## Problem Set #1 - Answers
## Due January 26, 2000

[Numbers in brackets are the points allocated in the grading.  There are 46 points total.]

1.  [13]The figure at the right shows the budget line, $AB$, and an initial indifference curve, $U_1$, for a consumer who chooses between goods X and Y, where good Y is an aggregate of all of the other goods that he or she might consume.  A second indifference curve, labeled $U_2$, is also shown.  A magnified version of the figure is reproduced on the next page, where you should show your answers.
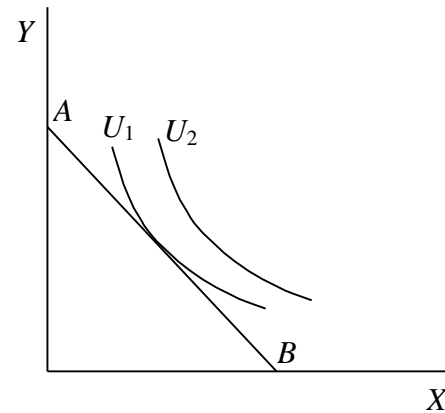
    Letting the price of good Y be $p_Y=1$, the price of good X be $p_X$, and the consumer's income be $I$,

    a.  [2]Label the two intercepts of the budget line.

    b.  [2]Identify in the figure the initial levels of consumption at these prices of goods X and Y:  $C_X$ and $C_Y$.

    c.  [3]Suppose now that the price of good X changes to $p_X'$, income and the price of good Y remaining unchanged, and that the price change is just the right amount for the consumer to choose consumption on indifference curve $U_2$.  Add to the figure the new budget line that prevails after the price change, label its intercepts, and show the new levels of consumption of both goods:  $C_X'$ and $C_Y'$.

    d.  [1]Did the price of X rise or fall in part (c)?

    e.  [1]Is the elasticity of demand for good X by this consumer larger or smaller than one?

    f.  [2]Find in the figure the change in income that would have been exactly equivalent to this price change, in the sense that the consumer's well being would have been exactly the same if they have gotten that instead of the price change.  Label it "EV."  (This is the "equivalent variation.")

    g.  [2]Find in the figure the change in income that, if it accompanied the price change, would leave the consumer exactly as well off as they were before.  Label it "CV." (This is the "compensating variation.")
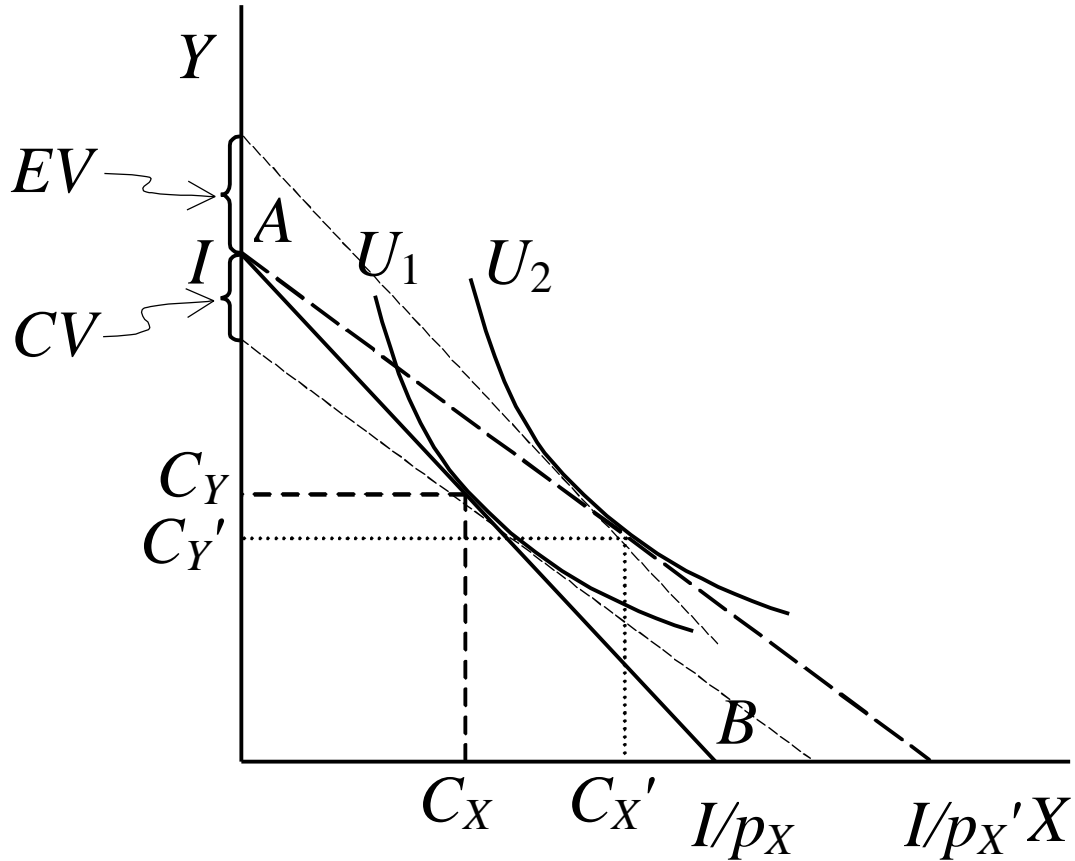
d)  The price of X (circle one):

