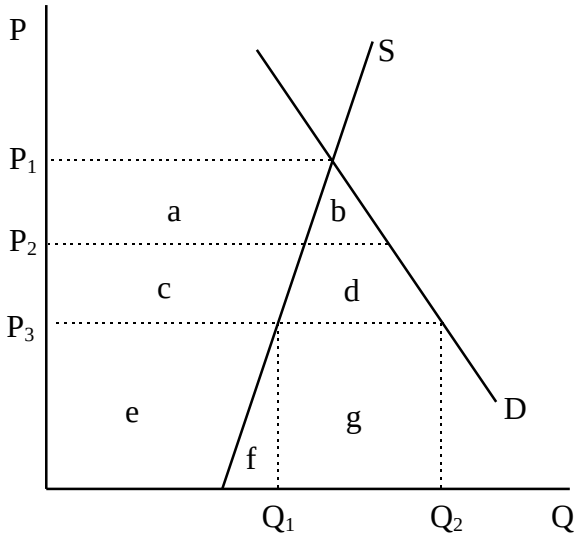
Problem Set #2 - Answers
Due February 2, 2000

[Numbers in brackets are the points allocated in the grading. There are 103 points total]

1. [19]In the figure at the right are a supply curve, S, and a demand curve, D, together with several labeled prices, quantities, and areas. Identify the following:



   a) [1]The market equilibrium price for this market alone.

   P_1

   b) [1]Total producer surplus in that market equilibrium

   a + c + e

   Suppose now that this market is not alone, but that buyers and sellers have access to another market (the world market) which is much larger and in which they can both buy and sell at will at price P_3. As a result, price in this market becomes P_3, with buyers and/or sellers using the world market to make up any difference between supply and demand. Identify the following:

   c) [4]The quantities of the good produced and consumed after opening. What is the nature and quantity of the trade between this market and the world?

   Q_1 is produced, Q_2 is consumed. Since $Q_2 > Q_1$, the country imports $Q_2 - Q_1$.

   d) [2]The revenue of suppliers and the expenditure by demanders.

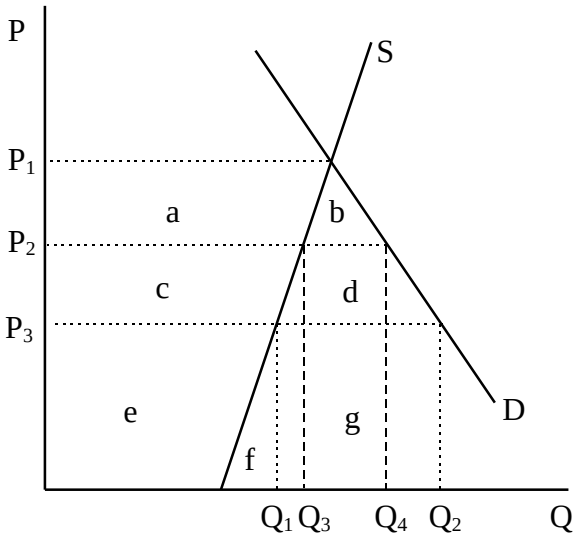
   Revenue = e + f. Expenditure = e + f + g.

   e) [4]The changes in producer and consumer surplus caused by opening up to this world market.

   Change in producer surplus = – (a + c)

   Change in consumer surplus = + (a + b + c + d)

Suppose finally that the price on the world market now rises, from P_3 to P_2. What changes occur, as a result of this price increase, in the following variables?



f) [3]Quantities supplied, demanded, and traded?

Supply rises to Q_3

Demand falls to Q_4

Imports fall to $(Q_4 - Q_3)$

g) [4]Producer and consumer surplus?

