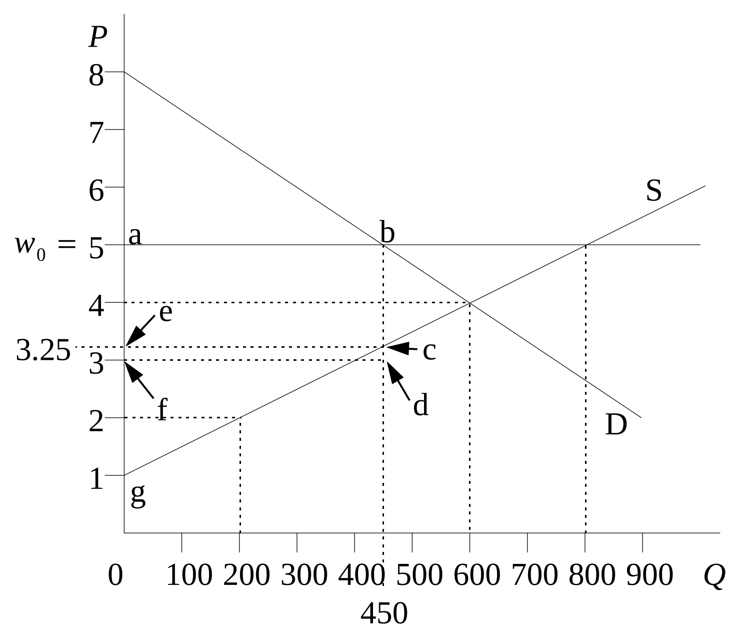
Problem Set #4 - Answers
Due March 8, 2000
(March 9, really, at 5:00 PM)

[Numbers in brackets are the points allocated in the grading. There are 94 points total.]

1. [34]The figure below shows supply and demand in a labor market where a minimum wage is enforced at the level $w_0 = \$5.00$. Labor is measured by number of workers, and it is assumed that all employed workers have only one job, working standard hours. The upward sloping supply curve reflects the different willingness to work of different potential workers, worker No. 1 being willing to work at a wage of $1, worker No. 200 at a wage of $2, worker No. 800 at a wage of $5, and so forth as indicated by the supply curve, S. Answer each of the questions below three times, once for each of the following mechanisms for allocating scarce jobs whenever there is excess supply of labor:

   i) Random allocation among those willing to work
   i) Queuing, with all workers having the same cost per hour of waiting in line
   ii) Bribery of hiring agents



a) [6]Identify, by their numbers and whatever words are necessary, which workers will be employed in the situation shown in the figure.

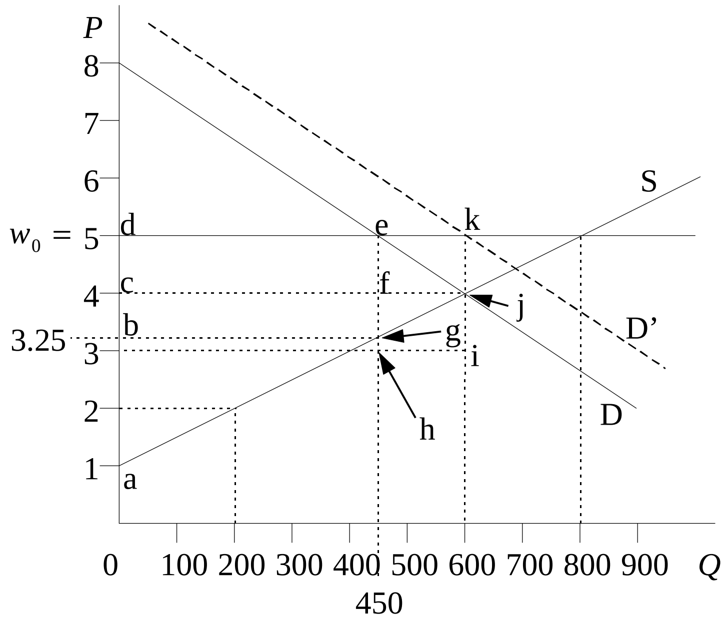
[You are not asked in this part to identify welfare effects, but I will do them anyway to make answering parts (b) and (c) easier.]

