Problem Set #3 - Answers
Due February 9, 2000

[Numbers in brackets are the points allocated in the grading. There are 75 points total]

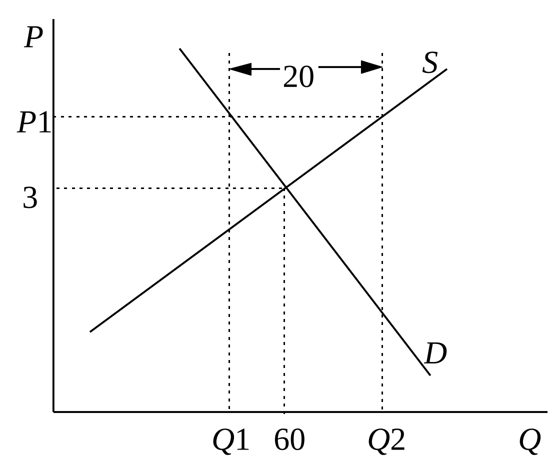
1. [48]The University of Michigan, concerned about the nutritional deficiencies of students' diets, is considering providing a healthy lunch to all students free of charge every day. A small-scale pilot project has already been conducted, and it determined that a standardized "healthy lunch" can be provided at current prices for $3. Students (who are, remarkably, identical) would only be willing to pay $1.25 for this healthy lunch, but the benefits to society as a whole (including the students themselves, their parents, the university health service, etc.) are three times this large, or $3.75. U of M has therefore decided tentatively to go ahead with the project, providing 20,000 free lunches each day at several locations around campus.
   However, they have recently realized that this much demand for healthy food on the Ann Arbor market might raise its price, and if so this might render the project less beneficial than was thought. Unhealthy food is available at a constant cost that will remain unchanged by this policy, but the market for healthy food is smaller and might be affected by this large a purchase by U of M. Ann Arbor currently sells the equivalent of only 60,000 healthy lunches per day, all bought by non-students.
   Your job, therefore, is to conduct a sensitivity analysis of the free lunch project with respect to the assumed elasticities of supply and demand in Ann Arbor for healthy food. That is, for each elasticity combination listed below, first find the new equilibrium price of healthy food after the U of M adds its own demand for 20,000 lunches to the existing demand. Then calculate the various welfare effects indicated. Report your results in a table like the one below, but also accompany it with the details of how you got your results.

   Ans:
   To solve this problem in general, let $x=\%\Delta p=\Delta p/p$ be the unknown percent change in price, and solve for that from the market equilibrium condition that equates $S=D$, or $\%\Delta S=\%\Delta D$. By definition, $\%\Delta S=ESx$, where ES is the elasticity of supply. The change in demand includes both the new purchase of 20,000 lunches by UM, which is a 1/3 increase, plus a response to price based on the elasticity of demand, ED: $\%\Delta D=(1/3)-EDx$. Thus equilibrium requires

   $$ESx = (1/3) - EDx$$

   or

   $$\%\Delta P = x = \frac{1}{3(ES+ED)}$$



ES=4, *ED*=0:

$\%\Delta P=0$
$P1=3(1+\%\Delta P)=3$
$\Delta CS= -(P1-3)(Q1+60)/2=0$
$\Delta PS=(P1-3)(Q2+60)/2=0$
$Cost=P1(20)= -60$
$Benefit=3.75(20)=75$
$Net=\Delta CS+\Delta PS-Cost+Ben=0+0-60+75= +15$

ES=4, *ED*=1:

$\%\Delta P=0$
$P1=3(1+\%\Delta P)=3$
$\Delta CS= -(p1-3)(Q1+60)/2=0$
$\Delta PS=(P1-3)(Q2+60)/2=0$
$Cost=P1(20)= -60$
$Benefit=3.75(20)=75$
$Net=\Delta CS+\Delta PS-Cost+Ben=0+0-60+75= +15$

ES=1, *ED*=0:

$\%\Delta P=1/3(1+0)=1/3$
$P1=3(1+\%\Delta P)=3(1.33)=4$
$Q1=60(1-ED\%\Delta P)=60$
$Q2=60(1+ES\%\Delta P)=60(1+1/3)=80$
$\Delta CS= -(P1-3)(Q1+60)/2= -(4-3)60= -60$
$\Delta PS=(P1-3)(Q2+60)/2=(4-3)70=70$
$Cost=P1(20)= 4(20)=80$
$Benefit=3.75(20)=75$
$Net=\Delta CS+\Delta PS-Cost+Ben= -60+70-80+75= +5$

ES=1, *ED*=1:

$\%\Delta P=1/3(1+1)=1/6$
$P1=3(1+\%\Delta P)=3(1+1/6)= 3.5$
$Q1=60(1-ED\%\Delta P)=60(1-1/6)=50$
$Q2=60(1+ES\%\Delta P)=60(1+1/6)=70$
$\Delta CS= -(P1-3)(Q1+60)/2= -(3.5-3)55= -27.5$
$\Delta PS=(P1-3)(Q2+60)/2=(3.5-3)65=32.5$
$Cost=P1(20)= 3.5(20)=70$
$Benefit=3.75(20)=75$
$Net=\Delta CS+\Delta PS-Cost+Ben= -27.5+32.5-70+75= +10$

ES=1, ED=2:

%ΔP=1/3(1+2)=1/9
P1=3(1+%ΔP)=3(1+1/9)= 3.33
Q1=60(1–ED%ΔP)=60(1–2/9)=46.67
Q2=60(1+ES%ΔP)=60(1+1/9)=66.67
ΔCS= – (P1–3)(Q1+60)/2= –(3.33–3)(46.67+60)/2= –17.78
ΔPS=(P1–3)(Q2+60)/2=(3.33–3)(66.67+60)/2=21.11
Cost=P1(20)= 3.33(20)=66.67
Benefit=3.75(20)=75
Net=ΔCS+ΔPS–Cost+Ben= –17.78+21.11–66.67+75= +11.66

ES=0.5, ED=0:

%ΔP=1/3(0.5+0)=1/1.5=2/3
P1=3(1+%ΔP)=3(1+2/3)=5
Q1=60(1–ED%ΔP)=60
Q2=60(1+ES%ΔP)=60(1+0.5(2/3))=80
ΔCS= – (P1–3)(Q1+60)/2= –(5–3)60= –120
ΔPS=(P1–3)(Q2+60)/2=(5–3)70=140
Cost=P1(20)= 5(20)=100
Benefit=3.75(20)=75
Net=ΔCS+ΔPS–Cost+Ben= –120+140–100+75= –5

ES=0.5, ED=1:

%ΔP=1/3(0.5+1)=1/4.5=2/9
P1=3(1+%ΔP)=3(1+2/9)=3.67
Q1=60(1–ED%ΔP)=60(1–2/9)=46.67
Q2=60(1+ES%ΔP)=60(1+0.5(2/9))=66.67
ΔCS= – (P1–3)(Q1+60)/2= –(3.67–3)(46.67+60)/2= –35.56
ΔPS=(P1–3)(Q2+60)/2=(3.67–3)(66.67+60)/2=42.22
Cost=P1(20)= 3.67(20)=73.33
Benefit=3.75(20)=75
Net=ΔCS+ΔPS–Cost+Ben= –35.56+42.22–73.33+75= 8.33

ES=0.5, ED=2:

%ΔP=1/3(0.5+2)=1/7.5=2/15
P1=3(1+%ΔP)=3(1+2/15)=3.4
Q1=60(1–ED%ΔP)=60(1–4/15)=44
Q2=60(1+ES%ΔP)=60(1+0.5(2/15))=64
ΔCS= – (P1–3)(Q1+60)/2= –(3.4–3)(44+60)/2= –20.8
ΔPS=(P1–3)(Q2+60)/2=(3.4–3)(64+60)/2=24.8
Cost=P1(20)= 3.4(20)=68
Benefit=3.75(20)=75
Net=ΔCS+ΔPS–Cost+Ben= –20.8+24.8–68+75= 11

Elasticity of Supply:	Infinite		1.0			0.5		
Elasticity of Non-Student Demand:	0.0	1.0	0.0	1.0	2.0	0.0	1.0	2.0
New Equilibrium Price [a]	3	3	4	3.5	3.33	5	3.67	3.4
Change in Non-Student Consumer Surplus [b]	0	0	–60	–27.5	–17.78	–120	–35.56	–20.8
Change in Healthy Food Producer Surplus [b]	0	0	70	32.5	21.11	140	42.22	24.8
Cost to UM [b]	–60	–60	80	70	66.67	100	73.33	68
Benefit to Students, Parents, etc. [b]	75	75	75	75	75	75	75	75
Net Benefit to Society [b]	15	15	5	10	11.66	–5	8.33	11

[a] $ per lunch. [b] Thousands of $ per day.

2. [10]The University has fielded many complaints from students who would like to go to home sports events but who cannot because regular tickets are too expensive and only a few seats are set aside for students at the lower student ticket price. Reserving additional seats for students means adding an additional athletics fee to each student's tuition.

