Homework #3 - *Answers*
Not Due, Ever – Just FYI

1. The equations below represent the Open Economy Model of Mankiw's Chapter 5, which determines the endogenous variables Y, C, I, NX, and ε in terms of the exogenous variables $\overline{K}$, $\overline{L}$, r^*, $\overline{G}$, and $\overline{T}$.

Production function:	$Y = F(\overline{K}, \overline{L})$	(1)
Consumption:	$C = C(Y - \overline{T})$	(2)
Investment:	$I = I(r^*)$	(3)
Net Exports:	$NX = NX(\varepsilon)$	(4)
Goods market equilibrium:	$Y = C + I + \overline{G} + NX$	(5)

Here the $I(\cdot)$ and $NX(\cdot)$ functions are both downward sloping, $F(\cdot)$ is increasing in both K and L, and $C(\cdot)$ is increasing in disposable income with $0<MPC<1$.

a. Use the equations of the model and the known properties (slopes) of the functions to determine the effect of an increase in taxes, $\overline{T}$, on the real exchange rate, ε. (By "use the equations" I mean totally differentiate them and solve for $d\varepsilon$ in terms of $d\overline{T}$.)

Ans: Holding constant all exogenous variables except $\overline{T}$, nothing changes in equations (1) and (3), so these may be ignored, and dY=dI=0. Differentiating what remains, we get:

$$dC = -C'd\overline{T}$$
$$dNX = NX'd\varepsilon$$
$$0 = dC + dNX$$

Where C' and NX' are the slopes of the respective functions. This is solved as follows:

$$d\varepsilon = (1/NX')dNX = (1/NX')(-dC) = (1/NX')C'd\overline{T}$$

Since $NX' < 0$ and $C' > 0$, $d\overline{T} > 0 \Rightarrow d\varepsilon < 0$. That is, a rise in taxes causes the real exchange rate to depreciate.

(In case the above is too abbreviated, the full procedure for solving the model this way is as follows. Start by differentiating the entire system of equations totally with respect to all exogenous and endogenous variables:

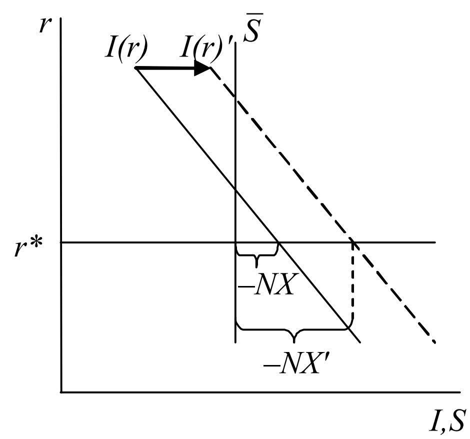
$$dY = F_K d\overline{K} + F_L d\overline{L})$$

$dC = C'dY - C'd\overline{T})$
$dI = I'dr*$
$dNX = NX'd\varepsilon$
$dY = dC + dI + d\overline{G} + dNX$

Then solve this system of linear equations treating the differentials (dY, $d\overline{K}$, etc.) as variables. Set the differentials of those exogenous variables that are not changing to zero (in this case everything except $d\overline{T}$), and solve for the changes in whatever endogenous variables you are interested in (in this case dε).)

b. Use a representation of the model in terms of a diagram or diagrams to work out the effect of a major new investment opportunity (which causes a higher level of investment for every value of the interest rate) on net exports. Assume that before this change the economy is running a trade deficit. (In this case, explain in words why a particular curve or curves shifts, and then use the diagrams to demonstrate how equilibrium will change as a result.)

Ans: The new investment opportunity shifts the I(r) curve shown at the right to the right. Since the interest rate is unchanged at r, investment increases. Savings does not change, however, since it is fixed by the fixed levels of income and taxes. So net exports, which must equal S–I, decrease. Since we start with a trade deficit, and thus I>S, the trade deficit becomes larger, as shown.*



c. Write a paragraph, using words only (not equations, graphs, or symbols), to explain to a policymaker of a country that is running a trade deficit how a rise in the foreign price level will affect the size of this deficit.