Change in producer surplus = + c

Change in consumer surplus = – (c + d)

2. [23]The demand for noodles in Hong Kong per day is given by

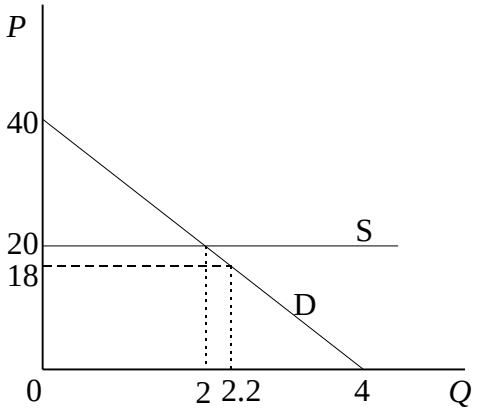
$$P = 40 - 10Q$$

where Q is the quantity of noodles sold per day, in millions of bowls, and P is the price in HK$ per bowl. Suppliers of noodles make them available at a constant cost of HK$20 per bowl. Recognizing the unique health benefits of noodles, the Hong Kong government is considering providing a subsidy to their production of HK$2 per bowl.

a) [6]Without the subsidy, calculate the market equilibrium quantity and price of noodles in Hong Kong.

$P = 40 - 10Q \quad \Rightarrow \quad Q = 4 - 0.1P$

and thus the demand curve is as drawn:



Supply is horizontal at a price of 20, since that is the constant (marginal) cost. Thus equilibrium price equals that marginal cost, or HK$20 per bowl. Equilibrium quantity is found by substituting that price in the equation above.
$Q^e = 4 - 0.1(20) = 4 - 2 = 2$. Thus

P^e = HK$20 per bowl

Q^e = 2 million bowls per day

b) [3]Hong Kong's GDP is in the neighborhood of US$100 billion. (Note this is US$, not HK$. The exchange rate is about HK$8=US$1) Approximately what percentage of Hong Kong's GDP is noodles, if the above information is correct? (Make explicit any additional assumptions that you make.)

Value of noodles produced and sold in US$ per year is

$$\begin{aligned} &20(HK\$/bowl)\times 2(mil.bowls/day)\times 365(days/yr)\times(1/8)(US\$/HK\$) \\ &\quad = 1825(mil.US\$/yr) \\ &\quad = 1.825(bil.US\$/yr) \end{aligned}$$

which is 1.8% of Hong Kong's US$100 bil. GDP. (Assumes 365 days per year.)

c) [4]If the proposed subsidy is provided, what will be the new quantity of noodles produced and consumed, how much will consumers pay per bowl, and how much will suppliers receive?

New quantity = $4-0.1(18)=4-1.8=2.2$ mil. bowls produced and consumed per day. Consumers pay HK$18 per bowl, while producers still receive HK$20 per bowl.

d) [6]Calculate the changes in consumer surplus and producer surplus due to the subsidy. How much will the subsidy cost the government?

Change in consumer surplus = $2\times\left(\frac{2.0+2.2}{2}\right)=2\times 2.1=4.2$ HK$ mil. per year

Change in producer surplus = 0, since they still receive their (constant) marginal cost.

Cost to government = $2\times 2.2=4.4$ HK$ mil. per year

e) [2]If consumers are unaware of these health benefits, how much would the benefits have to be worth in order for this subsidy to be a socially desirable policy?

Excluding any health benefits, the net social cost of this policy is the sum of the changes in consumer and producer surplus minus the cost to the government. This is 4.2 – 4.4 = HK$0.2 mil. per year.

If consumers are unaware of these health benefits, then they are not included in the marginal private benefits that they reflect in their demand curve, and therefore they constitute an additional benefit not included in the above. Thus if the health benefits are worth more than HK$0.2 mil. per year, then this policy is worth doing.

f) [2]How would your answer to part (e) change if the health benefits of noodles *are* correctly perceived by consumers?

If consumers *are* aware of the health benefits, then these are included in their private marginal benefit from consuming noodles, and they are already a part of the increase in consumer surplus that we have calculated. Therefore the fact that these health benefits exist does not provide any *additional* benefit above what we have already calculated. Since the analysis above shows that the policy imposes a net social cost, the policy is *not* desirable regardless of how large the health benefits may be. (Of course, as a check on this assumption, one might want to estimate the health benefits directly, if possible, and compare them to the change in consumer surplus. If the health benefits are known to be larger than the HK$4.2 mil. per year change in consumer surplus, then something may be wrong. The health benefits in this case cannot be included in the demand curve, unless perhaps consumption of noodles is in fact unpleasant, and people eat them only for their health benefits.)