   The University is considering three options:

   - WB: Reserve 1,000 more seats for students at women's basketball games and charge each student a fee
   - MH: Reserve 1,000 more seats for students at men's hockey games and charge each student a fee
   - DN: Do nothing and charge no additional fee.

   Three groups on campus have weighed in on the issue (assume equal membership of 100 students in each group and 100% voting). Their preferences are:

	Diehard Wolverines (DW)	Moderate Fans (MF)	Sleepyheads (SH)
1st choice	MH	WB	DN
2nd choice	WB	DN	MH
3rd choice	DN	MH	WB

a) [2]Suppose there were a series of elections between successive pairs of these options, the winner in each pairwise election running against the third option until one option has beaten both of the others. What would be the result?

The elections would never end. Under these circumstances, there can be no decision, since the winner in any election will lose the next election:

	Choices			
Elections	WB	MH	DN	Results
WB v. MH	MF	**DW, SH**	XXXXXX	MH
MH v. DN	XXXXXXX	DW	**MF, SH**	DN
WB v. DN	**DW, MF**	XXXXXX	SH	WB
			RESULT:	**NONE**

b) [4]Suppose U of M decides to have just two elections rather than a series of them: an election is held between two of the options, and the winner of that election is run off against the remaining option.

i) If the first election is WB v. DN, what are the options in the SECOND election, and what is the result?

As the table above shows, WB wins a vote between WB and DN. Therefore, the second and final election is WB v. MH, and MH wins.

ii) Does the order of elections matter? Why or why not?

Yes. Since each election has a different outcome, whichever one is completed first determines which option is eliminated from consideration and therefore which option is ultimately selected. Whichever option is NOT in the first election will win the second.

Election 1	**Election 2**	**Outcome**
WB v. DN	WB v. MH	**MH**
WB v. MH	MH v. DN	**DN**
MH v. DN	WB v. DN	**WB**

c) [4]Until now you have assumed an equal number of students, 100, in each group. If you now change the number in one group, the Sleepyheads, is it possible that the problems you have identified will go away. That is, if you increase or decrease the number of Sleepyheads, does there come a point at which one of the options becomes able to win (not tie) elections against both alternatives? If so, what number of Sleepyheads accomplishes this, and which option wins? Consider both i) increasing the number of Sleepyheads, and ii) decreasing them.

i) increasing

If you increase the number of Sleepyheads to 201, then option DN will win both elections against MH (301 to 100) and against WB (201 to 200).

ii) decreasing

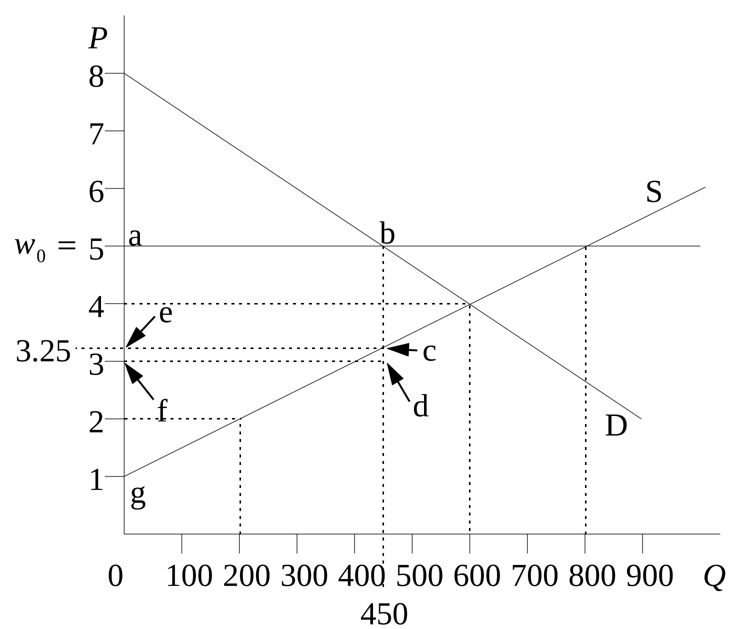
If you decrease the number of Sleepyheads, the problem will not disappear. As long as there are any Sleepyheads at all, even only one, they will tip the balance in elections of DN over MH and of MH over WB. Yet WB will win overwhelmingly over DN. If the number of Sleepyheads drops to zero, then MH and WB would tie.

