Midterm Exam No. 2 - *Answers*
July 29, 2002

Answer all questions, in blue book. Plan ahead and budget your time. The questions are worth a total of 60 points, as indicated. You will have 80 minutes to complete the exam.

1. [19 points] In the diagram on page 2 are drawn unit-value isoquants for a world that produces three goods, X_1, X_2, and X_3, using two factors, K and L, at free trade prices p_1, p_2, and p_3 measured in dollars. To assist you, I have drawn two straight lines tangent to pairs of the isoquants, showing the points of tangency as open dots, and also drawing the rays from the origin through these points of tangency. Factor endowments are shown for seven countries, represented by the solid dots labeled $E^1,\ldots,E^7$. Answer the following questions about these countries. Feel free to tear off this front page of the exam so that you can look at it and the diagram together.

   a. (4 points) Which country (or countries, if there are ties) has the largest absolute stock of capital? Which has the most labor? Which countries have the largest and the smallest *ratios* of capital to labor in their endowments?

      Country 2 has the largest capital stock, K_2.
      Country 7 has the largest labor force, L_7.
      Country 1 has the largest capital-labor ratio, $k_1 = K_1/L_1$ (since it is to the left of the steepest ray drawn, and all other countries are to the right of that).
      Country 7 has the smallest capital-labor ratio.

   b. (4 points) Which country (or countries) has the highest rental price of capital? Which has the lowest? Which pairs or groups of countries, if any, have the same wage rate of labor?

      Countries 6 and 7 both have the highest rental price of capital, $r_6=r_7$.
      Country 1 has the lowest rental price of capital, r_1.
      Factor price equalization holds in the two cones, and therefore countries 2 and 4 have the same wage of labor, $w_2=w_4$, and so do countries 6 and 7, $w_6=w_7$ ($<w_2$).

   c. (2 points) Which country produces the most good 1? Which produces the most good 3?

      Only country 7 produces good 1 at all, so it must produce the most. (It employs the vector of factors v_1^7 in producing good 1.)

      Countries 1, 2, and 4 all produce good 3, country 1 producing only that, while countries 2 and 4 produce both good 3 and good 2. They employ vectors v_3^2 and v_3^4 in producing good 3. Clearly, country 2 employs more of both factors in the sector than either of the other countries, so it produces the most good 3.

d. (4 points) What does country 6 produce, what does it export and import, and how do its factor prices compare to factor prices in the other countries?

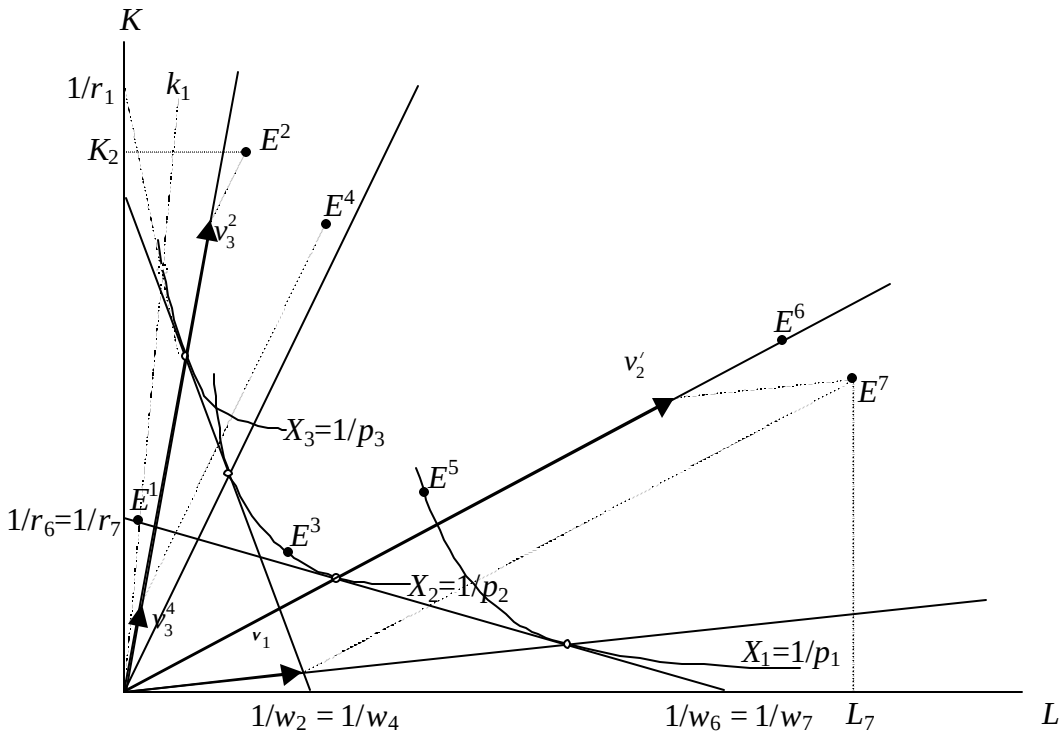
Country 6 on the edge of a cone. It produces only good 2 (since to produce any of good 1 would leave it the wrong amount of factors to fully employ in good 2). It therefore exports good 2, while it imports goods 1 and 3. Its factor prices are the same as country 7's, r_7 and w_7, and thus its wage is lower than in all of countries 1-5 and its rental on capital is higher than in countries 1-5.