Ans: A rise in the foreign price level, if the exchange rate does not change, makes foreign goods relatively more costly compared to domestically produced goods. This would stimulate demand for exports, reduce demand for imports, and reduce the trade deficit, if only the exchange rate would remain unchanged. But it won't. Before this change, the trade deficit was matched by a capital inflow, reflecting the fact that your country was spending more than its income and had to rely on capital from abroad. The rise in the foreign price level does nothing, in the long run, to reduce your country's spending or to increase its income, so the capital inflow must continue unchanged. So if the trade deficit were to decline, this would mean an excess demand for the country's currency, which would therefore rise in value. In fact, for the exchange market to remain in equilibrium, the currency must appreciate by the same amount that the foreign price level has gone up, so as to leave relative prices of domestic and foreign goods unchanged.

Only then will the net demand for foreign currency to buy foreign goods equal the net supply of foreign currency that is accompanying the capital inflow.

2. In homework #2 question 4, you used the closed economy long-run model of Mankiw's Chapter 3 to find the effects of the model's exogenous variables on its endogenous variables. Which of the answers that you gave for the closed economy would be changed in the open economy model?

Ans: The answers for the closed economy were as follows:

	Effects on:				
Due to:	*Y*	*W*	*r*	*C*	*I*
$\bar{K}$	+	+	–	+	+
$\bar{L}$	+	–	–	+	+
$\bar{G}$	*0*	*0*	+	*0*	–
$\bar{T}$	*0*	*0*	–	–	+

The answers for Y and W are unchanged, since these depend solely on the production function. The answers for C are also unchanged, for essentially the same reason. The answers for r and I are all now changed: they are all zero, since r=r and changes only for a change in r*, and since I depends only on r.*

3. The following is a version of Mankiw's unemployment model, expanded slightly to make explicit how U and E change over time.

Labor force: $L = E + U$ (1)

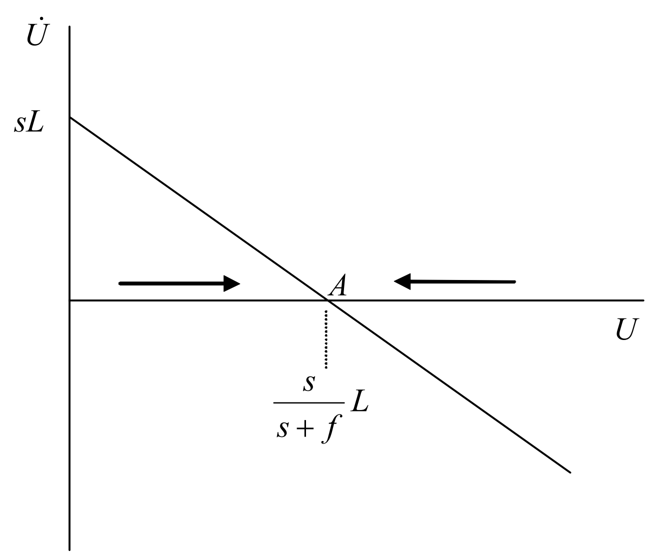
Unemployment dynamics: $\dot{U} = dU/dt = sE - fU(= -\dot{E})$ (2)

Combine these equations to express $\dot{U}$ in terms only of U and exogenous variables. Then graph this function, on the axes below. From this graph, explain whether, or under what circumstances, the unemployment rate will converge over time to the equilibrium rate described by Mankiw.

Ans: Substituting E=L–U from (1) into (2), we get

$$\dot{U} = sL - (s+f)U$$

which is graphed here. Since $\dot{U} > 0$ *to the left of point A and* $\dot{U} < 0$ *to the right of point A, U will change over time as shown by the arrows, converging on point A at which the unemployment rate is u=U/L=s/(s+f).*



4. Using Mankiw's framework for explaining steady-state unemployment in terms of rates of job separation, *s*, and job finding, *f*, indicate how you would expect that one, both, or neither of these rates would be affected by the following, and also how as a result the steady-state unemployment rate, *u*, would change. Explain.

   a. A reduction in family sizes makes it more difficult for the unemployed to be supported by family members.

