Problem Set #4
Due October 22, 1997

1. The Ann Arbor Arboretum (the Arb) is a large park that is currently available free to all users. At the zero price, users currently make 250,000 visits to the Arb each year. The city spends $375,000 per year maintaining the Arb, of which $125,000 is considered to be a fixed cost that would be needed regardless of the number of visits, and the rest is an estimated variable cost of $1 per visit. It is believed that demand for visits to the Arb is linear, and that demand would be positive for any price per visit below $5 but zero at prices of $5 or higher.

a) Calculate the total consumer surplus enjoyed by visitors to the Arb each year. How much is this per visit? How does it compare to the city's average cost per visit?

b) Suppose now that the Arb were turned over to a private firm that was allowed to charge admission to the park. Fixing the price of admission at the average cost per visit currently observed, what would be the *change in* consumer surplus and what would be the profit (or loss) of the firm?

c) If free to charge any price, what would the firm charge, and what would be the *level of* total consumer surplus and profit in that situation?

d) Suppose that, at the current price of zero, the 100,000 residents of Ann Arbor are in three groups: 45,000 are Couch Potatoes who never visit the Arb at all; 25,000 are Token Tree Huggers who visit it twice a year; and 30,000 are Nature Nuts who visit it five times a year. The remaining visits are by nonresidents who visit only once a year. All visitors to the Arb have linear demand curves that cross the price axis at $5. (Visits are perfectly divisible, so that, for example, a nonresident facing a price of $1 would visit 0.8 times a year.) What is the socially efficient price to charge each of these groups of visitors (society being defined as the world as a whole, not just Ann Arbor)?

e) Assuming that the current cost to the city of maintaining the Arb is shared equally among all 100,000 residents, what is the net benefit to each type of person of having the Arb (compared to closing it) and sharing the cost? Compared to this, how much would each type gain or lose if the Arb were privatized as in part (c)? Also compared to free admission, how much would each type gain or lose if the socially efficient price of part (d) were charged and costs were shared equally among Ann Arbor residents? Record your results in a table like that on the following page:

	Net Benefit if Free Admission	Gain from Privatization	Gain from Efficient Pricing
Couch Potatoes			
Token Tree Huggers			
Nature Nuts			
Nonresidents			

f) What are the net benefits to Ann Arbor and to society as a whole of each of these options?

g) Which of these options, if any, would be selected by majority voting?

2. Calculate the present discounted value of the projects listed in the table below, which reports for each of four projects, a, b, c, and d, the relevant interest rate, r, and the benefits (positive) and costs (negative) in the present (t=0), and each of t years from the present.

Project	Interest rate	Benefits (+) and Costs (–) in present (0) and future years, t= 0	1	2	3	4…9	10	11	12…∞
a)	5%	–700	300	400					
b)	3%	5	–5	–5	–5	–5	–5		
c)	7%	–200	14	14	14	14	14	14	14
d)	10%							100	100
e)	6%	–50	–50	–50	6	2*t	20	75	
f)	–2%	–1000	100	100	100	100	100		
g) x=1.03	4%		10x	$10*x^2$	$10*x^3$	$10*x^t$	$10*x^{10}$	$10*x^{11}$	$10*x^t$
h)	1%		10	10	10	10	10	10	10

3. Lee is tired of sitting in the cold (but refreshing) Michigan winter drafts and is considering insulating his home. It would cost him $5000 this year to insulate (he's got a big house), and the estimated reduction in his yearly fuel costs at this year's prices would be $350. The insulation can be installed immediately, so that he saves this amount on fuel even in the current year. Assume that the nominal interest rate is 9% a year and that the rate of inflation is 2% a year on everything, including fuel unless specified otherwise. Lee plans to live in the house until he retires, 23 years from today, at which point he will sell the house and move to someplace warmer. He has already consulted a real estate agent for advice and was told that insulation will not add anything at all to the value of the house when it is sold.

(a) What is the net present value of insulating Lee's humble abode? Should Lee insulate?

(b) If fuel prices were expected to rise at a rate of 5% per year instead of the 2% rate of the general price level, what would be the net present value of Lee's insulation project? Should Lee insulate in this case?

(c) Suppose that the real estate agent is wrong, and that insulation can, after all, add to the value of a house when it is sold. Letting the price of Lee's house today (without insulation) be $600,000, calculate

i) The market value of the house without insulation 23 years from today if housing prices rise at the rate of inflation.

ii) The annual fuel cost saving 23 years from today if fuel costs also rise at the rate of inflation.

iii) The present value as of 23 years from today of the cost saving due to insulation from there on into the indefinite future.

iv) The market value of the house 23 years from today if it includes the then-present value of the insulation.

v) The present value, today, of the increased price of the house 23 years from today due to the insulation.

vi) The net present value of insulating Lee's house today, allowing both for Lee's own cost saving as in part (a) and for the increased value of the house when he sells it.

d) Calculate the present value of insulating Lee's house if he were to decide never to sell it, but rather to keep it for himself and his descendants forever. How does your answer compare to your answer in part (c-vi), and why?