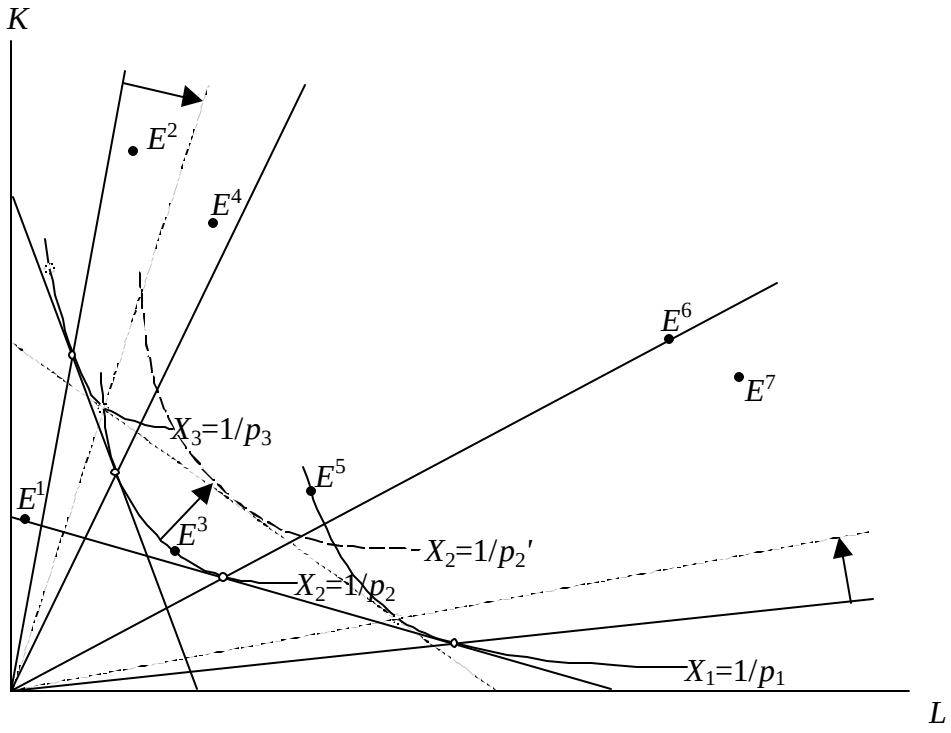
e. (2 points) In what industry does country 7 employ the largest fraction of its total labor force? Is country 7's output of good 1 worth more than \$1, less than \$1, or exactly \$1?

Country 7 employs vector v_2^7 in sector 2 and v_1^7 in sector 1, from which it employs more labor in sector 2 than in sector 1, hence a larger fraction of its labor force. Country 7's inputs to sector 1 lie well inside its unit isocost line (the straight line from $1/r_7$ to $1/w_7$, and therefore these inputs, and also its output, are worth less than \$1

f. (3 points) Which country or countries (if any) produce all three goods? In what direction would the price of good 2 have to change (holding the prices of goods 1 and 3 constant) in order to make it possible for some country that currently does not produce all three goods to do so? If that price change happened, would factor prices then be equal in all seven countries?

No country produces all three goods. In a 2-cone equilibrium such as this, prices are such that no country can produce all three goods in free trade. To change that, all three unit-value isoquants would have to be tangent to a single straight line. To achieve that by changing only p_2, we would need to reduce p_2 until its unit-value isoquant had shifted out to the new position shown in the lower diagram below. There would then be the single cone shown, instead of two cones, and countries 1 and 2 would lie outside of it. So factor prices would not be equal across all seven countries. (Indeed, factor prices in country 1 would continue to be what they were before.)