3. [29]There are 10 households in Upper Middle Centrebrook, each with an annual demand for electricity of $q = 50 - P$. Upper Middle Centrebrook Hydro (UMCH), which generates electricity for the city, has a total cost curve of $TC = 500 + Q$.

a) In the past, UMCH has been required to give away electricity for nothing, satisfying the demands of all ten households. Its costs were covered by a lump-sum grant from the city, financed by equal lump-sum taxes on each of the ten households.

i) [2]How much electricity was produced and consumed each year?

At $P=0$, demand per household is $50 - P = 50$, and with 10 households, 500 units are produced and consumed.

ii) [2]How much tax did each household have to pay?

Since $TC=500+Q=500+500=1000$, each household must pay a tax of $1000/10=100$.

b) An SPP intern at the office of the Upper Middle Centrebrook City Administrator has pointed out that this method of providing electricity is not efficient.

i) [2]If the regulators of UMCH want to make sure that the market for electricity is efficient, what price will they force UMCH to charge?

Marginal cost of an extra unit of electricity is 1:

$$MC = \frac{dTC}{dQ} = \frac{d}{dQ}(500 + Q) = 1$$

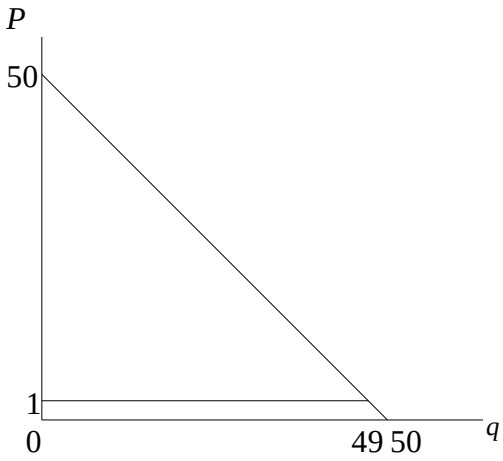
Therefore the price to charge in order to induce efficient use of electricity (where MB=MC) is 1.

ii) [2]What will output be in that case, if UMCH is also required to satisfy all demand at that price?

Demand, and therefore output, will be $10 \times (50 - 1) = 490$

iii) [4]Calculate the change in consumer surplus and the change in UMCH's profit, due to moving from the zero price to the efficient price. Assume that the same lump-sum taxes and the grant to UMCH from part (a) continue.

Change in consumer surplus for one household $= \Delta CS = \Delta P \times \frac{q_1 + q_2}{2} = 1 \times 49.5 = 49.5$. For ten households this is 495.



Change in profit includes the increase in revenue of 490 (1 for each unit now sold) plus a reduction in cost of 10 (the marginal cost of the 50^{th} units that are no longer produced for each of 10 consumers), for a total increase in profit of 500.

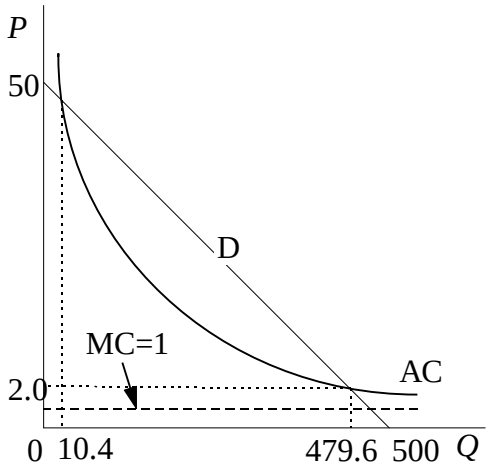
iv) [2]What would be the profit of UMCH, charging the efficient price, if the taxes and grant were discontinued?

