation can be implemented with entry (subscription) fees, but with lowered efficiency

Scheduling Problems

- Autonomous agents with:
  - conflicting scheduling requirements
  - private local information
  - messaging capability
- Scheduling optimization is hard:
  - discrete
  - complementarities
  - even with public information it's typically a knapsack problem

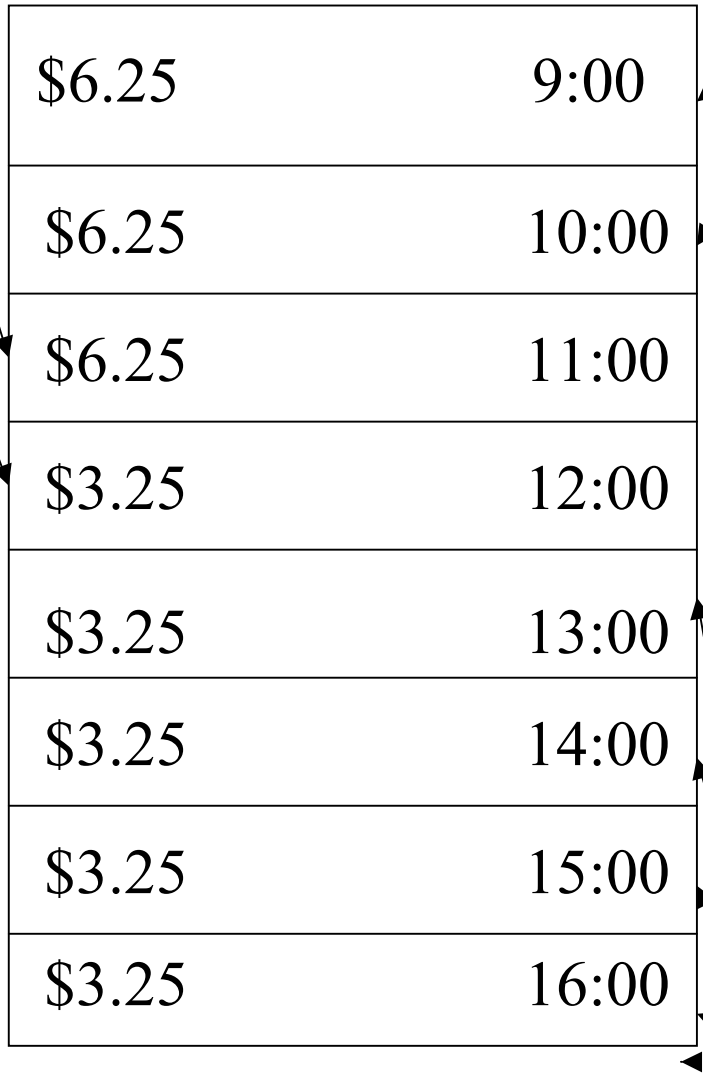
Our challenges

- Protocol design:
  - how to organize message passing, reach closure, determine outcome?
- Economic design:
  - how to elicit messages containing info needed for good outcomes?

Example

A1
v=$10
L=2 hrs
by 12:00

A2
v=$16
L=2 hrs
by 11:00



A4
v=$6
L=1 hrs
by 11:00

A3
v=$14.5
L=4 hrs
by 16:00

Min price = $3

Markets can be good

- Theorem 1: If there exists a **p** such that *f* is a market equilibrium, then *f* is Pareto optimal
- But mkt equilib may not exist

	Length	Deadline	Value
Agent 1	2	2	3
Agent 2	1	2	2

What do we know in general?

- Lemma: The single unit problem always has a unique minimum equilibrium price vector
- Theorem 3: Any optimal solution to a single unit scheduling problem is supported by a price equilibrium
- But now: How to *find* good solutions even for single unit problems?