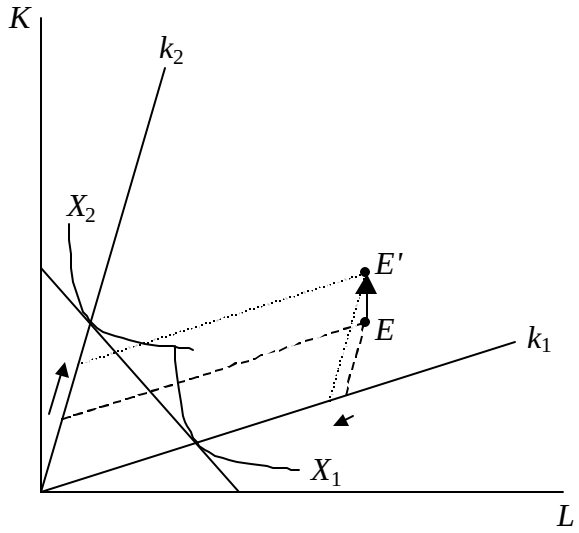

K
1/r1
k1
K2
E2
E4
v32
E6
v2′
E7
X3=1/p3
E5
E1
1/r6=1/r7
E3
X2=1/p2
v34
v1
X1=1/p1
1/w2 = 1/w4
1/w6 = 1/w7
L7
L


K
E2
E4
E6
E7
X3=1/p3
E5
E1
E3
X2=1/p2′
X2=1/p2
X1=1/p1
L

2. [8 points] As a start toward learning how economic development may affect the world economy, analyze the effect of capital accumulation in labor-abundant countries on world output of goods, as follows: Suppose that the world economy is characterized by free trade in either the 2-good-1-cone HO model or the 3-good-2-cone HO model, as indicated below, with the factor endowments of various countries ranging from least developed, with very little capital per worker, to most developed, with a lot of capital per worker. Then determine the effect, *at constant prices*, of a developing country in the stated situation increasing its capital stock while holding its labor force constant. What will happen, as a result, to that country's outputs (and therefore the world's outputs) of each of the goods in the model? (Assume that capital accumulation is not enough to change the country's pattern of specialization.)

   a. (2 points) The 2-good-1-cone HO model: capital accumulation in a country that produces both goods – labor-intensive X_1 and capital-intensive X_2 – and that exports X_1.

      This is the standard Rybczynski result that, as shown below, capital accumulation at constant prices causes the output of the capital-intensive good to increase and that of the labor-intensive good to decrease. Thus output of X_2 (the most capital-intensive good) rises and of output of X_1 falls.



   b. (3 points) The 3-good-2-cone HO Model: capital accumulation in a country that specializes completely in producing the most labor-intensive good, X_1.

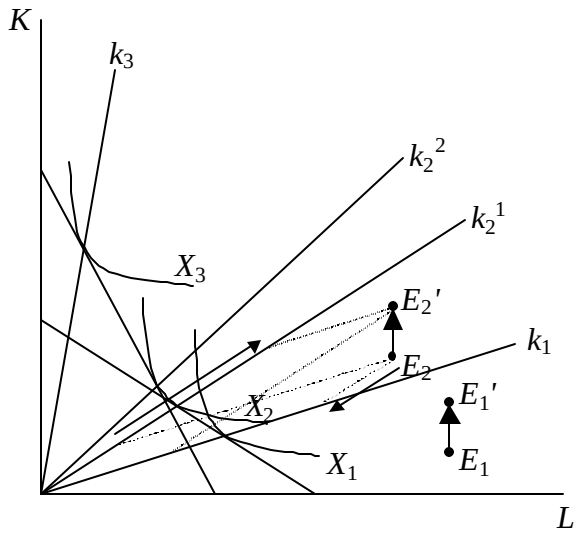
   c. (3 points) The 3-good-2-cone HO Model: capital accumulation in a country that produces both the most labor-intensive good, X_1, and the good of intermediate factor intensity, X_2, and that exports X_1.

      The 2-cone equilibrium, shown below, behaves quite differently. For a country specialized in producing only good 1, shown as the endowment E_1, capital

accumulation just means that it can produce more of good 1 itself, and output of that good rises while outputs of the other two goods remain unchanged at constant prices.

But if the country is inside the lower cone, producing both goods 1 and 2, then Rybczynski effects cause it, at constant prices, to produce more of good 2 and less of good 1. Thus output of good 2 rises, output of good 1 falls, and output of good 3 stays the same.

Summarizing these three cases, capital accumulation in a developing country can lead to an increase in the world output of (only) the most capital-intensive good, the most labor-intensive good, or the good of intermediate capital-intensity, depending on the nature of the trading equilibrium (1-cone versus 2-cone) and the pattern of specialization of the growing country.