      Ans: The unemployed will be less reluctant to take undesirable jobs, knowing that they have less support at home, and they will therefore become re-employed more quickly. If this is all that happens, s does not change, f rises, and consequently u falls. It is also possible that both employers and employees will be more reluctant to initiate separations, so that s falls, reinforcing the fall in u.

   b. Lawsuits for wrongful dismissal make firms reluctant to fire workers.

      Ans: Firms will keep workers longer, which lowers the rate of separation, but they may also become more selective in hiring workers in order to be more sure that they won't want to fire them later. So f may fall too. If so, then the effect on unemployment is ambiguous.

   c. The internet makes it easier for workers to find and apply for jobs.

      Ans: This just speeds up the process and therefore increases f, reducing u.

   d. A one-time wave of corporate mergers causes an unusual number of workers to be laid off.

      Ans: This will lay off a lot of workers temporarily, but it should not alter the rate at which workers continue to be separated from jobs, or unemployed workers to find them. Thus there is no change in s or f, and therefore no change in the long-run equilibrium rate of unemployment. (Of course, in the short run unemployment presumably rises, but the model here is about the long run.)

5. The following is a version of the Solow Growth Model in discrete time with an explicit "Cobb-Douglas" functional form for the production function. For simplicity, the population of this country does not grow.

$Y_t = K_t^{0.5} L_t^{0.5}$	Production function
$L_t = 100$	Constant population
$K_{t+1} - K_t = I_t - 0.05K_t$	Constant rate of depreciation, 0.05
$I_t = S_t = s_t Y_t$	Constant rate of savings out of income, s_t

   a. Starting with a constant savings rate, s=0.1, show that the long-run levels of income and the capital stock in this economy are Y=200 and K=400. (In this case, because the population does not grow, the "steady state" involves a stationary capital stock and income.) What is the corresponding level of consumption, $C=Y-S$, in this long-run equilibrium?

Ans: In the steady state with $K_{t+1}=K_t$, we must have $I=0.05K$. Substituting for $I=sY=s(K^{0.5}100^{0.5})=10sK^{0.5}$ we get $10sK^{0.5} = 0.05K$, or $200sK^{0.5}=K$. Squaring this and canceling a K, this simplifies to