3. [17]The figure below shows supply and demand in a labor market where a minimum wage is enforced at the level $w_0 = \$5.00$. Labor is measured by number of workers, and it is assumed that all employed workers have only one job, working standard hours. The upward sloping supply curve reflects the different willingness to work of different potential workers, worker No. 1 being willing to work at a wage of $1, worker No. 200 at a wage of $2, worker No. 800 at a wage of $5, and so forth as indicated by the supply curve, S. Answer each of the questions below three times, once for each of the following mechanisms for allocating scarce jobs whenever there is excess supply of labor:

i) Random allocation among those willing to work

i) Queuing, with all workers having the same cost per hour of waiting in line

ii) Bribery of hiring agents



a) [3]Identify, by their numbers and whatever words are necessary, which workers will be employed in the situation shown in the figure.

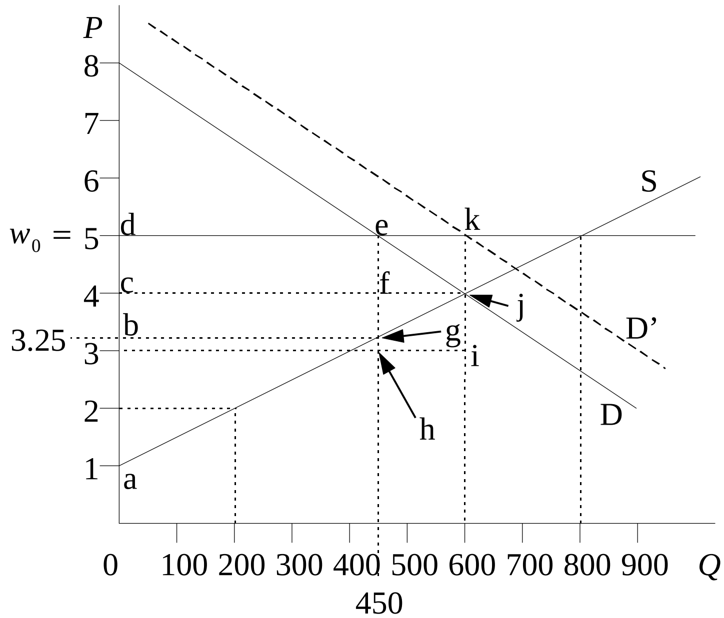
[You are not asked in this part to identify welfare effects, but I will do them anyway to make answering parts (b) and (c) easier.]

i) Random: There are 450 workers employed, chosen randomly from the group of workers labeled 1-800. Since their marginal opportunity cost of working ranges linearly from $1 to $5, these workers have an average opportunity cost of ($5+$1)/2=$3. Since all are paid $5, they earn a total surplus of the area abdf in the figure above, or $2 times 450 workers, for a total of $950 (per hour, presumably, if the wages are expressed per hour).

ii) Queuing: Workers numbered 1 to 450 are employed. The marginal worker has an opportunity cost of working of $3.25 (given by the supply function), and therefore the market clears when that worker has a queuing cost equivalent to a wage differential of $5-3.25=$1.75. With certainty about the outcome, the rest of workers 1-450 will join the line at the same time, all bearing a queuing cost of $1.75 (since their costs of waiting are the same), for a total queuing cost of $1.75(450)=$787.50. Workers then enjoy a total surplus of the triangle ecg=$2.25(450)/2=$506.25. If there is uncertainty about the outcome, then each worker may line up earlier, equating the difference between the wage and their own marginal cost of working to the cost of waiting. In that case, workers end up with no surplus, and the cost of waiting is the entire potential surplus, the trapezoid abcg=450($4+$1.75)/2=$1293.75. (This is actually not quite right, since with uncertainty workers 1-450, not knowing they will get jobs, will wait less, but workers above 450 will also wait. The total waiting cost for all workers, therefore may be above or below this, but it is a good estimate.) The net benefit to workers, then, is only $506.25 with certainty and zero with uncertainty.

iii) Bribery: Workers numbered 1 to 450 are again employed. The equilibrium bribe (assuming that hiring agents are unable to bribe-discriminate among potential workers and therefore get a single size bribe from all) is again equivalent to a wage differential of $5-3.25=$1.75. Workers again get the surplus triangle ecg=$2.25(450)/2=$506.25, while the hiring agents get total bribes of $1.75(450)=$787.50. The total surplus to workers and hiring agents as a group is therefore $506.25+787.50=$1293.75.

b) [6]Calculate the benefits and costs to all actual and potential workers in this market if the government now intervenes by hiring 150 workers in addition to whatever the private sector may demand.



i) Random: The additional 150 workers are chosen at random from the pool of workers numbered 0-800. Since their average marginal cost of working is $3, their expected surplus is the area ekih=$2(150) =$300.