1. Ascending Single-Good Auction

- **Goods**: 1 auction for each slot
- **Rules**:
  - min. bid increment ε
  - no bid withdrawal
  - closure when bidding stops
- **Strategies**:
  - agent j bids for set of slots to max surplus at current p
  - or drop out if no set of slots has positive surplus
  - N.B. For single-slot problem, this is dominant strategy
  - N.B. For multi-slot, not regret proof

ASGA: Single-slot problems

- Thm 3 says equilib exists, but this protocol not guaranteed to find it

	1pm	2pm	3pm
A1	6	6	
A2	7	7	7
Min	4	3	3

A2 first: could bid b2=3
A1 then could bid b1=4: s1->A1, s2->A2, v=16
But at p2=3 A1 prefers s2 so not an equilib

ASGA single-slot(cont)

- **Theorem 5**: p_j will differ from the min. unique equilib price by at most $\kappa\varepsilon$, where $\kappa = \min(n,m)$
- **Theorem 6**: *v(f)* will differ from optimal by at most $\kappa\varepsilon(1+\kappa)$

- Single-slot demand summary:
  - equilib exists
  - ASGA deviate from optimal by bounded amount, proportional to ε

AGSA: Multi-Slot Demands

- (b.1) p can differ from equilib by arbitrarily large amount
- (b.2) *v(f)* can differ from optimal by arbitrarily large amount

	Length	Deadline	Value
Agent 1	1	1	3
Agent 2	2	2	11

Min prices: (1,9)
Suppose A2 bids: b1=1, b2=9
Suppose A1 bids: b1=2
Then: s1->A1, s2->A2, v(f)=3
But optimum: s1->A1, s2->⊥, v(f)=12

Example cont.

- Can make inefficiency arbitrarily large
- Is it reasonable for agent 2 to stop bidding?
  - By bidding b1=3, it can do better than if auction stopped
  - However, better still is someone else bids away s2
  - No dominant strategy exists that makes A2 always better off

2. Combinatorial Auctions

- Looking for price equilibria for multi-slot problems
- Redefine concerns: combinations of goods
  - Banks et al. RAND 1989
  - Rassenti et al. Bell 1982
  - Rothkopt et al. Mgmt Sci 1998
- **Problem**: Exponential complexity (2^n combinations)
- Need feasible mechanisms that work “pretty well”

Combinatorial Auctions (cont.)

- New goods definition: pair(y,z) is y slots by deadline z, contains z
- Complexity = O(nL), L = max schedule length
- Finds equilibria for some problems not solved by ascending auction:

	Length	Deadline	Value
Agent 1	2	2	3
Agent 2	1	2	2

Consider: p(1,1)=p(1,2)=2.1, p(2,2)=2.9
A1 gets (2,2), A2 gets nothing
A1 can't do better with single goods
A2 doesn't want anything at these prices

Combinatorial Auctions Performance

- But CA price equilibria are not necessarily efficient

	Length	Deadline	Value
Agent 1	2	2	3
Agent 2	1	2	2
Agent 3	1	2	2

Equilib: p(1,1)=p(1,2)=1.6, p(2,2)>=3.2
Another: p(1,1)=p(2,1)=2.1, p(2,2)=2.9
(same as previous example)
Second is suboptimal -- no useful bound on deviation from optimality!

Combinatorial Auction: Summary

- All single-good price equilibria are also (not nec. unique) combinatorial equilibria
- Some problems with no single-good price equilibria **do** have combinatorial equilibria
- But, combinatorial equilibria (that are not single-good eqa.) are not necessarily efficient

Generalized Vickrey Auction

- Varian and MacKie-Mason (1994)
- **Goods** definition: Identical to agent concerns
- **Rules**: Agents announce $r_j()$, alleged utility function
  - GVA computes optimal allocation for the r()
  - GVA computes payments to agents $P_j(r_{-j})$

GVA properties

- Pareto efficient
- If a price equilibrium exists, GVA payments are no higher than payments under minimum equilibrium prices (Gul & Stacchetti)
- If minimum reserve prices known, GVA for scheduling is participatory efficient in dominant strategies
  - If not, participatory efficient for M-1 agents (market maker may run deficit)

GVA computation cost

- Single-slot optimal solutions can be found by a simple centralized greedy algorithm (Neapolitan & Naimipour) in $\Theta(m \log m)$
- GVA needs to solve m+1 of these: one for global optimum, plus one for each of agent payments, so $\Theta(m^2 \log m)$
- Same m-factor increase for multi-slot, but the underlying optimization is now NP-complete Integer Knapsack

SUMMARY

- We have explored a spectrum of mechanisms for scheduling problems:
- single good <-> combi <-> GVA
- increasing ---------->
  - scope of concerns
  - cases in which (near) efficient outcome found
  - computational complexity
- Applying to multi-stage network scheduling problems
- Proposed: “metamechanisms”