3. [12 points] Use the Reciprocal Dumping Model to determine the effect of a fall in the cost of transportation on the *foreign* market, as follows. Find the changes in

   a. total sales in the foreign market,

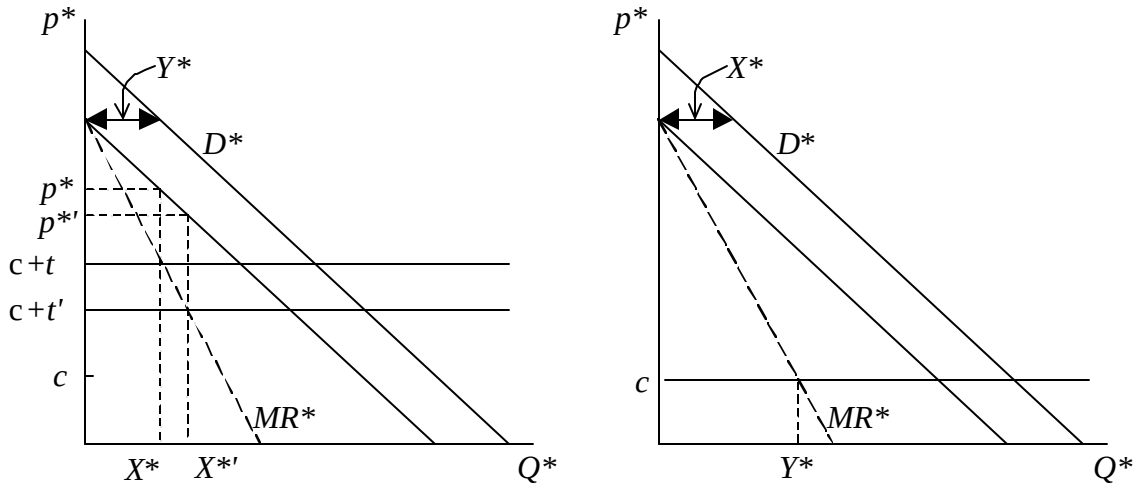
   b. sales there by the Home-country firm,

   c. sales there by the Foreign-country firm,

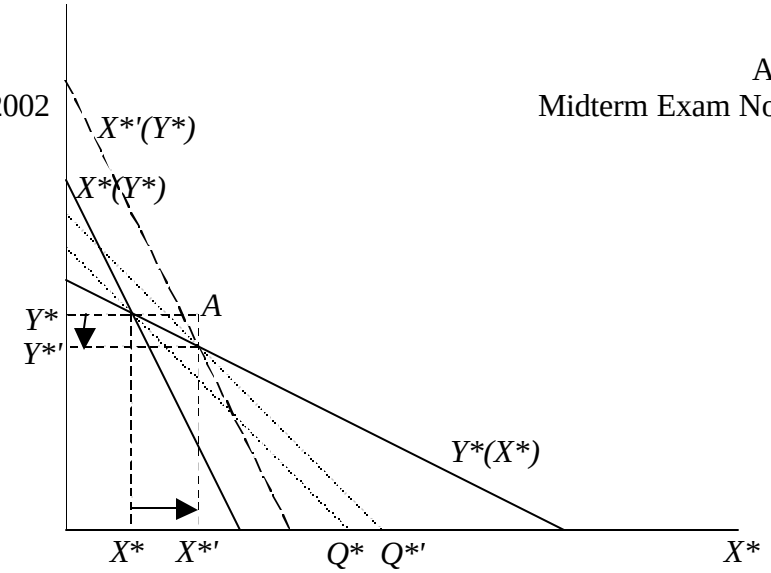
   d. the well-being of foreign consumers,

   It is enough that you determine the direction of each of these effects, not necessarily their size. Feel free to use either diagrams or equations for your analysis, or even a mixture of the two, so long as you do it carefully and completely.

