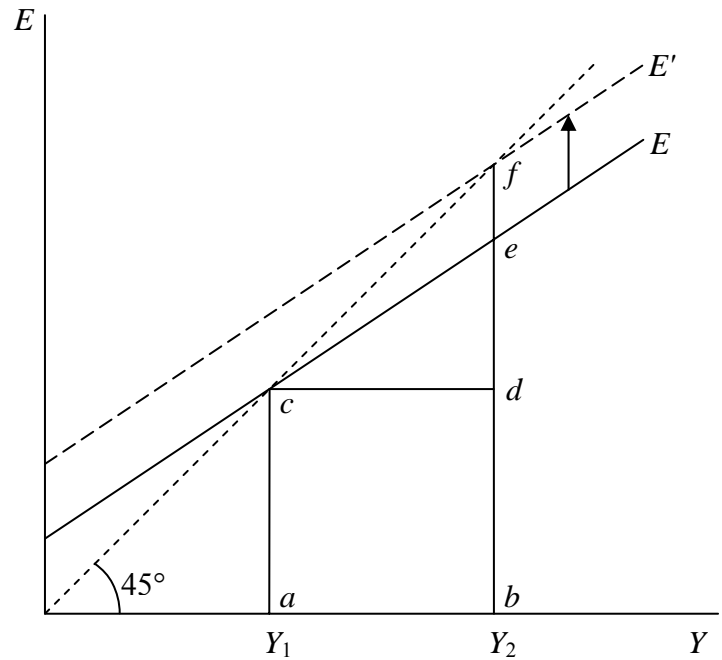


Homework #4 - Answers
The IS-LM Model
Due Mar 18

1. Fun with the Keynesian Cross:

- a. Use the geometry of the Keynesian Cross diagram shown at the right to derive that the government purchases multiplier is $1/(1-MPC)$, where MPC is the slope of the planned expenditure line, E . In the figure, planned expenditure has increased (for any given income) by the amount of an increase in government purchases, ΔG . Use the labels in the figure (a, b, c, \dots) to denote the distances involved (e.g., ab is the distance from Y_1 to Y_2 , or ΔY). First, identify the following:

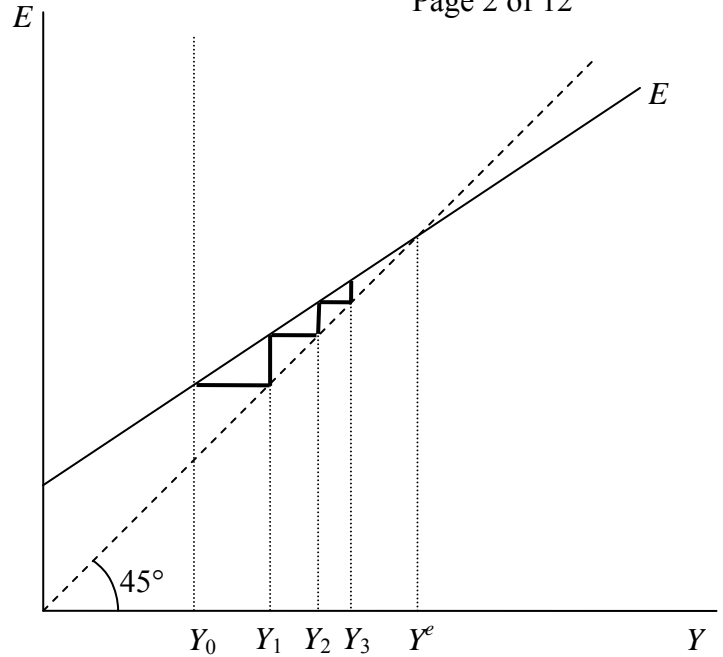


ΔY	ab
ΔG	ef
MPC	de/cd
Two other distances that = ab	cd, df

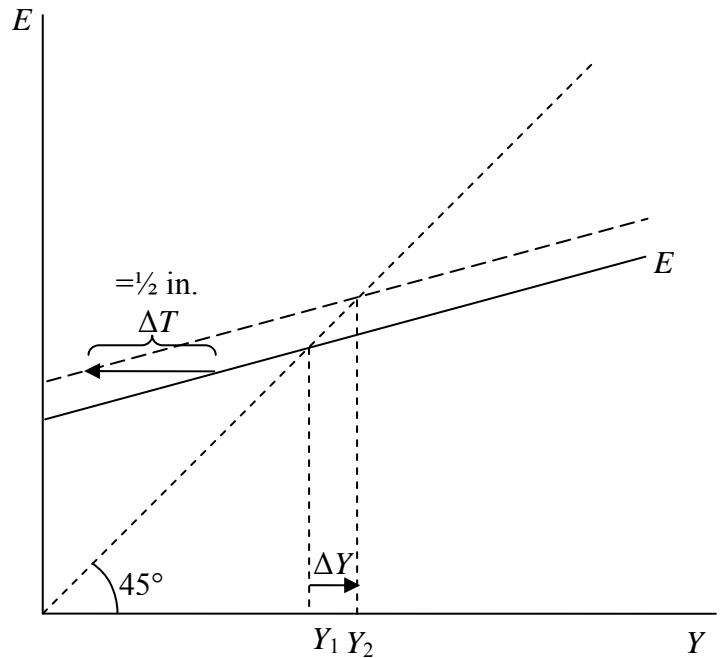
Then use these to derive the government purchases multiplier:

$$\frac{\Delta Y}{\Delta G} = \frac{ab}{ef} = \frac{cd}{(df - de)} = \frac{1}{\left(\frac{df}{cd} - \frac{de}{cd}\right)} = \frac{1}{1 - MPC}$$

- b. In the diagram at the right, suppose that the economy starts, in time period $t=0$, out of equilibrium at income Y_0 . Suppose also the following dynamic process for adjusting when out of equilibrium: in each period, t , the economy produces an output equal to the level of planned expenditure from the *previous* period. Find and label in the figure the levels of income for time periods 1, 2, and 3.



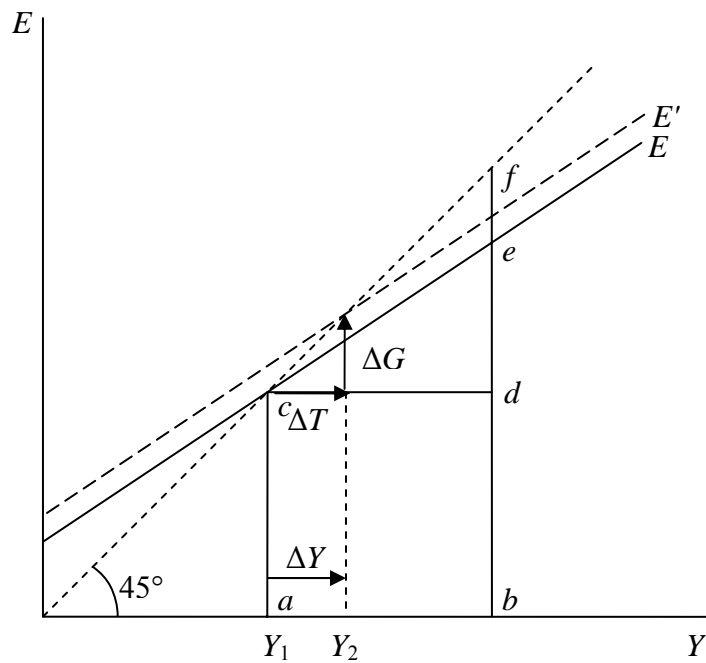
- c. In the diagram at the right, draw the new planned expenditure curve that will exist if there is a *cut* in taxes of $\Delta T = \frac{1}{2}$ inch. Label ΔT so as to show how you determined where the new curve should be. (Hint: If taxes are cut by some amount, say \$1000, then at any total income Y consumers will buy the same amount that they would have bought before if their income had been $Y+1000$.) Identify also the old and new equilibrium levels of income, Y_1 and Y_2 and the change in income ΔY .



Here ΔY is quite a bit smaller than ΔT . Note that the multiplier for a tax cut is $-MPC/(1-MPC)$, which is smaller (in absolute value) than -1 if $MPC < 1/2$.

