Auction Protocols for Decentralized Scheduling

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Coordinates

- My research: http://www-personal.umich.edu/~jmm/
- Program for Research on the Information Economy http://www.si.umich.edu/~prie/
- "Auction Protocols for Decentralized Scheduling" (with Wellman, Walsh and Wurman). *Games and Economic Behavior* (forthcoming 2001).

Overview

- Designing market-like mechanisms for complex resource allocations
  - Mechanism design
  - Scheduling problems
  - Methodology for protocol analysis

I. Markets are not a panacea

- Design space is huge
- The set of existing instantiations -- real and artificial -- has measure zero
- Failures
  - indivisibilities
  - complementarities
  - transaction costs
  - asymmetric information and strategic behavior

Creating new market spaces: Mechanism design

- Need to systematically map more of the design space
- Standard design loop:
  - construct market mechanisms for classes of allocation problems
  - analyze their performance
  - design better mechanisms

Mechanism Design is "Hard"

- Typical design goals:
  - Pareto efficiency: No feasible re-allocation benefits at least one agent without harming at least one other agent
  - Participatory efficiency: willingness; budget balance
  - Agent strategies are plausible
- Generally **cannot satisfy all three** for large regions of design space

Under dominant agent strategies?

- Gibbard-Satterthwaite (1973)
  - If agent preferences are ordinal but otherwise unrestricted, and
  - If there are at least three outcomes that are each sometimes socially desirable
  - THEN only dictatorial mechanisms can implement Pareto efficiency in dominant strategies

Under Bayesian-Nash strategies?

- Defn: Agents play strategies that provide best expected payoff if other agents also play Bayesian-Nash strategies
- Myerson-Satterthwaite (1980):
  - If agent valuations drawn from overlapping intervals
  - And trade is bilateral
  - THEN NO mechanism can implement both Pareto and participatory efficiency

Mechanism Design Search

- Which subsets of goals can we satisfy under what conditions?
- E.g.: bilateral trade can be implemented with ex post voluntary participation if the market maker tolerates a deficit (GVA)
- E.g., Commit-in-advance participation can be implemented with entry (subscription) fees, but with lowered efficiency

II. Scheduling problems

- Require coordination of multiple agents
- Require allocation of scarce resources among competing uses
- Must respect autonomous agent interests and private information
- For these reasons: We are interested in decentralized mechanisms

