Homework #3
Not Due, Ever – Just FYI

1. The equations below represent the Open Economy Model of Mankiw's Chapter 5, which determines the endogenous variables Y, C, I, NX, and ε in terms of the exogenous variables $\overline{K}$, $\overline{L}$, r^*, $\overline{G}$, and $\overline{T}$.

Production function:	$Y = F(\overline{K},\overline{L})$	(1)
Consumption:	$C = C(Y - \overline{T})$	(2)
Investment:	$I = I(r^*)$	(3)
Net Exports:	$NX = NX(\varepsilon)$	(4)
Goods market equilibrium:	$Y = C + I + \overline{G} + NX$	(5)

Here the $I(\cdot)$ and $NX(\cdot)$ functions are both downward sloping, $F(\cdot)$ is increasing in both K and L, and $C(\cdot)$ is increasing in disposable income with $0<MPC<1$.

a. Use the equations of the model and the known properties (slopes) of the functions to determine the effect of an increase in taxes, $\overline{T}$, on the real exchange rate, ε. (By "use the equations" I mean totally differentiate them and solve for $d\varepsilon$ in terms of $d\overline{T}$.)

b. Use a representation of the model in terms of a diagram or diagrams to work out the effect of a major new investment opportunity (which causes a higher level of investment for every value of the interest rate) on net exports. Assume that before this change the economy is running a trade deficit. (In this case, explain in words why a particular curve or curves shifts, and then use the diagrams to demonstrate how equilibrium will change as a result.)

c. Write a paragraph, using words only (not equations, graphs, or symbols), to explain to a policymaker of a country that is running a trade deficit how a rise in the foreign price level will affect the size of this deficit.

2. In homework #2 question 4, you used the closed economy long-run model of Mankiw's Chapter 3 to find the effects of the model's exogenous variables on its endogenous variables. Which of the answers that you gave for the closed economy would be changed in the open economy model?

3. The following is a version of Mankiw's unemployment model, expanded slightly to make explicit how U and E change over time.

Labor force:	$L = E + U$	(1)
Unemployment dynamics:	$\dot{U} = dU / dt = sE - fU (= -\dot{E})$	(2)

   Combine these equations to express $\dot{U}$ in terms only of U and exogenous variables. Then graph this function, on the axes below. From this graph, explain whether, or under what circumstances, the unemployment rate will converge over time to the equilibrium rate described by Mankiw.

4. Using Mankiw's framework for explaining steady-state unemployment in terms of rates of job separation, s, and job finding, f, indicate how you would expect that one, both, or neither of these rates would be affected by the following, and also how as a result the steady-state unemployment rate, u, would change. Explain.

   a. A reduction in family sizes makes it more difficult for the unemployed to be supported by family members.

   b. Lawsuits for wrongful dismissal make firms reluctant to fire workers.

   c. The internet makes it easier for workers to find and apply for jobs.

   d. A one-time wave of corporate mergers causes an unusual number of workers to be laid off.

5. The following is a version of the Solow Growth Model in discrete time with an explicit "Cobb-Douglas" functional form for the production function. For simplicity, the population of this country does not grow.

$Y_t = K_t^{0.5} L_t^{0.5}$	Production function
$L_t = 100$	Constant population
$K_{t+1} - K_t = I_t - 0.05K_t$	Constant rate of depreciation, 0.05
$I_t = S_t = s_t Y_t$	Constant rate of savings out of income, s_t

a. Starting with a constant savings rate, s=0.1, show that the long-run levels of income and the capital stock in this economy are Y=200 and K=400. (In this case, because the population does not grow, the "steady state" involves a stationary capital stock and income.) What is the corresponding level of consumption, C=Y-S, in this long-run equilibrium?

b. Starting now at time 0 in the long-run equilibrium from part (a), suppose that the economy continues in this equilibrium for 10 periods, but then, starting in period 11 the savings rate rises from 0.1 to 0.2 and stays there forever. Calculate the levels of Y_t, K_t, and C_t, for t=11,12,…,200. Graph each of these variables over time (starting at time 0 so that you can better see the change). (You'll find it easiest to use a spreadsheet for these calculations and graphs.)

c. What are the long-run equilibrium levels of Y, K, and C for the economy with this new higher savings rate? Does it appear from your solution in part (b) that these variables are approaching their new long-run values over time?

d. From your solutions in parts (a), (b), and (c), how many periods does it take for each of the variables Y, K, and C to move half way from the first long-run equilibrium to the second? How many periods does it take for consumption to surpass its level of the old long-run equilibrium?