$$K = 40{,}000s^2 = 40{,}000(0.01) = 400$$

Thus

$$Y = 400^{0.5}100^{0.5} = (20)(10) = 200$$

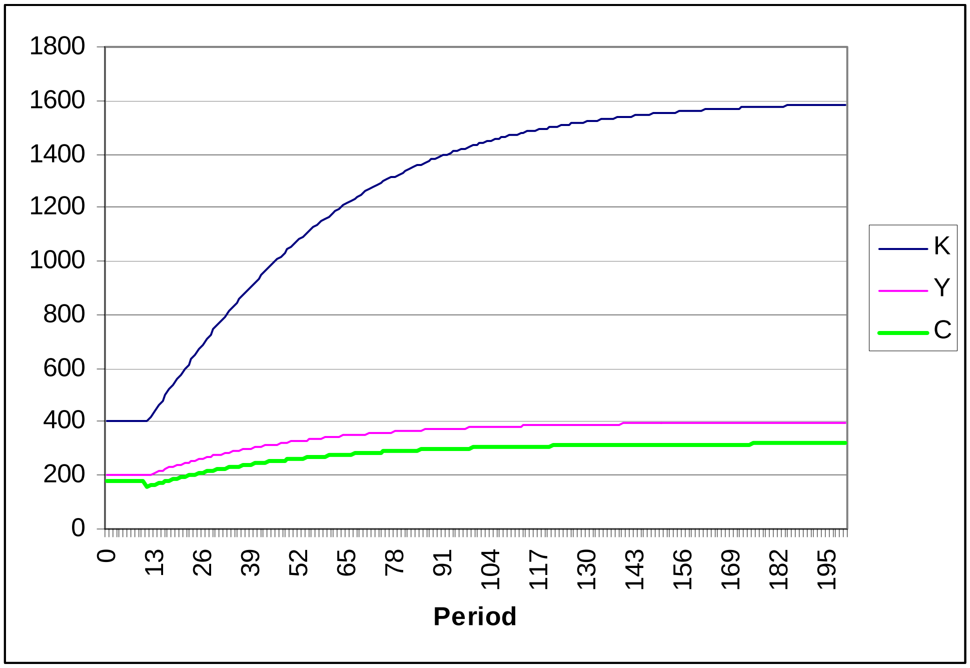
$$C = (1–0.1)(200) = 180$$

b. Starting now at time 0 in the long-run equilibrium from part (a), suppose that the economy continues in this equilibrium for 10 periods, but then, starting in period 11 the savings rate rises from 0.1 to 0.2 and stays there forever. Calculate the levels of Y_t, K_t, and C_t, for t=11,12,…,200. Graph each of these variables over time (starting at time 0 so that you can better see the change). (You'll find it easiest to use a spreadsheet for these calculations and graphs.)

The complete results are too big to include here, but the first 20 periods after the increase in the savings rate are shown in the table below, followed by the graph for all 200 periods.

t	L	K	Y	S	C
10	100	400	200	20	180
11	100	400	200	40	160
12	100	420	205	41	164
13	100	440	210	42	168
14	100	460	214	43	172
15	100	480	219	44	175
16	100	500	224	45	179
17	100	519	228	46	182
18	100	539	232	46	186
19	100	558	236	47	189
20	100	578	240	48	192
21	100	597	244	49	195
22	100	616	248	50	199
23	100	635	252	50	202
24	100	654	256	51	205
25	100	672	259	52	207
26	100	690	263	53	210
27	100	708	266	53	213
28	100	726	269	54	216
29	100	744	273	55	218

30	100	761	276	55	221



c. What are the long-run equilibrium levels of Y, K, and C for the economy with this new higher savings rate? Does it appear from your solution in part (b) that these variables are approaching their new long-run values over time?

Ans: Using s=0.2 in the result derived in part (a), $K = 40{,}000(0.2)^2 = 40{,}000(0.04) = 1600$. $Y = 1600^{0.5}100^{0.5} = (40)(10) = 400$. $C = (1\text{-}0.2)(400) = 320$. *Yes, the numbers in (b) did approach these values.*

d. From your solutions in parts (a), (b), and (c), how many periods does it take for each of the variables Y, K, and C to move half way from the first long-run equilibrium to the second? How many periods does it take for consumption to surpass its level of the old long-run equilibrium?

Ans: Steady state Y changes from 200 to 400, and it gets half way there, to 300, after 29 periods (in period 39, since the process started in period 11). Steady state K changes from 400 to 1600, and reaches its half-way point, 1000, after 36 periods. Consumption changes from 180 to 320, reaching half way (250) after 34 periods.

Consumption falls at first, from 180 to 160, because of the increased saving. It rises back to 180 – 182 actually – in period 7, and from then on rises even higher.

6. The following are the equations of the Solow Growth Model with technical progress:

$Y = F(K, EL)$	Production function (constant returns to scale)
$\dot{L} = dL/dt = nL$	Constant rate of population growth, n
$\dot{E} = dE/dt = gE$	Constant rate of technical progress, g
$\dot{K} = dK/dt = I - \delta K$	Constant rate of depreciation, δ
$I = S = sY$	Constant rate of savings out of income, s

Here, E represents "efficiency units of labor per worker," and one can think of the model as determining output, wages, etc. per efficiency unit. Then, even if for example the wage per efficiency unit is constant over time, since workers are acquiring more efficiency units at the growth rate g, their wage per worker will be growing.

a. Let $k=K/EL$ be the capital-labor ratio with labor in efficiency units, and use the model to derive its equation of motion:

$$\dot{k} = sf(k) - (n + \delta + g)k$$

where $f(k) = F(k,1) = Y/EL$ is output per efficiency unit.