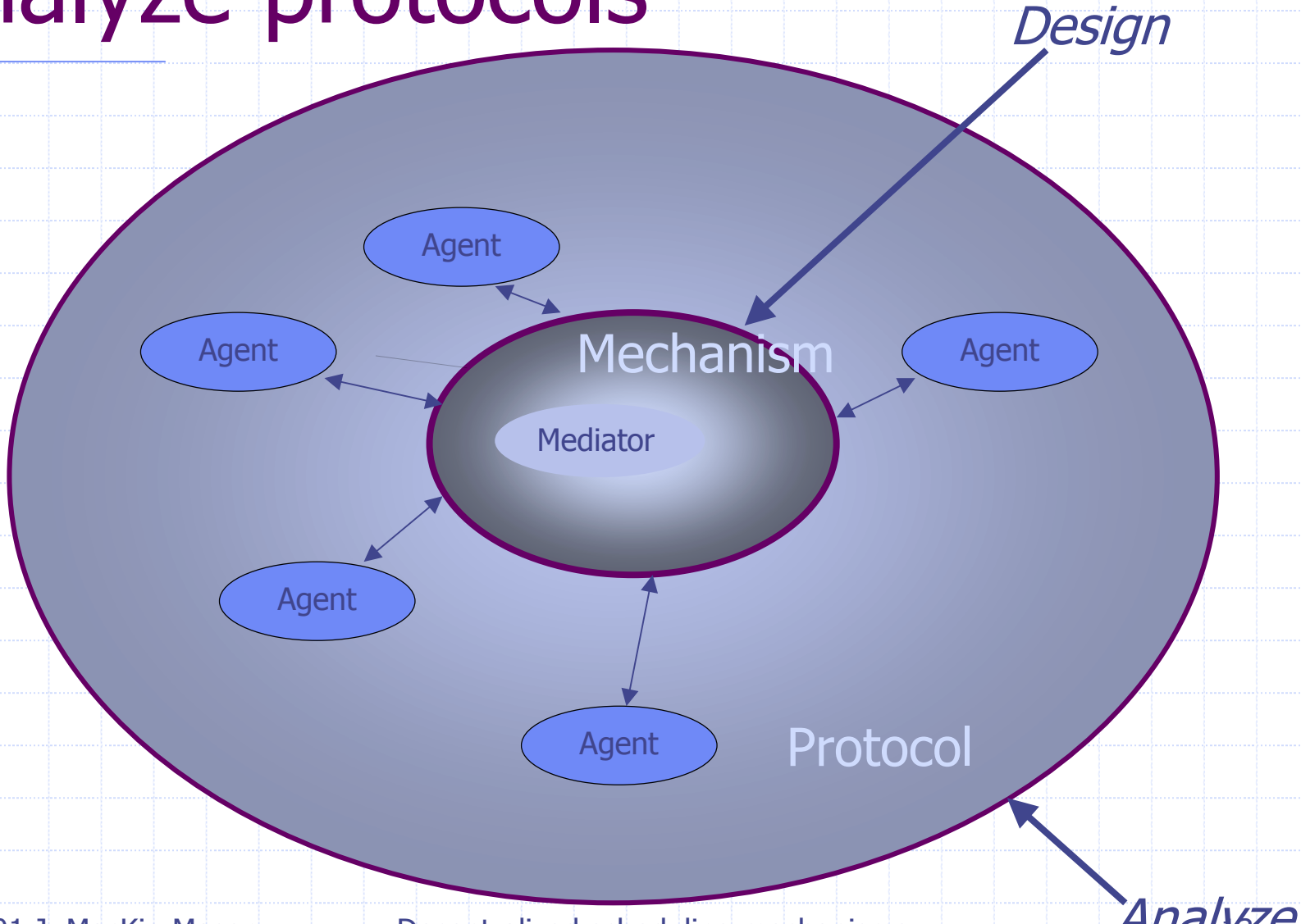
Agents

- **Autonomy** over own decisions
- **Attitudes**: preferences, beliefs, intentions
- **Rationality**: Behavior will be consistent with beliefs and preferences in some way
- Possess **private information**

Mechanisms and Protocols

- Mechanisms
  - Comprise the rules of a social choice process
  - As a function of messages received from participating agents, specify:
    - information revealed
    - outcomes (resource allocations)
  - Mediated if some entity distinct from the participants manages the communication and implements the rules
- Protocols
  - Mechanism plus specification of agent strategies for interacting with it are a complete protocol for a mediator
    - Mediator specification must respect agent's private information, and thus cannot define agent-specific behavior