i) Random: There are 450 workers employed, chosen randomly from the group of workers labeled 1-800. Since their marginal opportunity cost of working ranges linearly from $1 to $5, these workers have an average opportunity cost of ($5+$1)/2=$3. Since all are paid $5, they earn a total surplus of the area abdf in the figure above, or $2 times 450 workers, for a total of $950 (per hour, presumably, if the wages are expressed per hour).

ii) Queuing: Workers numbered 1 to 450 are employed. The marginal worker has an opportunity cost of working of $3.25 (given by the supply function), and therefore the market clears when that worker has a queuing cost equivalent to a wage differential of $5-3.25=$1.75. With certainty about the outcome, the rest of workers 1-450 will join the line at the same time, all bearing a queuing cost of $1.75 (since their costs of waiting are the same), for a total queuing cost of $1.75(450)=$787.50. Workers then enjoy a total surplus of the triangle ecg=$2.25(450)/2=$506.25. If there is uncertainty about the outcome, then each worker may line up earlier, equating the difference between the wage and their own marginal cost of working to the cost of waiting. In that case, workers end up with no surplus, and the cost of waiting is the entire potential surplus, the trapezoid abcg=450($4+$1.75)/2=$1293.75. (This is actually not quite right, since with uncertainty workers 1-450, not knowing they will get jobs, will wait less, but workers above 450 will also wait. The total waiting cost for all workers, therefore, may be above or below this, but it is a good estimate.) The net benefit to workers, then, is only $506.25 with certainty and zero with uncertainty.

iii) Bribery: Workers numbered 1 to 450 are again employed. The equilibrium bribe (assuming that hiring agents are unable to bribe-discriminate among potential workers and therefore get a single size bribe from all) is again equivalent to a wage differential of $5-3.25=$1.75. Workers again get the surplus triangle ecg=$2.25(450)/2=$506.25, while the hiring agents get total bribes of $1.75(450)=$787.50. The total surplus to workers and hiring agents as a group is therefore $506.25+787.50=$1293.75.

b) [12]Calculate the benefits and costs to all actual and potential workers in this market if the government now intervenes by hiring 150 workers in addition to whatever the private sector may demand.



i) Random: The additional 150 workers are chosen at random from the pool of workers numbered 0-800. Since their average marginal cost of working is \$3, their expected surplus is the area ekih=\$2(150) =\$300.

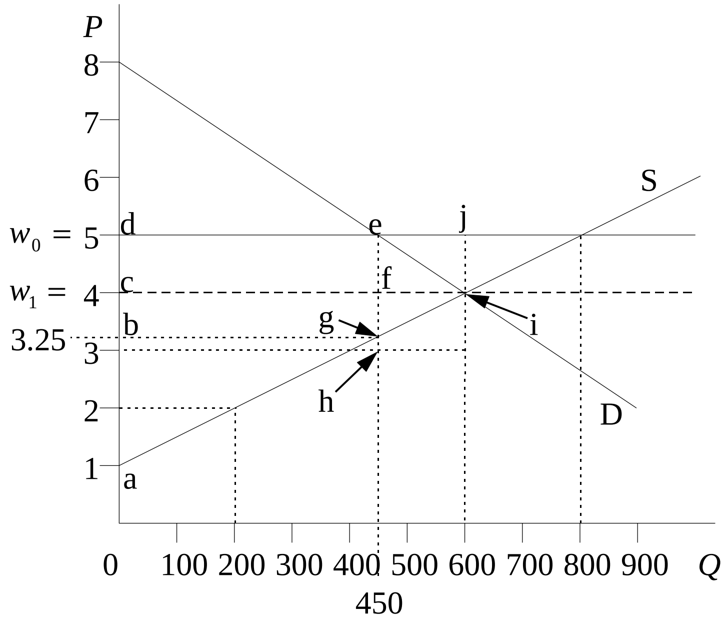
ii) Queuing: The marginal cost of the marginal worker, number 600, is \$4, so that the surplus of that worker is now only \$5-4=\$1. Therefore the queue will form later, so that that worker, at least, only bears a waiting cost of \$1. With certainty, this will be true of all workers, whose surplus collectively will therefore increase by the area of the trapezoid bgjc=\$.75(450+600)/2=\$393.75. Waiting cost falls by area cfgb=\$.75(450)=\$337.50 for the 450 workers who were employed before, but rises by area ekjf=\$1(150)=\$150 for the workers newly hired by the government. The total waiting cost therefore declines by \$337.50-\$150=\$187.50. With uncertainty there is again no surplus, and total waiting cost merely increases by the trapezoid ekjg=(150)(\$1.75+\$1)/2=\$206.25.