Ans: Differentiating k we get

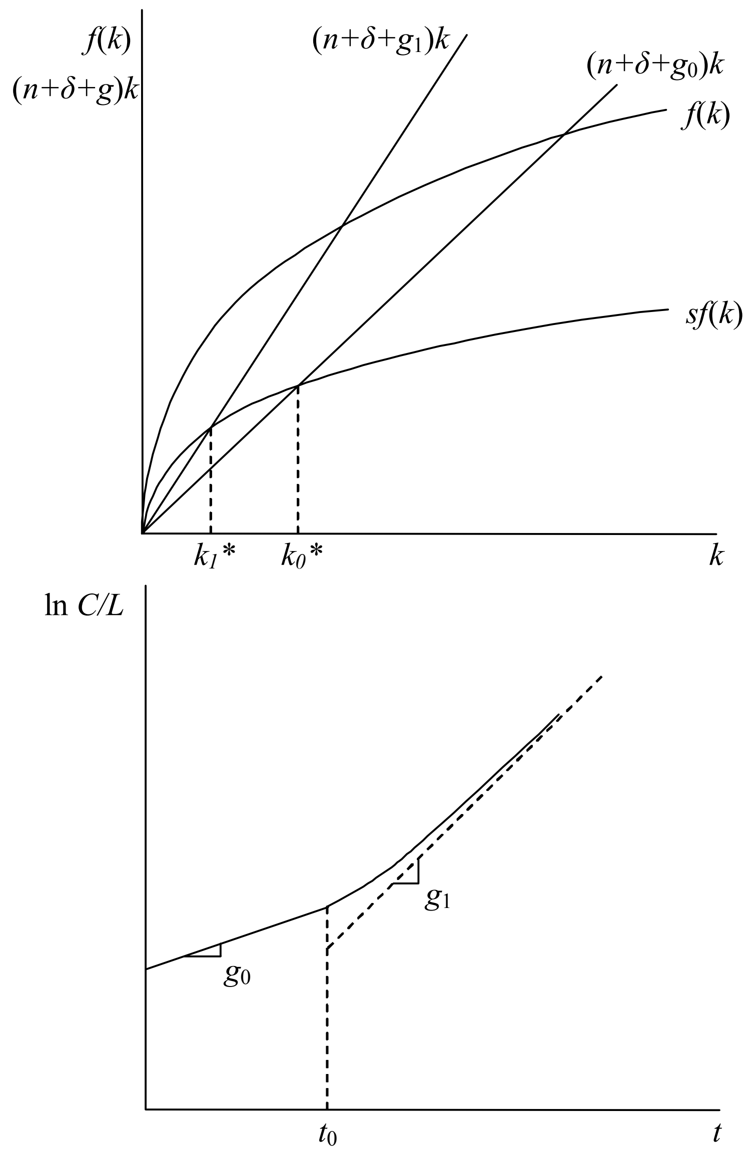
$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{E}}{E} - \frac{\dot{L}}{L} = \frac{sY - \delta K}{K} - g - n = \frac{sY/EL}{K/EL} - \delta - g - n = \frac{sf(k)}{k} - (n + \delta + g)$$

Multiplying through by k yields the result.

b. In a steady state of this model, what are the growth rates of the following variables?

Y	*n+g*
Y/L	*g*
I	*n+g*
C/L=(Y–S)/L	*g*
K/EL	*0*

c. Starting from a steady state in this model, suppose that at some time t_0 the rate of technical progress now suddenly and permanently rises, from g_0 to g_1. Find the new steady state in the first diagram below, and then, on the axes below that, continue drawing an approximate time path for per capita consumption, C/L, over time after t_0. (To make your life easier, I've labeled the vertical axis with the logarithm of per capita consumption, ln C/L, because when a variable grows at a constant rate, its logarithm follows a straight line.)



Per capita consumption grows at the constant rate g_0 before time t_0. At time t_0 technology begins to grow at the faster rate g_1, but this does not immediately change anything. Over time, however, the current level of investment no longer keeps up with the faster growing efficiency units of labor, so that k, and therefore Y/EL and C/EL, both decline. They approach the new lower steady state, *however, at which output and therefore C grow at the faster rate* $n+g_1>n+g_0$. *In effect, they approach the lower but steeper steady-state growth path that they would have already been on if the capital stock had somehow fallen instantaneously at time* t_0 *to its new steady-state level.*

d. Augment the model now to include the Quantity Equation from Chapter 4 to determine the price level. Assuming that velocity of money is constant at V_0 and that the central bank expands the money supply at the constant annual rate *m*, write an expression for the steady-state rate of inflation, $\pi = \dot{P}/P$, in terms of any or all of *m*, *s*, *n*, δ, and *g*.