Profit = Revenue – Cost = 490 – (500 + 490) = –500

c) Suppose now that citizens object to paying both a lump-sum tax and a positive price for electricity.

i) [3]If the regulators want to make sure that UMCH won't lose money in the absence of any grant from the city, what is the lowest price they can impose?

The lowest price that will avoid the firm losing money is average cost, $P = AC = TC / Q = (500 + Q) / Q = 1 + 500 / Q$. To be also on the demand curve, price and quantity must satisfy $Q = 10q = 10(50 - P) = 500 - 10P$. Thus, we must set a price that will put the firm on one of the intersections of AC and D, as shown at the right. Combining the two equations for AC and D and solving (using the quadratic equation), we get



$$Q = 500 - 10\left(1 + \frac{500}{Q}\right)$$

$$\Rightarrow \quad Q^2 - 490Q + 5000 = 0$$

$$\Rightarrow \quad Q = \frac{490 \pm \sqrt{490^2 - 4(5000)}}{2} = \frac{490 \pm \sqrt{220100}}{2}$$

$$= \frac{490 \pm 469.148}{2} = 479.574 \text{ or } 10.42$$

These two answers indicate the two intersections of the average cost curve with the demand curve. Of them, only the larger quantity, 479.574, is relevant, since that corresponds to the lower price. The corresponding price is $P = 1 + (500 / 479.574) = 2.043$.

ii) [4]Calculate output, consumer surplus, and profit in that case.

Profit is zero by design, and output has already been calculated as 479.574. Total consumer surplus is $CS = (1/2)(50 - 2.043)(479.574) = 11{,}499.465$.

iii) [8]How does this solution compare, in terms of total social welfare, to the other two?

Case A: Free Electricity

Consumer surplus	$CS = (1/2)(50)(500) =$	12,500	
Tax		−1000	
Net Households			11,500
Profit	Rev−TC = 0 − 1000	−1000	
Grant		1000	
Net UMCH			0
Social welfare			11,500

Case B: Efficient Pricing, with tax and grant

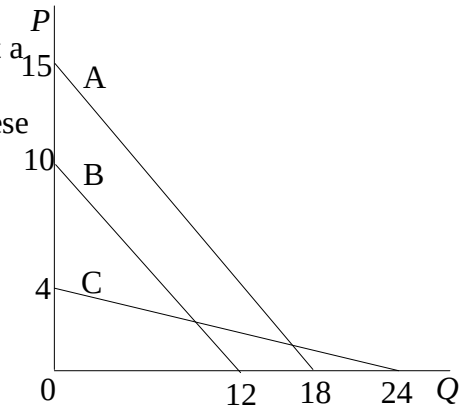
Consumer surplus	$CS = (1/2)(50-1)(490) =$	12,005	
Tax		−1000	
Net Households			11,005
Profit	Rev−TC = 490 − 990	−500	
Grant		1000	
Net UMCH			500
Social welfare			11,505

Case C: Minimum Break-Even Pricing

Consumer surplus		11,499	
Tax		0	
Net Households			11,499
Profit	Rev−TC = 2.043(479)−979	0	
Grant		0	
Net UMCH			0
Social welfare			11,499

Thus, the greatest social welfare, 11,505, is achieved with efficient pricing. This is greater than welfare with zero pricing (11,500) and with minimum break-even pricing (11,499).

4. [32]A community has three households, A, B, and C, each with a different demand curve for police protection, as shown below. As a group, they could purchase police protection for all of them together at a marginal cost of 18. Individually they could purchase it for just themselves at a marginal cost of 9. Assume that the same household demand curves apply for police protection in both of these forms.



a) [13]Graph this community's willingness to pay for community police protection as a public good, carefully labeling intercepts and kinks. What is the socially optimal level of community protection that they should select? If they divide the cost of that optimal protection equally among the households, how much will each pay and how much consumer surplus, if any, will each enjoy?