iii) Bribery: The equilibrium bribe falls to \$1. As in the case of queuing with certainty, the surplus of workers rises by trapezoid bgjc=\$393.75. The total bribes collected by the hiring agents falls (as the queuing cost did above) by \$187.50. Therefore the net gain to workers and hiring agents together is \$393.75-\$187.50=\$206.25

c) [see below]Calculate the effects, instead, of reducing the minimum wage from \$5.00 to \$4.00.

[The question neglected to ask specifically for *all* effects here, so I will require only that you get the effects on actual and potential workers as in part (b). Be sure, however, that you also understand how to get effects on others.]

In all cases, the minimum wage is no longer binding, since it is set at the market-clearing level. Also in all cases, since the wage paid by demanders falls from $5 to $4, there is a gain in consumer surplus of the trapezoid deic=$1(450+600)/2 =$525.



i) [8]Random: Of the 450 workers initially employed at the $5 minimum wage, since they were chosen randomly, we may expect that ¼ of them, 112.5, were from the group numbered 600-800 who are now not willing to work at the wage $4. They quit. Since their average marginal cost of working was $4.50 (the average of wages over the interval $4 to $5), they lose a total surplus of $.50(112.5) =$56.25. The remaining 337.5 previous workers stay on the job, and they lose $1 of wage each, or a total of $337.50. Of workers numbered 0 to 600, there were previously also 600-337.5=262.5 workers who were not employed, and they now become employed at the $4 wage. Their average marginal cost of working was $2.50 (the average over the interval $1 to $4), so they gain an average surplus of $4-$2.50=$1.50, for a total of ($1.50)262.50=$393.75. The net effect on society is therefore +$525 (firms) – $56.25 (quitters) – $337.50 (continuing workers) + $393.75 (new workers) = $525.

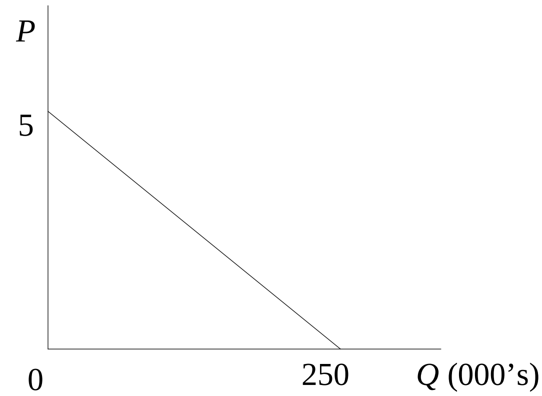
ii) [4]Queuing: Queuing is no longer necessary, so the total surplus of workers is now just the usual triangle aic=($4–$1)600=$1800. If there previously was certainty, then workers gain the trapezoid cigb=$.75(450+600)/2=$393.75 and society gains +$525 (firms) + $393.75 (workers) = $918.75. If there previously was uncertainty so that workers got no surplus at all, then they gain the full $1800, and society gains $2325.

iii) [4]Bribery: Here the workers no longer pay any bribe at all, and their surplus rises again by trapezoid cigb=$393.75. The hiring agents lose all the bribes they were getting, or –$787.50. Society therefore gains +$525 (firms) + $393.75 (workers) –$787.50 (hiring agents) = $131.25.