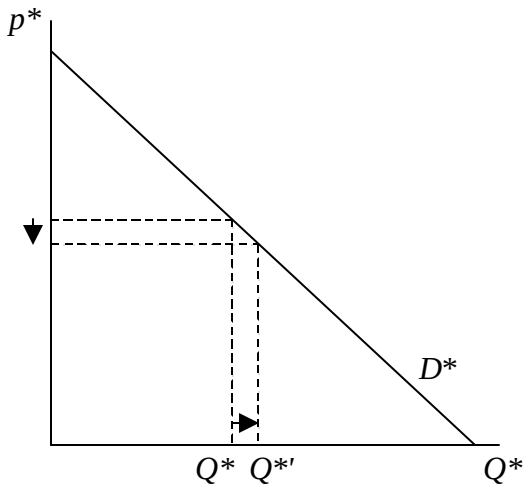
   The reaction curve of the Home firm selling in the foreign market, X(Y*), is determined by its profit maximization subject to a demand D*–Y* and a cost c+t, as shown on the left below. The reaction curve of the Foreign firm selling in the foreign market, Y*(X*), is determined by its profit maximization subject to a demand D*–X* and cost c as shown on the right below. A fall in the transport cost, t, causes the Home firm to expand its sales for given Y* to X*', while it has no effect on the Foreign firm's optimal sales Y* given X*.*



   From this, the reaction curve for the Home firm, X(Y*), shifts out, while the reaction curve for the Foreign firm remains unchanged. The result, as shown below, is that the Nash equilibrium sales of the Foreign firm decline, while those of the Home firm expand. From the dotted 45^o lines along which X*+Y* is constant, we can also see that total sales expands from Q* to Q*'. (This uses the fact that X*(Y*) is steeper than the 45^o line, while Y*(X*) is flatter than it – the slopes are –2 and –1/2 respectively.)*



Since total sales increases, price must fall along the demand curve, as below, and this benefits consumers in the foreign market.



For those who prefer to solve the problem algebraically, the model is

X maximizes (p*–c–t)X* = (a–b(X*+Y*) –c–t)X**

Y maximizes (p*–c)Y* = (a–b(X*+Y*)–c)Y**

The first-order condition for X is*

$$0 = a - bY^* - c - t - 2bX^*$$

yielding the reaction curve for X:*

$$X^* = \frac{a-c-t}{2b} - \frac{Y^*}{2}$$

The first-order condition for Y is*

$$0 = a - bX^* - c - 2bY^*$$

yielding the reaction curve for Y:*

$$Y^* = \frac{a-c}{2b} - \frac{X^*}{2}$$

Together these give the Nash equilibrium:

$$X^{*N} = \frac{a-c-t}{2b} - \frac{Y^{*N}}{2} = \frac{a-c-t}{2b} - \frac{a-c}{4b} + \frac{X^{*N}}{4}$$

$$X^{*N} = \frac{4}{3}\left(\frac{a-c-t}{2b} - \frac{a-c}{4b}\right) = \frac{2(a-c-t)}{3b} - \frac{(a-c)}{3b} = \frac{(a-c)-2t}{3b}$$

$$Y^{*N} = \frac{a-c}{2b} - \frac{(a-c)-2t}{6b} = \frac{2(a-c)+2t}{6b} = \frac{(a-c)+t}{3b}$$

Other variables in the Nash equilibrium are:

$$Q^{*N} = X^{*N} + Y^{*N} = \frac{(a-c)-2t}{3b} + \frac{(a-c)+t}{3b} = \frac{2(a-c)-t}{3b}$$

$$p^{*N} = a - b(X^{*N} + Y^{*N}) = a - b\frac{2(a-c)-t}{3b} = \frac{3a}{3} - \frac{2(a-c)-t}{3} = \frac{a+2c+t}{3}$$

$$CS^* = \frac{1}{2}(a-p^*)Q^* = \frac{1}{2}\left(a - \frac{a+2c+t}{3}\right)\left(\frac{2(a-c)-t}{3b}\right) = \frac{(2(a-c)-t)^2}{18b}$$

From these we get immediately that

$$\frac{dX^{*N}}{dt} = \frac{-2}{3b} < 0; \quad \frac{dY^{*N}}{dt} = \frac{1}{3b} > 0; \quad \frac{dQ^{*N}}{dt} = \frac{-1}{3b} < 0; \quad \frac{dp^{*N}}{dt} = \frac{1}{3} > 0$$

$$\frac{dCS^*}{dt} = \frac{-2(2(a-c)-t)}{18b} = -\frac{(2(a-c)-t)}{9b} < 0 \text{ since } t < (a-c)/2$$

from which a fall in transport cost increases X, decreases Y*, increases Q*, decreases p*, and increases CS*.*