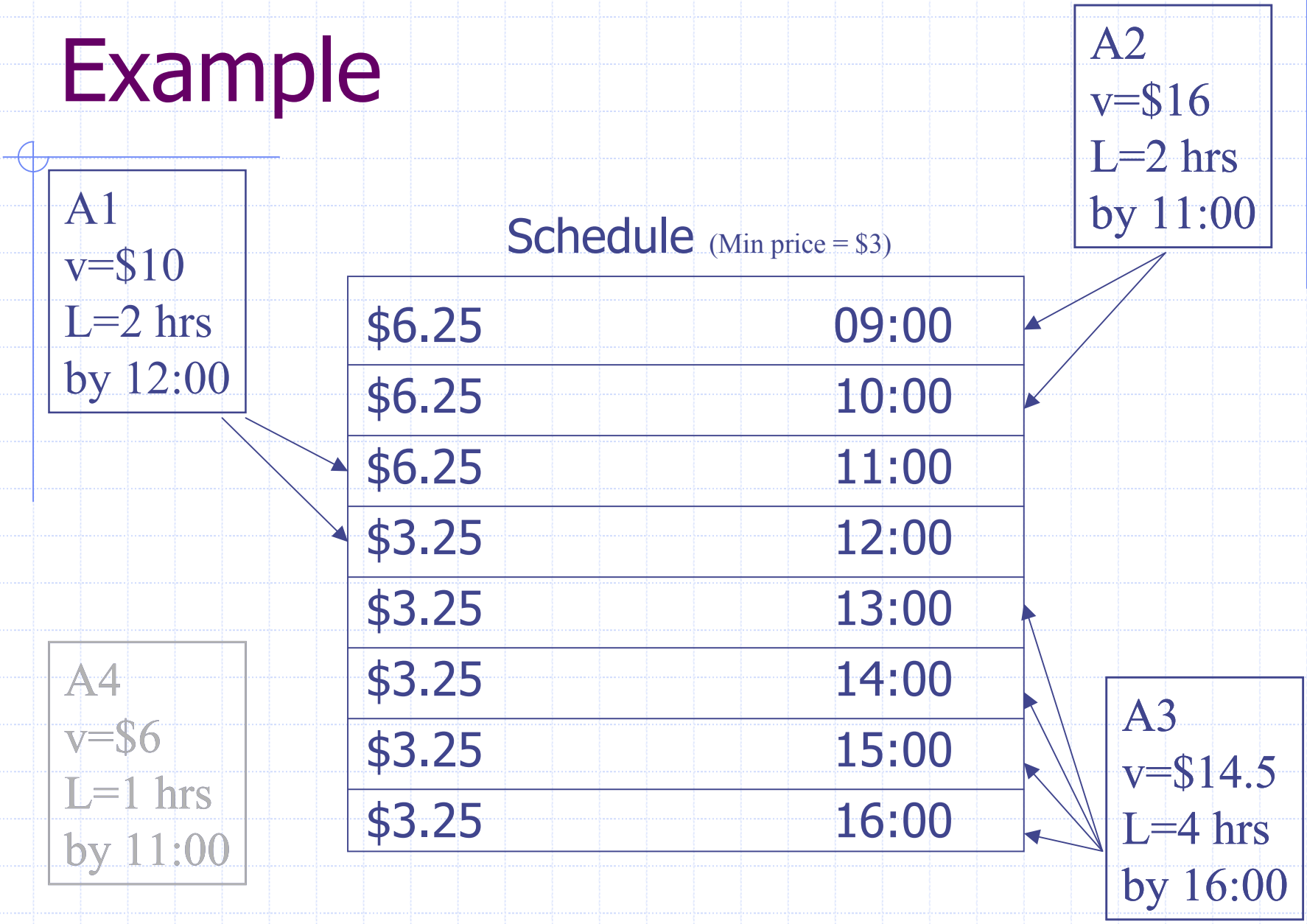
Design mechanisms, Analyze protocols



Scheduling is "Hard"

- Harder than basic allocation problems
  - Time-dependency transforms single into multiple-resource problems
  - Agent valuations may have dependencies across time
    - Deadlines
    - Value for consecutive time slots
    - Nonseparable value for multiple slots
    - Interactions between different uses
      - Example: Michael can attend meetings at 2 and 3 if in same building; if across campus needs a 15 minute break)
- These complex constraints typically create non-convex optimizations that are NP-hard

Example


A1
v=$10
L=2 hrs
by 12:00
A2
v=$16
L=2 hrs
by 11:00
Schedule (Min price = $3)
$6.25 09:00
$6.25 10:00
$6.25 11:00
$3.25 12:00
$3.25 13:00
$3.25 14:00
$3.25 15:00
$3.25 16:00
A4
v=$6
L=1 hrs
by 11:00
A3
v=$14.5
L=4 hrs
by 16:00

Markets can be good

- Theorem 1: If there exists a **p** such that *f* is a market equilibrium, then *f* is Pareto optimal
- But mkt equilib may not exist

	Length	Deadline	Value
Agent 1	2	2	3
Agent 2	1	2	2

What do we know in general?

- Lemma: The single unit problem always has a unique minimum equilibrium price vector
- Theorem 3: Any optimal solution to a single unit scheduling problem is supported by a price equilibrium
- But now: How to ***find*** good solutions even for single unit problems?

1. Ascending Single-Good Auction

- **Goods**: 1 auction for each slot
- **Rules**:
  - min. bid increment ε
  - no bid withdrawal
  - closure when bidding stops
- **Strategies**:
  - agent j bids for set of slots to max surplus at current p
  - or drop out if no set of slots has positive surplus
  - N.B. For single-slot problem, this is dominant strategy
  - N.B. For multi-slot, not regret proof

ASGA: Single-slot problems

- Thm 3 says equilib exists, but this protocol not guaranteed to find it

	1pm	2pm	3pm
A1	6	6	
A2	7	7	7
Min	4	3	3

A_2 first: could bid b_2=3
A_1 then could bid b_1=4: $s_1 \rightarrow A_1$, $s_2 \rightarrow A_2$, v=16
But at p_2=3, A_1 prefers s_2 so not an equilib. Also not optimal.