A: $P = 15 - (5/6)Q \quad Q = 18 - (6/5)P$

B: $P = 10 - (5/6)Q \quad Q = 12 - (6/5)P$

C: $P = 4 - (1/6)Q \quad Q = 24 - 6P$

Willingness to pay (W):

Q=0: $W = 29$

0<Q<12: $W = P_A + P_B + P_C = 29 - (11/6)Q$

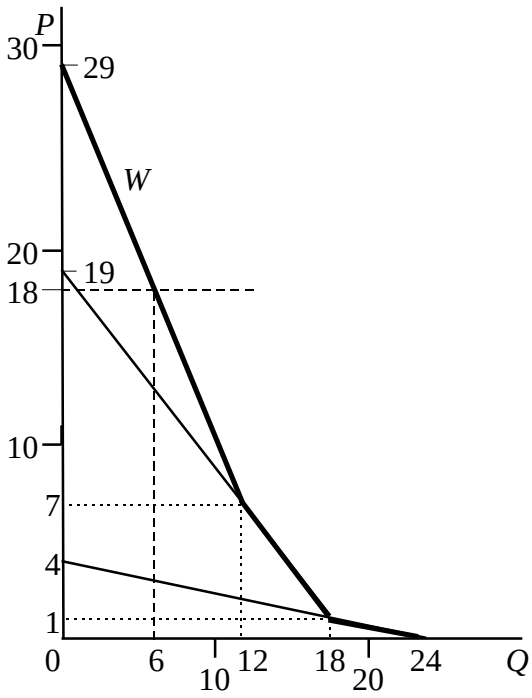
Q=12: $W = 7$

12<Q<18: $W = P_A + P_C = 19 - Q$

Q=18: $W = 1$

18<Q<24: $W = P_C = 4 - (1/6)Q$

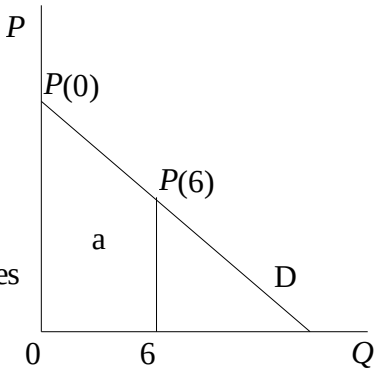
Q=24: $W = 0$



The community's willingness to pay, collectively, for the public good is the heavy line shown in the figure above. The social optimum is found where the marginal cost of the public good, 18, is equal to the total marginal benefit or willingness to pay of the three consumers added together, along this heavy line. Since a horizontal line at 18

crosses the line in its highest segment (above the kink at W=7), the equation for willingness to pay at that point is $18 = W = P_A + P_B + P_C = 29 - (11/6)Q$ or $(11/6)Q = 29 - 18 = 11$ or Q=6. Thus the socially optimal level of community protection is 6.

At this level of protection, the total cost is $18 \times 6 = 108$, and divided three ways each household pays 108/3=36. Each household gets a different level of benefit from the protection due to their different demand curves. The total willingness to pay of each is given by the area under its demand curve from zero to six units, as shown in the figure at the right for a generic demand curve that could belong to any of the three households. That area, marked a, is measured by its base times the average height at $P(0)$ and $P(6)$: $6\frac{P(0)+P(6)}{2}$.



Calculating this area for each household and subtracting their common cost of 36 yields the surplus of each:

A: $CS_A = 6\frac{15+10}{2} - 36 = 75 - 36 = 39$

B: $CS_B = 6\frac{10+5}{2} - 36 = 45 - 36 = 9$

C: $CS_C = 6\frac{4+3}{2} - 36 = 21 - 36 = -15$

a) [15]Graph the community's demand for police protection as a private good and determine the total amount that the three households together would buy at the price of 9. How much consumer surplus does each household get when it buys police protection privately. Is each better or worse off than in part (a)?