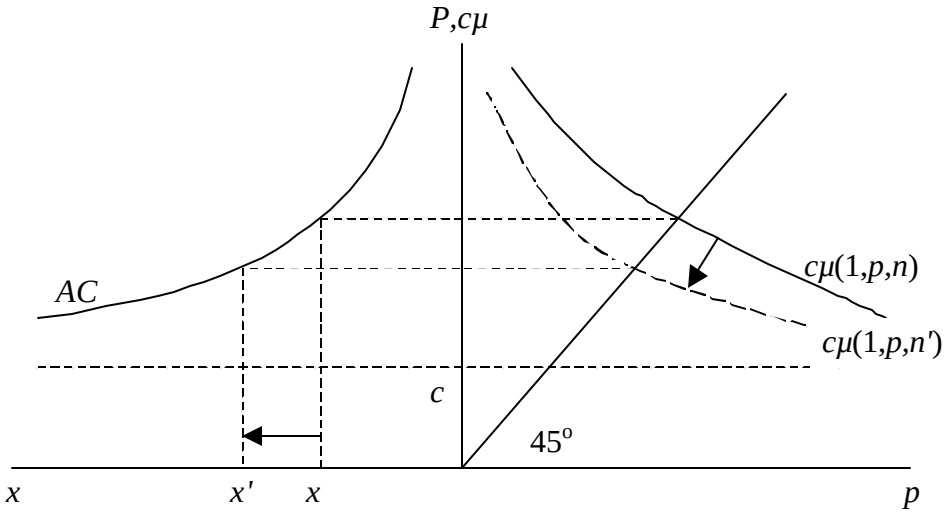
4. [13 Points] In our partial-equilibrium model of monopolistic competition, we assumed that the two countries were of different sizes – that is, the market in the home country in autarky was larger than the market in the foreign country. We also

assumed that the elasticity of demand faced by individual firms would increase with a rise in price and/or with an increase in the number of varieties. With these assumptions we showed that free trade between the two countries would cause a fall in price in both countries. Verify this result, and then determine whether it continues to be true if each of these assumptions is relaxed. That is:

a. (4 points) Show, under the stated assumptions, that the price of the good falls in both countries due to trade.

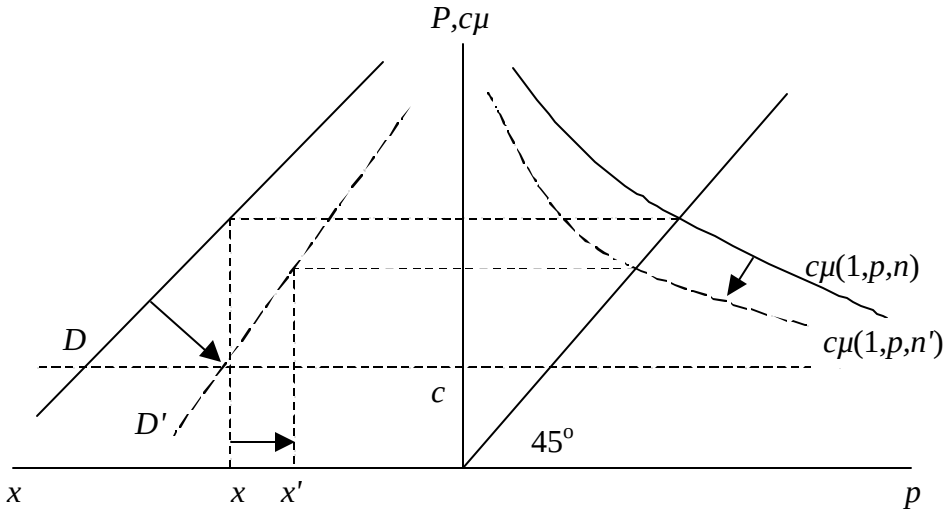
Under the stated assumptions, since elasticity ? rises with price and falls with variety, the optimal markup, μ=p/c= ?/(?–1) does just the opposite. Thus p=cμ is a downward sloping function of price and shifts down with an increase in the number of varieties, n. This can be used to derive an upward sloping relationship between n and the output per firm, x, that must hold for zero profits. This is shown in the top diagram below, where an increase in n shifts the cμ curve down, lowering the price (on the 45^o line, where price set by one firm equals prices set by others), and requiring movement down the average cost curve to higher output per firm, x. This also implies a downward sloping relationship between the same two variables that must hold for output per firm to equal demand for the firm's output, as in the bottom diagram. Here, the same rise in n both lowers cμ and shifts the demand curve in, leading to a fall in x. The latter relationship, but not the former, will lead to higher x as market size is increased, raising the demand for each variety.

Based on these results, equilibrium is found at the intersection of an upward sloping zero profit curve, ZZ, and a downward sloping market equilibrium curve, EE. With two countries of unequal size, the larger country will have an EE curve with larger x for each n, and thus in autarky, interacting with the same ZZ curve, will have both more firms and more output per firm than the smaller country.