ASGA single-slot (cont)

- **Theorem 5**: p_j will differ from the min. unique equilib price by at most $\kappa\varepsilon$, where $\kappa = \min(n,m)$
- **Theorem 6**: *v(f)* will differ from optimal by at most $\kappa\varepsilon(1+\kappa)$
- Single-slot demand **summary**:
  - equilib exists
  - ASGA deviates from optimal by bounded amount, proportional to ε

AGSA: Multi-Slot Demands

- p can differ from equilib by arbitrarily large amount
- *v(f)* can differ from optimal by arbitrarily large amount

Min prices: (1,9)
Suppose A_2 bids: $b_1=1$, $b_2=9$
Suppose A_1 bids: $b_1=2$
Then: $s_1 \rightarrow A_1$, $s_2 \rightarrow A_2$, $v(f)=3$
But optimum: $s_1 \rightarrow A_1$, $s_2 \rightarrow \perp$, $v(f)=12$

	Length	Deadline	Value
Agent 1	1	1	3
Agent 2	2	2	11

To increase gap: raise $\min(s_2)$ and v_2 in lock-step

Example cont.

- Can make inefficiency arbitrarily large
- N.B. Agent 2's strategy is not optimal: Is it reasonable for agent 2 to stop bidding?
  - By bidding b_1=3, it can do better than if auction stopped (π = (11-12) = -1, vs. π = (0-9) = -9)
  - However, better still if someone else (mediator?) bids away s_2
  - No dominant strategy exists that makes A_2 always better off

2. Combinatorial Auctions

- Looking for price equilibria for multi-slot problems
- Redefine concerns: combinations of goods
  - Banks et al. RAND 1989
  - Rassenti et al. Bell 1982
  - Rothkopf et al. Mgmt Sci 1998
  - Wurman (UM diss.) 2000, Parkes (Penn St diss. 2001)
- **Problem**: Exponential complexity (2^n combinations)
- Need feasible mechanisms that work "pretty well"

Combinatorial Auctions (cont.)

- Our new goods definition: pair(y,z) is y slots by deadline z, contains z
- Complexity = O(nL), L = max schedule length
- Finds equilibria for some problems not solved by ascending SG auction:

	Length	Deadline	Value
Agent 1	2	2	3
Agent 2	1	2	2

Consider: p(1,1)=p(1,2)=2.1, p(2,2)=2.9
A_1 gets (2,2), A_2 gets nothing
A_1 can't do better with single goods
A_2 doesn't want anything at these prices

Combinatorial Auctions Performance

- But CA price equilibria are not necessarily efficient

	Length	Deadline	Value
Agent 1	2	2	3
Agent 2	1	2	2
Agent 3	1	2	2

Equilib: p(1,1)=p(1,2)=1.6, p(2,2)>=3.2
$s_1 \rightarrow A_2$, $s_2 \rightarrow A_3$, v(f)=4
Another: p(1,1)=p(2,1)=2.1, p(2,2)=2.9
v(f)=3 (same as previous example)
Second is suboptimal -- no useful bound on deviation from optimality!

Combinatorial Auction: Summary

- All single-good price equilibria are also (not nec. unique) combinatorial equilibria
- Some problems with no single-good price equilibria **do** have combinatorial equilibria
- But, combinatorial equilibria (that are not single-good eqa.) are not necessarily efficient

Generalized Vickrey Auction (MacKie-Mason and Varian 1994)

- Agent α has utility $u_\alpha(x)$, asserts (reports) that it is $r_\alpha(x)$
- mechanism computes $x^* = \text{argmax} \, \Sigma_\alpha \, r_\alpha(x)$, assigns x^* to agents
- Compute:

$$W_{-\alpha}(x) = \sum_{b \neq a} r_b(x^*)$$

$$G_\alpha(r_{-\alpha}) = \max_x \sum_{b \neq a} r_b(0, x_{-\alpha})$$

- Agent α receives payoff: $u_\alpha(x^*) - [G_\alpha(r_{-\alpha}) - W_{-\alpha}(x^*)]$, which is utility of optimal allocation less net cost α imposes on other agents by participating

GVA properties

- Pareto efficient
- If a price equilibrium exists, GVA payments are no higher than paymens under minimum equilibrium prices (Gul & Stacchetti)
- If minimum reserve prices known, GVA for scheduling is participatory efficient in dominant strategies
  - If not, participatory efficient for M-1 agents (market maker may run deficit)

GVA computation cost

- Single-slot optimal solutions can be found by a simple centralized greedy algorithm (Neapolitan & Naimipour) in $\Theta(m \log m)$
- GVA needs to solve m+1 of these: one for global optimum, plus one for each of agent payments, so $\Theta(m^2 \log m)$
- Same m-factor increase for multi-slot, but the underlying optimization is now NP-complete Integer Knapsack

SUMMARY

- We have explored a spectrum of mechanisms for scheduling problems:
- single good $\leftrightarrow$ combi $\leftrightarrow$ GVA

  with increasing $\longrightarrow$
  - scope of concerns
  - # of cases in which (near) efficient outcome found
  - computational complexity