Market Demand (D):

P=0: D=54

0<P<4: $D = Q_A + Q_B + Q_C = 54 - (42/5)P$

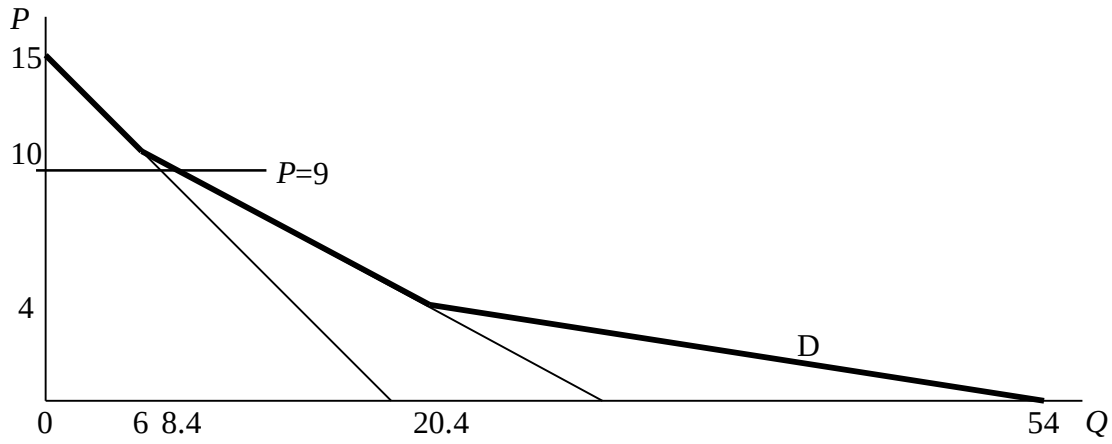
P=4: D=30–(48/5)=20.4

4<P<10: $D = Q_A + Q_B = 30 - (12/5)P$

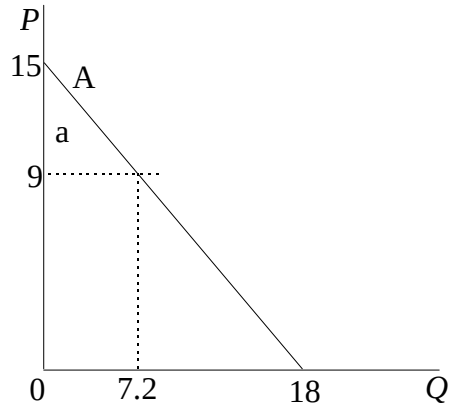
P=10: D=6

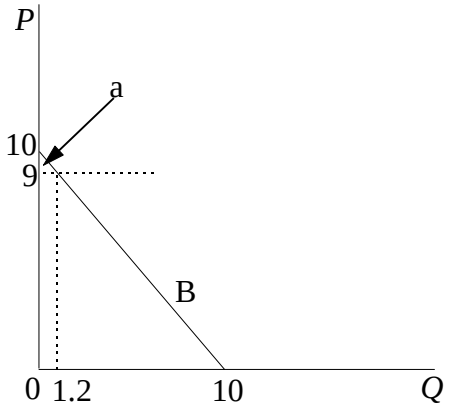
10<P<15: $D = Q_A = 18 - (6/5)P$

P=15: D=0



At P=9, these consumers (A and B only) demand $D = 30 - (12/5)9 = 8.4$.





Household A:

$Q = 18 - (6/5)9 = 7.2$

CS = *area a*

$= \frac{1}{2}(15-9)(7.2) = 21.6$

Household B:

$Q = 12 - (6/5)9 = 1.2$

CS = *area a*

$= \frac{1}{2}(10-9)(1.2) = 0.6$

Household C: They do not buy, and they get zero consumer surplus.

Households A and B are worse off than with the public good, household C is better off.

c) [4]How should police protection be provided to this community – as a public good or as a private good? What problems do you see with this solution?

Net social benefit is larger (36) when police protection is provided as a public good rather than as a private good, when the net social benefit is only 22.2. Therefore, it should be provided as a public good.

However, the difficulty is that one household (C) is made worse off if the good is provided as a public good and the cost is divided equally. By dividing the cost unequally this could be avoided, but then there is the problem of getting households truthfully to reveal their willingness to pay, since each will know that by understating their own willingness to pay they can avoid some of the cost.