- d. (Optional) Return to the diagram of part (a) and identify the planned expenditure curve that would reflect *both* the increase in government purchases *and* an accompanying tax increase sufficient to finance it. What is the multiplier ($\Delta Y / \Delta G = \Delta Y / \Delta T$) in that case? Can you show that in the figure?

As seen in part (c), a tax change shifts the E curve horizontally by the amount of the change in taxes. So here, with an increase in G accompanied by an equal increase in T, the E curve will shift both to the right by ΔT and up by ΔG , as shown below:



Considering point c, it moves first to the right and then up by the same amount, taking it back to the 45° line where it is therefore the new equilibrium. Therefore, in this case $\Delta Y = \Delta G = \Delta T$, or $\Delta Y / \Delta G = \Delta Y / \Delta T = 1$. It is sometimes said that the “balanced budget multiplier” is one.

2. The following are the equations of the IS-LM model, here including a feature that taxes are not simply given but depend on income through a tax function, $T(Y)$.

$$\text{IS Curve: } Y = C(Y - T(Y)) + I(r) + G$$

$$\text{LM Curve: } M / P = L(r, Y)$$

- a. Differentiate the model totally and solve for the government spending multiplier, dY/dG , in terms of the various slopes of the functions: C' = marginal propensity to consume out of disposable income, T' = marginal tax rate, I' = effect (derivative) of the interest rate on investment, and L_r, L_Y = the effects (partial derivatives) of the interest rate and income on liquidity preference.

Totally differentiate the two equations:

$$dY = C'(dY - T'dY) + I'dr + dG$$

$$dM / P - (M / P^2)dP = L_r dr + L_Y dY$$

For the current problem, $dM=dP=0$. Solve the second equation for dr :

$$dr = -(L_Y / L_r)dY$$

Substitute this for dr in the first equation:

$$dY = C'(1 - T')dY - (I'L_Y / L_r)dY + dG$$

Subtract the first two terms on the right from both sides and then divide by dG and by the coefficient of dY :

$$\frac{dY}{dG} = \frac{1}{1 - C'(1 - T') + (I'L_Y / L_r)}$$

- b. From this, what would the multiplier be if taxes did *not* depend on income? Does the income tax ($T' > 0$) make the multiplier larger or smaller?

If $T'=0$, the multiplier is

$$\frac{dY}{dG} = \frac{1}{1 - C' + (I'L_Y / L_r)}$$

Having $T' > 0$ makes the C' term smaller and, since that term appears with a minus sign in the denominator, makes the denominator larger and the multiplier smaller. An income tax makes the government spending multiplier smaller.

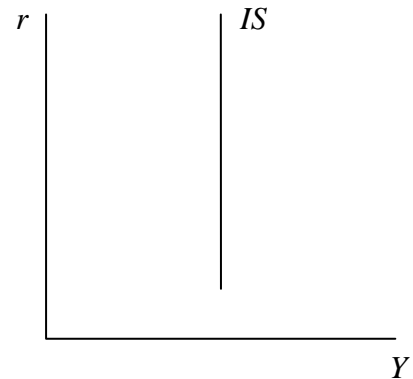
- c. Can you tell whether this multiplier from part (a) is larger or smaller than one? What features of behavior tend to make the multiplier smaller, and what make it larger?

No, we can't tell. We know that $I' < 0$, $L_r < 0$, and $L_Y > 0$, so the term $I'L_Y / L_r$ is positive. If it is larger than $C'(1 - T')$, then the denominator will be larger than one and the multiplier smaller than one. In general, the multiplier will be larger the larger in absolute value are C' and L_r and the smaller are T' , I' , and L_Y .

3. We normally draw the IS curve as downward sloping and the LM curve as upward sloping, as appropriate for our usual assumptions about behavior. How would either or both of these curves look different if the following *unusual* assumptions were made? Either describe or draw your answers, being sure in either case to make clear what you mean.

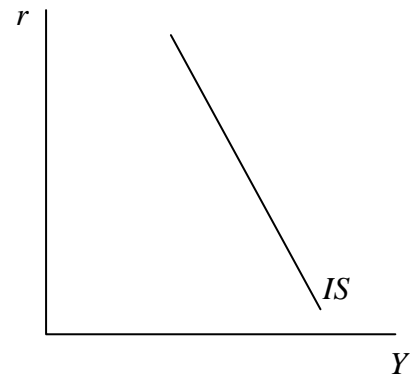
- a. Investment does not depend on the interest rate.