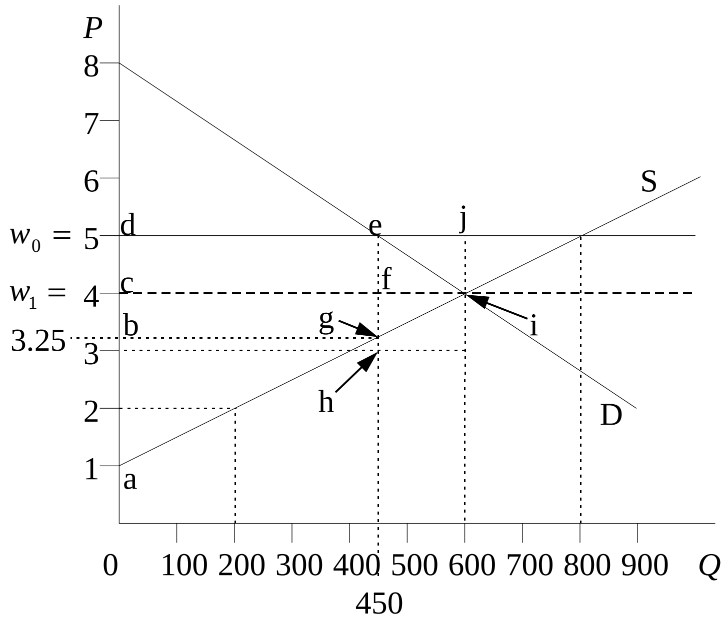
ii) Queuing: The marginal cost of the marginal worker, number 600, is $4, so that the surplus of that worker is now only $5-4=$1. Therefore the queue will form later, so that that worker, at least, only bears a waiting cost of $1. With certainty, this will be true of all workers, whose surplus collectively will therefore increase by the area of the trapezoid bgjc=$.75(450+600)/2=$393.75. Waiting cost falls by area cfgb=$.75(450)=$337.50 for the 450 workers who were employed before, but rises by area ekjf=$1(150)=$150 for the workers newly hired by the government. The total waiting cost therefore declines by $337.50-$150=$187.50. With uncertainty there is again no surplus, and total waiting cost merely increases by the trapezoid ekjg=(150)($1.75+$1)/2=$206.25.

iii) Bribery: The equilibrium bribe falls to $1. As in the case of queuing with certainty, the surplus of workers rises by trapezoid bgjc=$393.75. The total bribes collected by the hiring agents falls (as the queuing cost did above) by $187.50. Therefore the net gain to workers and hiring agents together is $393.75-$187.50=$206.25

c) [see below]Calculate the effects, instead, of reducing the minimum wage from $5.00 to $4.00.

[The question neglected to ask specifically for *all* effects here, so I will require only that you get the effects on actual and potential workers as in part (b). Be sure, however, that you also understand how to get effects on others.]

In all cases, the minimum wage is no longer binding, since it is set at the market-clearing level. Also in all cases, since the wage paid by demanders falls from $5 to $4, there is a gain in consumer surplus of the trapezoid deic=$1(450+600)/2 =$525.



i) [4]Random: Of the 450 workers initially employed at the $5 minimum wage, since they were chosen randomly, we may expect that ¼ of them, 112.5, were from the group numbered 600-800 who are now not willing to work at the wage $4. They quit. Since their average marginal cost of working was $4.50 (the average of wages over the interval $4 to $5), they lose a total surplus of $.50(112.5) =$56.25. The remaining 337.5 previous workers stay on the job, and they lose $1 of wage each, or a total of $337.50. Of workers numbered 0 to 600, there were previously also 600-337.5=262.5 workers who were not employed, and they now become employed at the $4 wage. Their average marginal cost of working was $2.50 (the average over the interval $1 to $4), so they gain an average surplus of $4-$2.50=$1.50, for a total of ($1.50)262.50=$393.75. The net effect on society is therefore +$525 (firms) – $56.25 (quitters) – $337.50 (continuing workers) + $393.75 (new workers) = $525.

ii) [2]Queuing: Queuing is no longer necessary, so the total surplus of workers is now just the usual triangle aic=($4–$1)600=$1800. If there previously was certainty, then workers gain the trapezoid cigb=$.75(450+600)/2=$393.75 and society gains +$525 (firms) + $393.75 (workers) = $918.75. If there previously was uncertainty so that workers got no surplus at all, then they gain the full $1800, and society gains $2325.

iii) [2]Bribery: Here the workers no longer pay any bribe at all, and their surplus rises again by trapezoid cigb=$393.75. The hiring agents lose all the bribes they were getting, or –$787.50. Society therefore gains +$525 (firms) + $393.75 (workers) –$787.50 (hiring agents) = $131.25.