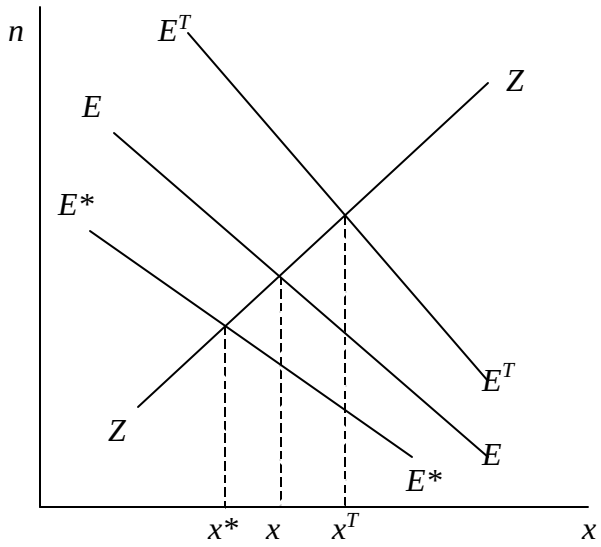
? $n>0$

All of this is shown in the diagram on the next page, which also includes the E^TE^T curve showing the market equilibrium for a world of the two countries trading freely. It is above the other two because of its larger market, which is all that we need for the current purpose. We see immediately that output per firm in both countries with trade, x^T, is larger than their output per firm in autarky, x and x respectively. Since price must equal average cost, and average cost declines with output per firm, it follows that price is lower with trade than in autarky for both*



? $n>0$

countries.



b. (3 points) Determine whether price still falls in both countries if they are the same size (with demand elasticity still an increasing function of price and number of varieties).

*If the countries are the same size, then their equilibrium curves, EE and E*E* above, will be the same. However, the market with trade is still larger than in either autarky economy (twice the size, in fact), so E^TE^T will still be above both EE and E*E*, and the argument holds. Price falls in both.*

c. (3 points) Determine whether price still falls in both countries if they are of different sizes but demand elasticity does not depend on price (but still does increase with the number of varieties.

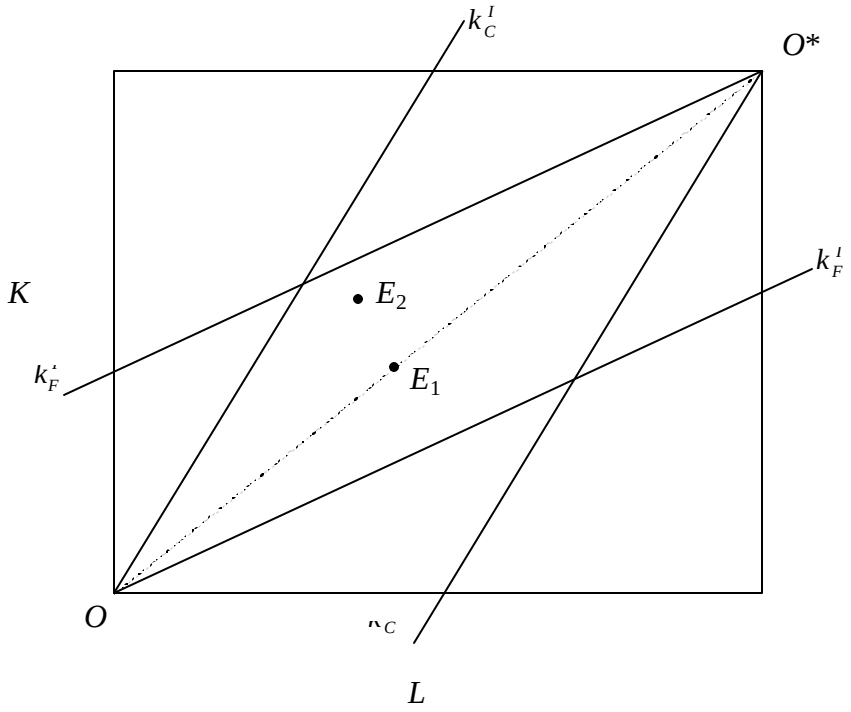
If demand elasticity does not depend on price, then the cμ curves in the diagrams on the previous page are horizontal, not downward sloping. However, as long as the elasticity does depend on then number of varieties, these horizontal curves will shift down when n increases, lowering the price at the intersection with the 45^o line, and the argument goes through. So price still falls in both countries even if elasticity does not depend on price.

d. (3 points) Determine whether price still falls in both countries if they are of different sizes but demand elasticity does not depend on the number of varieties (but still does increase with price).