IS curve is vertical (because now, as the interest rate falls, there is no component of aggregate demand that increases).



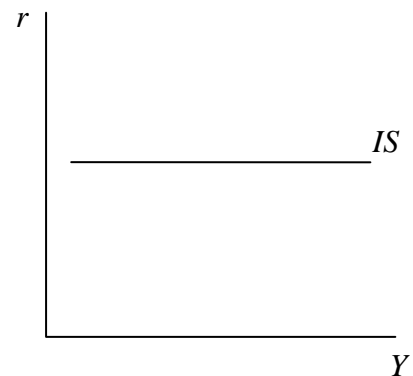
- b. The MPC is zero.

This makes the IS curve steeper than it would otherwise be, but it does not cause it to be vertical. It makes the multiplier equal to 1, rather than larger than one, but this is enough for a fall in the interest rate that increases investment to still cause an increase in income.



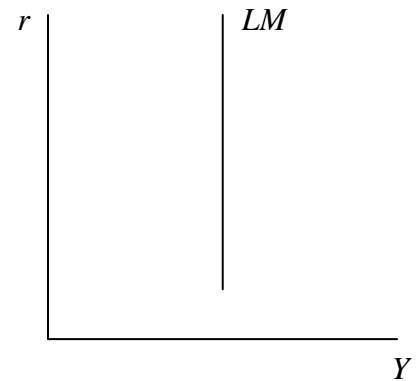
- c. The MPC is one.

This either causes there to be no equilibrium in the goods market at all, or it causes all values of Y to be equilibrium for a particular level of investment and thus the interest rate. The reason is that it makes the expenditure function have the same slope as the 45° line, and therefore either be parallel to it or coincide with it for all Y. Thus, if the IS curve exists at all, it is horizontal.



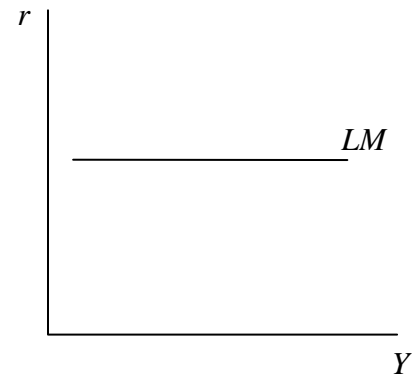
- d. Demand for money does not depend on the interest rate.

The LM curve is vertical (because now demand will equal supply of money only at the particular level of income, Y , for which that is true for all r).



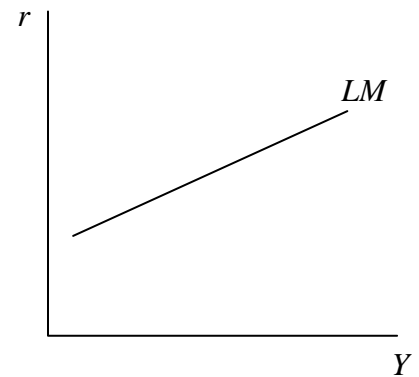
- e. Demand for money does not depend on income.

The LM curve is horizontal (because now demand will equal supply of money only at the particular level of the interest rate, r , for which that is true for all Y).



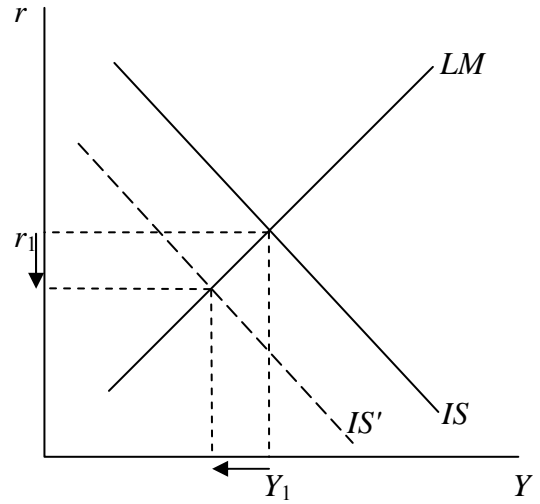
- f. Supply of money rises endogenously as a result of increases in the interest rate.