2. [36]The Ann Arbor Arboretum (the Arb) is a large park that is currently available free to all users. At the zero price, users currently make 250,000 visits to the Arb each year. The city spends $375,000 per year maintaining the Arb, of which $125,000 is considered to be a fixed cost that would be needed regardless of the number of visits, and the rest is an estimated variable cost of $1 per visit. It is believed that demand for visits to the Arb is linear, and that demand would be positive for any price per visit below $5 but zero at prices of $5 or higher.

   a) [4]Calculate the total consumer surplus enjoyed by visitors to the Arb each year. How much is this per visit? How does it compare to the city's average cost per visit?

The demand curve is as drawn:

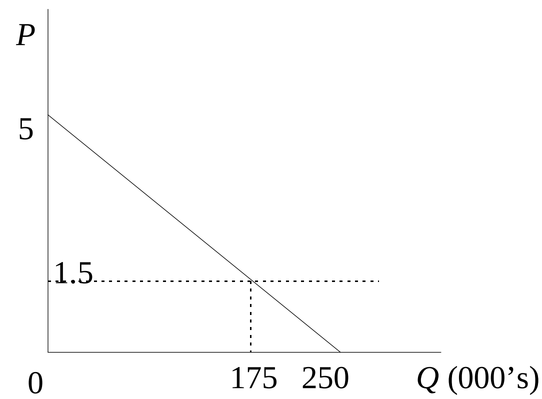


Consumer surplus is the entire area under the demand curve, since price is zero. This is 5(250)/2=625 or $625,000.

Per visit, this is 625/250=$2.50

The city's average cost is 375/250=$1.50 per visit, which is smaller.

   b) [4]Suppose now that the Arb were turned over to a private firm that was allowed to charge admission to the park. Fixing the price of admission at the average cost per visit currently observed, what would be the *change in* consumer surplus and what would be the profit (or loss) of the firm?

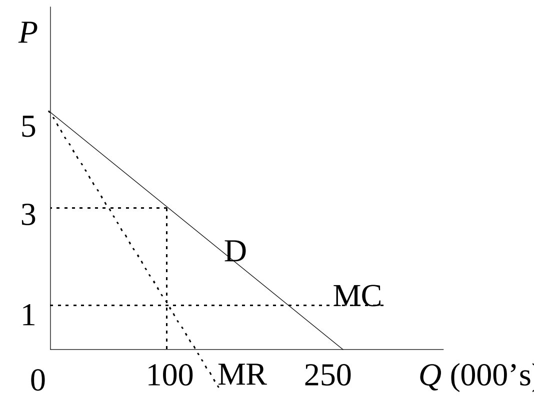


Fixing the price at $1.50 (the average cost per visit found in part (a)), the quantity demanded falls to 175,000 visits. (The demand curve is Q=250–50P, so Q=250–50(1.5)=175) The loss of consumer surplus is 1.5(175+250)/2=318.75 in thousands, or $318,750.

The firm earns revenue of 1.5(175)=262.5 and incurs cost of 125 (fixed) plus 1(175) (variable) = 300. Therefore the firm loses 300–262.5 = 37.5 = $37,500.

c) [6]If free to charge any price, what would the firm charge, and what would be the *level of* total consumer surplus and profit in that situation?

Marginal cost is $1 per visit. Marginal revenue is given by the straight line half way between the (straight line) demand curve and the vertical axis, or Q=125–25P, which cuts the MC curve at Q=125–25(1)=100. The profit maximizing monopoly firm will therefore set price so that Q=100, which means 100=250–50P or 50P=150 or P=3. That is, it charges $3 per visit and there are, as a result 100,000 visits.