III. Protocol Analysis Method: Two problems

- Recall: We can design mechanisms, but we can only analyze protocols
- But we often don't know optimal rational-agent strategies
- Even with known strategies (optimal or otherwise), may not be able to solve analytically for mechanism performance

The unavoidable a priori

- How will the agents interact with the mechanism?
  - For complex allocations Bayesian-Nash rational strategies often cannot be analytically derived
    - esp. with repeated interactions, complementarities, non-convexities
  - Even when dominant strategies known (rare!) may not be fully satisfactory (cognitive biases, bounded rationality)

Example: Strategy complexity

- Recall failure of multiple-slot AGSA:

Min prices: (1,9)
Suppose A_2 bids: b_1=1, b_2=9
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Then: $s_1 \rightarrow A_1$, $s_2 \rightarrow A_2$, v(f)=3
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	Length	Deadline	Value
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- Problem: A_2 got slot s_2 (value 0), when it would be better for mediator to retain s_2 (value 9).
- A_2's strategy problem:
  - Gamble on not getting caught w/only 1 slot or shade bids?
  - If de-commit allowed to "restore" efficiency, what effect on initial bids if don't have to commit?

Approaches

- **Analytic**: For some strategies, analyze efficiency (or other criteria) of mechanism
  - May do this for "NTS" strategies if Bayesian-Nash not known
- **Human subject experiments**: Evaluate performance of mechanisms conditional on actual strategies employed
  - Less rigorous (but more realistic?): tournaments, such as TAC (http://tac.eecs.umich.edu/)
- **Quasi experiments:** E.g., spectrum auctions
- **Simulation:** Even if have optimal strategies, may not be able to obtain analytic results on mechanism performance

Improving protocol analysis with NTS strategies

- We are developing an evolutionary game approach to explore strategy space
  - Posit a set of different NTS strategies, distributed over a population of agents
  - Let the agents compete against each other in a round of allocations
  - Let the population fractions replicate in accord with their relative success
  - Use "genetic" strategies to search strategy space
- Find "best" strategy(ies) and analyze protocol with those

Parameterizing strategy space for combinatorial auction

- Take base "myopically rational" strategy
  - Modifications:
    - Price monotonicity belief
    - Price profile equilibrium belief
    - Aggression
    - Risk management
  - Note: we can discuss characteristics of strategies, not derive these from characteristics of attitudes (preferences) since the problem is too complicated to derive optimal strategies

Price monotonicity belief

- In competitive equilibrium, if deadline values are weakly monotonic, then prices weakly monotonic
  - Useful bidding heuristic even though comp. eqm. may not exist, or mechanism may not converge to it?
  - Switch: Choose slots to bid assuming price for slot i is $\tilde{p}(i) = \max(p(i), \tilde{p}(i+1))$
  - Or bidding: If winning slot i, assume will lose it if $p(i) < \tilde{p}(i)$

Aggression

- Bid level aggression: If bidding on slot *i*, offer increment $\lambda\varepsilon$ rather than ε
  - Let $\lambda \rightarrow 1$ as agent believes it's closer to end of auction?
- Slot aggression: If at current ask prices agent needs total of *S* slots, and not winning on σ slots, bid on $1<s<S-\sigma$

Risk Management

- Without "de-commitment" rule, agents can be left holding slots they can't use (due to inseparabilities)
- Let agent have a threshold such that it stops bidding if worst case loss exceeds that threshold
  - Example:
    - Total value for 3 slots by 5 is 20.
    - Currently winning slot 5 @ p=7; prices on other slots are all 8
    - Let loss management threshold be 15
    - Don't bid 8+ ε on any new slot because worst case loss is > 15

Exploring strategy space

- Once parameterized, we can introduce variations on strategies to current population mix – basically a genetic algorithm search strategy
  - Mutation
  - Sexual reproduction (crossover)

Replicator dynamics

- Basic principle: If strategy type *s* does better than average, then its population fraction increases (phenomic, not genomic)
- Several different functions have been used, with different dynamics
  - Linear (growth rate = relative fitness)
  - Malthusian (proportional to relative fitness)
  - Difference
  - Threshold limited

Summary

- When resource allocation problems are necessarily decentralized, mechanism design theory offers principled approach to design
- Scheduling problems are in the class of hard allocation problems
  - possible to develop families of mechanisms that trade off efficiency for complexity
- Strong limitations on our ability to analyze the performance of complex new mechanisms
  - improved methodolgy is needed
  - evolution games offer one untested possibility