   rose        (fell)             remained unchanged

   (The slope of the budget line is $p_X$, and it has gotten flatter.)

e)  The elasticity of demand for good X is (circle one):

   (larger than one)      smaller than one      equal to one

   (We know that the elasticity is larger than one since expenditure on the good, which is the difference between income $I$ and consumption of the other good, $C_Y$, went up.)

2.  [15]The figure at the right, reproduced in
    magnified form on the next page where you
    should show your answers, shows the industry
    demand curve for a good, together with the
    industry marginal and average cost curves that
    prevail for a certain number of identical,
    profit-maximizing firms, $n$.
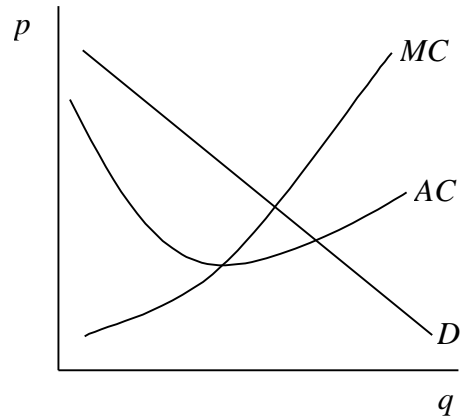
    a.  [3] If $n$ is large and fixed, find the
        equilibrium price and quantity and label
        them $p_1$ and $q_1$.  Identify also the profit per
        unit earned by these firms in that
        equilibrium, and label it p$_1$.

    b.  [3] If $n$ is fixed at one ($n=1$), find the equilibrium price and quantity and label them
        $p_2$ and $q_2$.  Identify also the profit per unit earned by this firm in that equilibrium,
        and label it p $_2$.

    c.  [3] If $n$ is large and variable, with free entry and exit, find the equilibrium price and
        quantity and label them $p_3$ and $q_3$.  Identify also the profit per unit earned by the
        industry in that equilibrium, and label it p $_3$.

    d.  [0 (omit this part)]If $n$ is fixed at two ($n=2$), without doing any formal analysis,
        write a sentence or two saying what you know about the equilibrium.

    Again without doing any formal analysis, state how your answers to parts (a), (b), and
        (c) would have been different if the curves in the figure above had instead looked
        like each of those below.