The LM curve is less steep than it would otherwise be, but it is still upward sloping. The reason is that now, as the interest rate increases, it increases supply of money in addition to decreasing demand for it, and it therefore requires a larger increase in income to offset these changes by increasing demand.



4. In each case below, you are given the IS-LM diagram with an initial equilibrium. Show how one or both of the curves change for the following exogenous changes in the model. Then, from that, determine the qualitative changes (+, -, 0, ?) in the indicated variables (where “?” means the variable could go either way). Give reasons for your results, where asked.

a. A reduction in government purchases



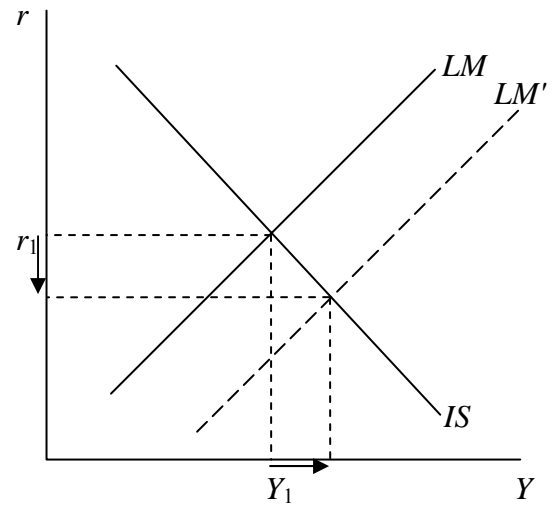
Y -

r -

C - Why? *Because income falls.*

M 0 Why? *Because M is exogenous and did not change.*

b. An increase in the money supply.



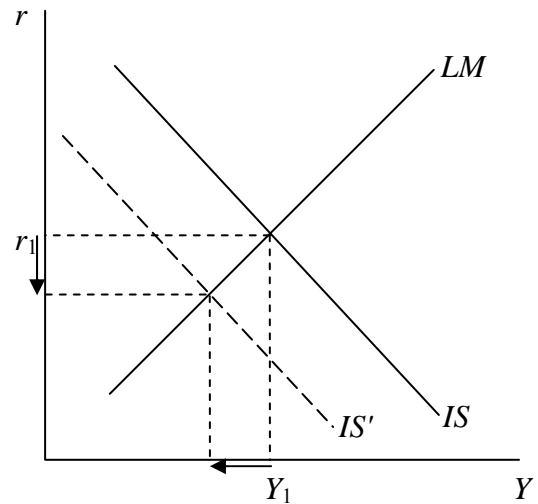
Y _____ + _____

r _____ - _____

I _____ + _____ Why? *Because the interest rate falls.*

L _____ + _____ Why? *Because $L=M$ and M went up. Or because r fell and Y rose, both of which raise L .*

c. A downward shift in the consumption function (less consumed at each $Y-T$).



Y _____ -

r _____ -

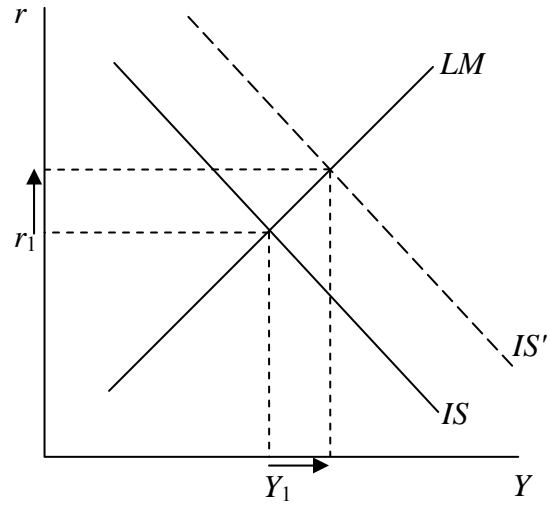
L _____ 0 Why?