Ans: The quantity equation is MV=PY which, since V is constant, implies that $\pi = \dot{P}/P = (\dot{M}/M) - (\dot{Y}/Y)$. *From the growth model, Y grows at a rate equal to population plus technology, or n+g. Thus* $\pi = m - n - g$.

e. How will an increase in the rate of technical progress, with other parameters constant (as in part c) affect the rate of inflation in part (d)? Do you think this would actually happen?

Ans: From part (d), a rise in g reduces the rate of inflation. Because it increases the growth rate of GDP, it increases the need for money. With no change in m, prices cannot rise as rapidly.

In fact, however, it is almost certain that if real income were to rise more rapidly, the Fed would provide for that need by increasing the rate of monetary expansion.

7. While the rate at which capital actually depreciates depends primarily on technology and is probably not sensitive to public policy, the rate at which firms *claim* that it depreciates is very much a matter of policy. Since it is usually in a firm's interest to exaggerate the rate of depreciation – so as to inflate its costs for tax purposes and reduce its tax – the government has rules regarding how rapidly firms can depreciate their capital. Recognizing this, describe what you would expect to be the effects on the performance of a growing economy of increasing the government limit on how rapidly firms can depreciate their capital. That is, in the broad context of the Solow growth model (but allowing, of course, for the presence of a government), describe what you would expect to happen, after such an increase, to the levels of savings and investment, and also to the growth rate of the economy, both initially and over time. (I'm not asking you to model this formally, but only to apply the insights that you've gotten from our various long-run models, including the Solow model, to this situation.)

Ans: Increasing the limit on claimed depreciation will not change how rapidly capital actually depreciates, so in spite of appearances, this policy will not *alter the* $(n+\delta)k$ *line in the graph of the Solow model.*

What the increase does *do is to permit firms to pay less in taxes, thus increasing the funds that either they or their stockholders have available for other purposes. Therefore, one effect of this policy change is that of a tax cut, increasing disposable income and therefore private savings.*

What this does to national savings, however, depends on how the government responds. If the tax cut is not accompanied by any change in government spending, then public saving falls by the full amount of the tax cut. Since the private sector saves only a fraction of any increase in disposable income, while the government is dis-saving all of it, it follows that national savings goes down. In the Solow model, this means a shift downward in the sf(k) curve. (One could instead assume, perhaps, that the government runs a balanced budget and therefore reduces spending by the full amount of the lost taxes. In that case, national savings does rise, and all of the answers below would be reversed.)

A second effect that one might expect would be to increase the real rate of return on physical capital, compared to other assets, since physical capital is subject to depreciation and permits this sort of tax dodge, while other assets (e.g. human capital) do not. Thus we might expect on this account for investment to be increased. However, in the Solow model it is really savings, not investment, that constrains growth, since investment is adjusted automatically (through the loanable funds market and the interest rate, presumably) to equal savings, and savings depends only on income, not on the interest rate. So if the change in depreciation rules were to increase desired investment relative to savings, this would just drive up the interest rate to offset it, leaving investment ultimately just equal to savings. In the end, then, it seems that this policy effects the economy primarily as a tax cut. It reduces national savings, and therefore investment, because the government reduces its saving by more than the increase by the private sector (or it increases both savings and investment if the government runs a balanced budget).

From here on, the Solow model tells us directly what to expect in the way of effects on growth. This downward shift in the savings curve causes an immediate decrease in investment, which is then below that needed to match (true) depreciation and population growth. The country's capital stock per person therefore begins to shrink. Thus the policy causes a lower growth rate.

However, over time, the decumulation of capital drives its marginal product up, and the growth rate rises back up. In the end, as always in the Solow model, the dampening of the growth rate is only temporary, and the growth rate returns to its prior value equal to population growth. Income per capita has gone down, however, and perhaps consumption per capita if the economy had not already surpassed its golden rule level. (Note that the depreciation rate referred to by the golden rule is the true one, and it is not altered by this change in accounting rules.)