    e.  [3]                                          f. [3]
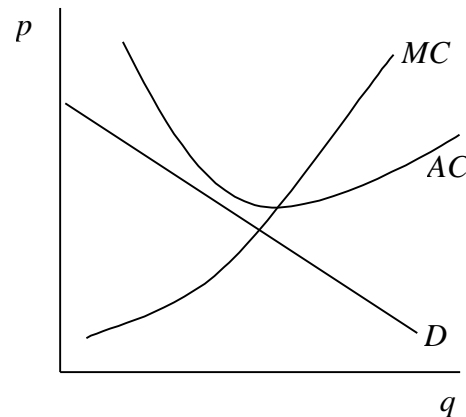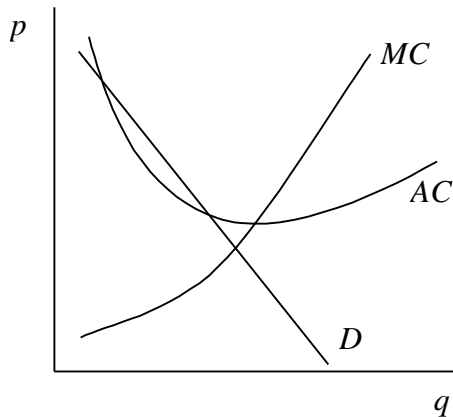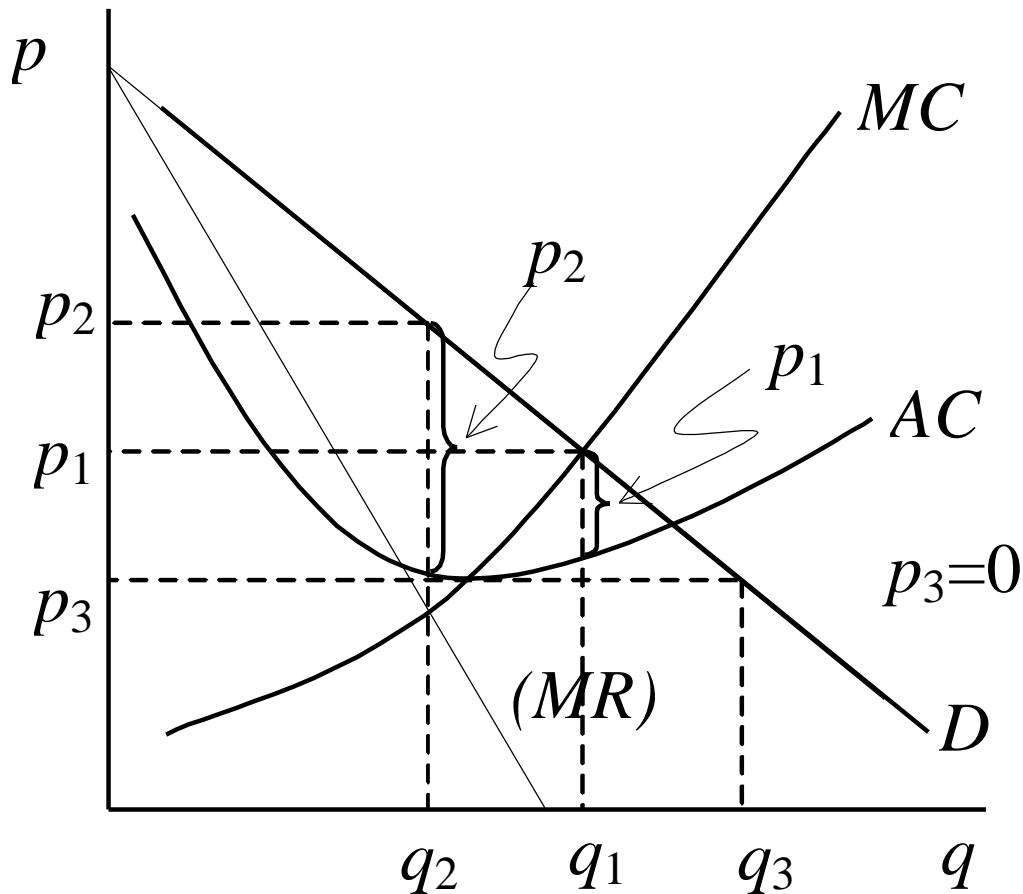
a.  If $n$ is large and fixed, then the market is perfectly competitive but without free entry
    and the market supply curve is the industry marginal cost curve.  Market equilibrium is
    therefore at the intersection of MC and D, as shown.  Profit per unit is the difference
    between the price $p_1$ and average cost, as shown.

b.  If $n$ is fixed at one, then this is a monopoly.  To find the equilibrium first construct the
    marginal revenue curve as a straight line halfway between he vertical axis and the
    demand curve.  Its intersection with the marginal cost curve gives the profit
    maximizing output, $q_2$, which then sells for the price above it on the demand curve, $p_2$.
    Profit per unit is again found as the distance between than price and average cost.

c.  If $n$ is large and variable, then the market is perfectly competitive and, in addition, the
    number of firms rises if there are profits, falls if there are losses, and reaches a long-
    run equilibrium only when profit is zero.  As noted in part (a), for the initial number of
    firms there is a positive profit, so there will be entry into the industry.  Each firm,
    including the new ones, has marginal and average cost curves that are just horizontally
    shrunken versions of the ones shown.  As more firms enter, these curves therefore
    expand horizontally to the right.  As the MC curve expands, the short-run competitive
    equilibrium moves down the demand curve and profit per unit shrinks.  This continues
    until profit is zero, which happens when price hits average cost.  However, the AC

curve is also expanding to the right with entry, and indeed the marginal cost curve always cuts the AC curve at its lowest point, so that zero profit is reached when both MC and AC cross each other and D at price $p_3$, as shown. In effect, the long run supply curve of the industry with free entry is a horizontal line at the minimum average cost, which is $p_3$.