If elasticity does not depend on number of varieties, but does depend on price, then the cμ curves are downward sloping, but they do not shift with n. Price therefore does not fall in deriving the zero-profit curve, and the ZZ curve is vertical. The EE curve will still be downward sloping, however, since the shift of the demand curve is enough to give a new x for higher n even with an unchanged p. But this won't help. The equilibrium of the trading world will be at the

intersection of a higher EE curve with the now-vertical ZZ curve, and therefore at the same output per firm as in the two economies in autarky. With the same output per firm, there will be the same average cost and therefore price. So price now does not *fall with trade in either country.*

5. [8 points] The graph below shows the possible allocations of world factor endowments to two countries, Home measured from *O* and Foreign measured from *O**, together with rays from both of these points showing the capital-labor ratios employed in the economy's two sectors in the Integrated World Economy, k_F^I and k_C^I. Two allocations are singled out by the points labeled E_1 and E_2.



a. (4 points) In the Heckscher-Ohlin (HO) Model, how do prices of factors in the two countries compare at allocation E_1? How do they compare at E_2, both across countries and compared to E_1?

In the HO model there is factor price equalization throughout the parallelogram formed by the k_F^I and k_C^I rays. Therefore, wages are the same in both countries at E_1 and also at E_2, and these wage levels, determined by the prices of the Integrated World Economy, are the same at E_1 as at E_2. The same is also true of rental prices of capital: they are the same in both countries, at both points, and also between both points.

b. (4 points) For the allocation E_1 (on the diagonal of the box), compare the volume of trade in the HO Model with the volume of trade in the Helpman-Krugman (HK) Model with monopolistic competition in the cloth sector (*C*) only.

At point E_1, because the two countries have the factors in the same proportions, they also produce the goods in the same proportions. Assuming identical and homothetic preferences so that all consumers everywhere consume the goods in the same proportions, both countries in the HO model are producing exactly what they consume, and therefore they do not trade. The volume of trade is zero.

In the HK Model with monopolistic competition in the cloth sector, on the other hand, although both countries produce the same amount of cloth that they consume, the cloth is now a differentiated product, and consumers everywhere want to buy some of each variety. Thus both countries' cloth producers sell part of their output to consumers in the other country, and there is trade. The volume of trade is therefore positive.

Extra Questions and Answers, Omitted from Exam:

1. g. (2 points) Which country or countries (if any) have a national income of exactly $1?

Only country 3 has an endowment, E_3, that is on the "isoquant for producing one dollar," which is the convex hull of the three unit-value isoquants shown. So only it has a national income of exactly $1. (Country 1's national income is quite a bit less than $1. Its endowment, E_1, happens to lie on the unit-isocost line of countries 6 and 7, and so would be worth $1 at their factor prices. But as we saw in part (b), its own factor prices are quite different.)

3. e. the profits earned by the Home-country firm in the foreign market,

f. and the profits earned by the Foreign-country firm in the foreign market.

The fall in price is necessarily less than the fall in transport cost, although this is not easy to see. To do so, note in the first diagram above that the fall in price from p to p*' – which is what would happen if X* rose to X*' and Y* did not change – is smaller than the drop in t (since MR is steeper than D). But with these levels of sales we would be at point A in the reaction curve diagram, at which total sales is larger, and therefore price is lower, than the final equilibrium X*',Y*'. Therefore price falls by less than transport cost.*

It follows that profit of the Home firm increases, since its cost falls by more than price, and it sells more.

It is also true that profit of the Foreign firm declines, and this can be seen more simply. Its cost is unchanged, and it is getting a lower price on smaller sales.

$$p_X^* = (p^* - c - t)X^{*N} = \left(\frac{a+2c+t}{3} - c - t\right)\left(\frac{a-c-2t}{3b}\right) = \frac{(a-c-2t)^2}{9b}$$

$$p_Y^* = (p^* - c)Y^{*N} = \left(\frac{a+2c+t}{3} - c\right)\left(\frac{a-c+t}{3b}\right) = \frac{(a-c+t)^2}{9b}$$

$$\frac{dp_X^*}{dt} = \frac{2(a-c-2t)}{9b}(-2) = -\frac{4(a-c-2t)}{9b} < 0 \quad since \quad t < (a-c)/2$$

$$\frac{dp_Y^*}{dt} = \frac{2(a-c+t)}{9b} > 0$$

from which a fall in transport cost increases p_X and decreases p_Y*.*