Because $L=M$ and M has not changed. (Note that you can't tell from Y and r , since the fall in Y lowers L while the fall in r raises L .)

C _____ - Why?

Because Y fell and the consumption function shifted down.

- d. An upward shift in the investment function (more investment at each interest rate).



Y +

r +

S
 (=national savings) + Why?

Since private savings depends positively on income, and income has gone up.

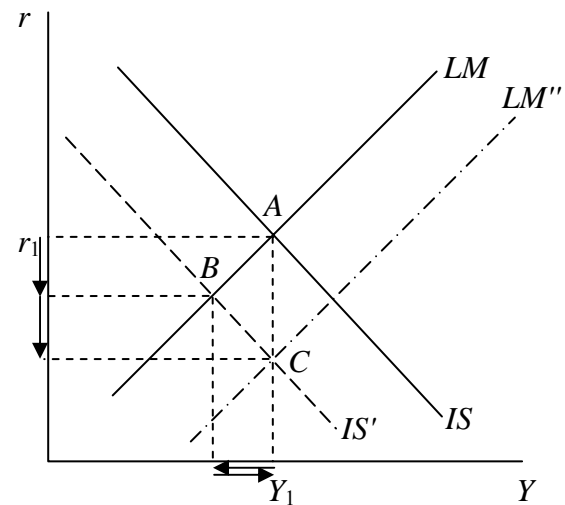
I + Why?

Because $I=S$ and S has risen. Note that you can not get it directly from the I function, since the shift of the function and the rise in r work in opposite directions.

5. In part (c) of question 4 above you found (I hope) that the downward shift in the consumption function caused a fall in output. Suppose now that policy makers try to prevent this by expansionary monetary or fiscal policy. For each of the policies indicated below, assume that the policy is used in exactly the right amount to offset the effect on Y , so that the combined effect of the shift of the consumption function and of the policy is for equilibrium Y not to change at all. Determine in each case how private savings, government savings, and national savings in the new equilibrium compare to what they were before all this happened.

- a. A change in the money supply

An increase in the money supply shifts the LM curve out to LM'' as shown and the interest rate falls further to a new equilibrium at point C. Thus investment is larger and private savings must also be larger than it was initially, since government savings is unchanged and S must equal I . (Private savings increased with the shift of the consumption function and is not further altered since Y is unchanged.)



- b. A change in government purchases

An increase in government purchases restores the old level of Y by shifting the IS curve back to its initial position, so equilibrium returns to point A. Thus there is no change in r , and therefore no change in I . Since $S=I$, national savings must be unchanged. But now the government is saving less, so private savings must be increased, exactly as in part (a).

- c. A change in taxes. How does the effect on the government budget here compare to part (b)?

A tax cut also shifts the IS curve back to its initial position and restores equilibrium at A, where investment and therefore national savings are at their initial levels. As in part (b), government savings is reduced, and therefore private savings must be higher. In fact, both of these changes must be larger than in part (b), because the tax cut raises private savings even more than the original shift of the consumption function. Indeed, the tax cut itself must be larger than the increase in government purchases in part (b) because the tax multiplier is smaller than the government spending multiplier.

6. Suppose that at a moment in time neither the goods market nor the money market is in equilibrium, so that the economy is neither on the IS curve nor on the LM curve.
- a. Based on what you know about how these two markets adjust toward equilibrium, which market would you expect to move more quickly towards equilibrium? Why?

The goods market adjusts through a rather cumbersome process: if expenditure doesn't match output, then firms respond by producing more or less. This changes incomes, which in turn cause a change in expenditure that also must be responded to by firms changing output. This process continues, and therefore can be expected to take some considerable time. The money market, on the other hand requires only that the interest rate adjust to whatever the levels of supply and demand for money happen to be. The interest rate is determined in the bond market, where bond prices (and thus interest rates) are determined moment to moment by bond traders. So the interest rate, and thus the money market can adjust very quickly. Therefore it makes most sense that an economy will move more rapidly to the LM curve than to the IS curve.

- b. Based on your answer to (a), show in the IS-LM diagram at the right, and then on the time axes below, how you would expect income and the interest rate to respond to a sudden large increase in the money supply that occurs at time t_0 .