d. If there are two firms, then the industry is a duopoly. How it behaves depends on the nature of the strategic interaction between the firms. Two possibilities are Cournot duopoly, in which each firm takes the other's output as given in setting its own, and Bertrand duopoly in which each takes the other's price as given. In general, equilibrium prices and outputs for duopoly lie somewhere between those for monopoly and for perfect competition.

e. In case (e), if the number of firms is fixed and large, then the fixed-$n$ competitive equilibrium (at the intersection of $MC$ and $D$) yields a price that is below average cost, so that the firms all make losses. A monopolist, on the other hand, will still be able to make a profit, since there exist quantities for which the price it can charge on the demand curve is above $AC$. If there is free entry and exit starting from the case shown of a large number of firms, then the losses noted above will cause some to exit and the cost curves will both shift to the left. This will continue until the intersection of the (moving) $MC$ and $AC$ curves hits the demand curve. At that point, a zero profit equilibrium is achieved at the same price $p_3$ given above.

f. Case (f) is the same as case (e) for large $n$, both fixed and variable. If this were a monopolist, however, it would simply shut down, since it cannot make a profit at any output. That is, for every output, the price that demanders are willing to pay is below its average cost.

3.  [6] Beth is the only consumer in an economy that produces only two goods:  books
    and hamburgers.  Beth's monthly income is $500.  Beth spends all her income on 5
    books and 21 hamburgers each week.  This bundle of goods makes Beth as happy as
    possible, given her budget.

    a.  [4] If hamburgers are $2.50 each, what is the marginal rate of substitution?

        If Beth's monthly income is $500, her weekly income $I_w = \$125$.  Let
                $P_h$ = price of hamburgers                $P_b$ = price of books
        Since Beth spends all her income each week on hamburgers and books,
                $$I_w = b \times P_b + h \times P_h$$
        Substitute in the given information:
                $$\$125 = 21 \times \$2.50 + 5 \times P_b$$
        Solve for $P_b$:
                $$P_b = \$14.50$$

        MRS = $P_b/P_h$ = $14.50/$2.50 = -5.8  (if you calculated $P_h/P_b$,
        MRS = 0.1724; both are correct.  It's just a question of which good went on
        which graph axis, and I didn't specify.)

    b.  [2] Given what we know about Beth's demand, what must be true about the
        marginal production costs of hamburgers and books for this economy to be in
        competitive equilibrium?

        For the economy to be in competitive equilibrium, price must equal marginal cost
        in both industries.  Therefore, $MC_h = P_h = \$2.50$, and $MC_b = P_b = \$14.50$.

4.  [12] The rosebush industry has a supply curve of S=2P-20, where S is the daily
    production of rosebushes.  Demand for rosebushes is given by D=100-0.5P.

    a.  [4] What is the price per rosebush at equilibrium?  How many rosebushes are
        produced daily at this equilibrium price?

        Set Q=S=D and solve for P.  Then substitute the result into either one of the
        original functions and solve:

                $100 - 0.5P = 2P - 20$
                $2.5P = 120$
                $P1 = 48$
                $S = 2(48) - 20$
                $Q = 96 - 20$
                $Q1 = 76$

    b.  [6] Suppose a government study finds that the growing of rosebushes enhances the
        value of properties adjacent to rosebush farms by creating pleasant smells and
        scenic views; however, the pesticides needed to produce rosebushes contaminate
        groundwater and cause increased rates of various illnesses in those that drink it.
        The net result is a negative externality of an estimated $45 per unit produced, so
        the government imposes a $45 per-unit tax on rosebush production.

        What is the new supply curve equation?  What is the new equilibrium price and
        quantity, given the tax?

        The initial supply curve could be solved for P to give the marginal cost curve:
        $P = 10 + 0.5S$.  The tax is a per-unit increase that can be viewed as increasing the
        marginal cost of firms, which therefore shifts up by $45, to $P = 55 + 0.5S$.
        Substituting this into the demand curve,
                $D = 100 - 0.5[55 + 0.5S] = 72.5 - 0.25S$.
        Setting Q=S=D and solving, $1.25Q = 72.5$, or
                $Q2 = 58$
        Including the tax, the price paid by demanders is $P = 55 + 0.5(58) = \$84$.  The net
        price received by suppliers is $84 - 45 = \$39$.

    c.  [2] How much of the tax is paid by rosebush producers?

        Since the price received by producers fell from $48 to $39, $9 of the $45 tax, or
        1/5 of it, can be said to have been paid by producers.