6. The following are the equations of the Solow Growth Model with technical progress:

$Y = F(K, EL)$	Production function (constant returns to scale)
$\dot{L} = dL/dt = nL$	Constant rate of population growth, n
$\dot{E} = dE/dt = gE$	Constant rate of technical progress, g
$\dot{K} = dK/dt = I - \delta K$	Constant rate of depreciation, δ
$I = S = sY$	Constant rate of savings out of income, s

Here, E represents "efficiency units of labor per worker," and one can think of the model as determining output, wages, etc. per efficiency unit. Then, even if for example the wage per efficiency unit is constant over time, since workers are acquiring more efficiency units at the growth rate g, their wage per worker will be growing.

a. Let $k=K/EL$ be the capital-labor ratio with labor in efficiency units, and use the model to derive its equation of motion:

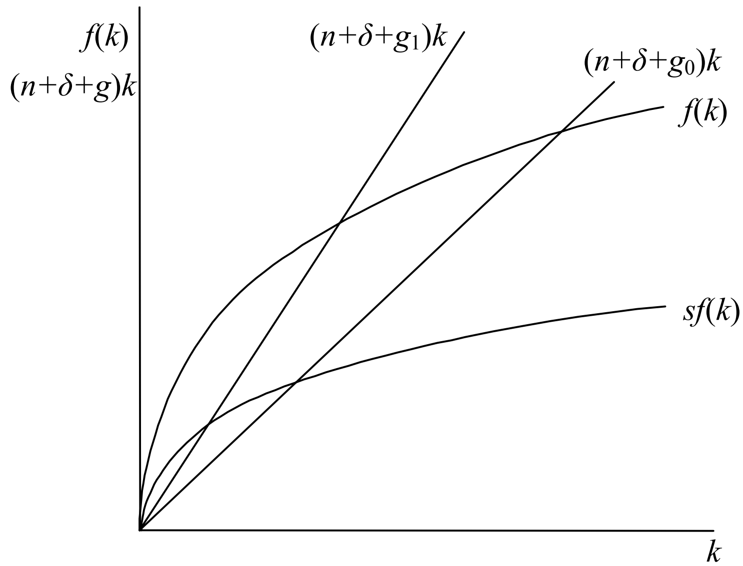
$$\dot{k} = sf(k) - (n + \delta + g)k$$

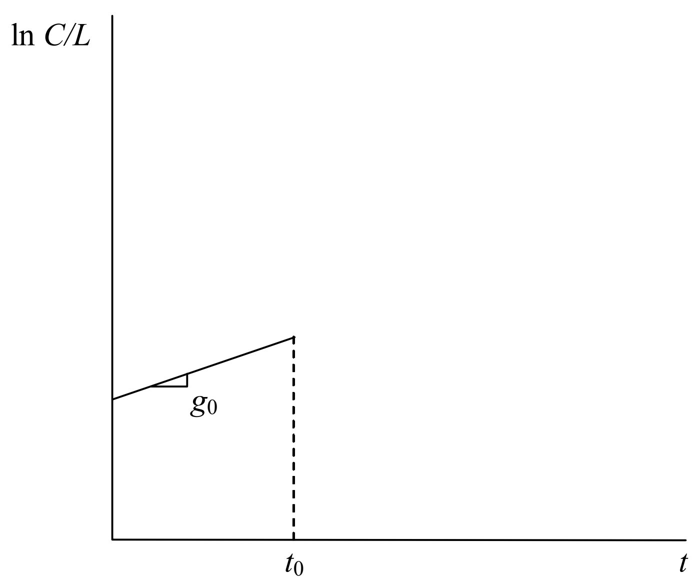
where $f(k) = F(k,1) = Y/EL$ is output per efficiency unit.

b. In a steady state of this model, what are the growth rates of the following variables?

Y	____________
Y/L	____________
I	____________
$C/L=(Y–S)/L$	____________
K/EL	____________

c. Starting from a steady state in this model, suppose that at some time t_0 the rate of technical progress now suddenly and permanently rises, from g_0 to g_1. Find the new steady state in the first diagram below, and then, on the axes below that, continue drawing an approximate time path for per capita consumption, C/L, over time after t_0. (To make your life easier, I've labeled the vertical axis with the logarithm of per capita consumption, ln C/L, because when a variable grows at a constant rate, its logarithm follows a straight line.)





d. Augment the model now to include the Quantity Equation from Chapter 4 to determine the price level. Assuming that velocity of money is constant at V_0 and that the central bank expands the money supply at the constant annual rate m, write an expression for the steady-state rate of inflation, $\pi = \dot{P}/P$, in terms of any or all of m, s, n, δ, and g.

e. How will an increase in the rate of technical progress, with other parameters constant (as in part c) affect the rate of inflation in part (d)? Do you think this would actually happen?

7. While the rate at which capital actually depreciates depends primarily on technology and is probably not sensitive to public policy, the rate at which firms *claim* that it depreciates is very much a matter of policy. Since it is usually in a firm's interest to exaggerate the rate of depreciation – so as to inflate its costs for tax purposes and reduce its tax – the government has rules regarding how rapidly firms can depreciate their capital. Recognizing this, describe what you would expect to be the effects on the performance of a growing economy of increasing the government limit on how rapidly firms can depreciate their capital. That is, in the broad context of the Solow growth model (but allowing, of course, for the presence of a government), describe what you would expect to happen, after such an increase, to the levels of savings and investment, and also to the growth rate of the economy, both initially and over time. (I'm not asking you to model this formally, but only to apply the insights that you've gotten from our various long-run models, including the Solow model, to this situation.)