The level of consumer surplus at a price of $3 is the triangle above the $3 price line, or (5–3)(100)/2=100=$100,000.

The profit in that situation is revenue minus cost, or 3(100)–(125+1(100))=300–225=75=$75,000.

d) [2]Suppose that, at the current price of zero, the 100,000 residents of Ann Arbor are in three groups: 45,000 are Couch Potatoes who never visit the Arb at all; 25,000 are Token Tree Huggers who visit it twice a year; and 30,000 are Nature Nuts who visit it five times a year. The remaining visits are by nonresidents who visit only once a year. All visitors to the Arb have linear demand curves that cross the price axis at $5. (Visits are perfectly divisible, so that, for example, a nonresident facing a price of $1 would visit 0.8 times a year.) What is the socially efficient price to charge each of these groups of visitors (society being defined as the world as a whole, not just Ann Arbor)?

The marginal cost of a visit is $1, so the socially efficient price to charge is also $1. The differences in the demand curves are irrelevant.

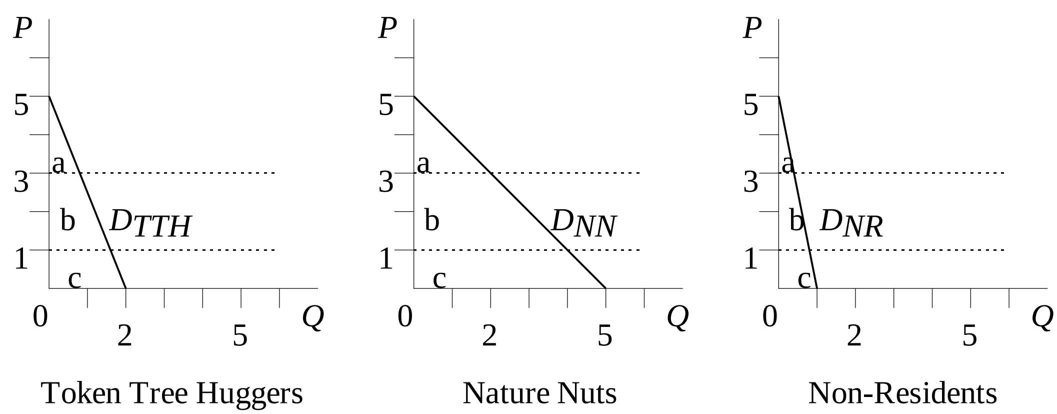
e) [12]Assuming that the current cost to the city of maintaining the Arb is shared equally among all 100,000 residents, what is the net benefit to each type of person of having the Arb (compared to closing it) and sharing the cost? Compared to this, how much would each type gain or lose if the Arb were privatized as in part (c)? Also compared to free admission, how much would each type gain or lose if the socially efficient price of part (d) were charged and costs were shared equally among Ann Arbor residents? Record your results in a table like that on the following page:

Table A
Individual Net Benefits

	Net Benefit if Free Admission	**Gain from Privatization***	**Gain from Efficient Pricing***
Couch Potatoes	–3.75	3.75	2.50
Token Tree Huggers	1.25	–0.45	0.70
Nature Nuts	8.75	–6.75	–2.00
Nonresidents	2.50	–2.10	–0.90

Units are $ per individual.
*Compared to free admission

The demand curves of three of the four types of individual are shown below (Couch Potatoes have zero demand at all prices), based upon our knowledge that each is linear with a price intercept of $5, together with the number of visits that they are observed



to make now at a price of zero. Also drawn are horizontal lines at the monopoly price $3 from part (c) and at the efficient price $1 from part (d).

With free admission each type of person gets consumer surplus of areas (a+b+c) (Couch Potatoes get zero). Residents also bear a cost of $3.75 per person, equal to the total cost divided by the number of residents. The net benefits of each are therefore as follows:

Couch Potatoes: Benefit=0, Cost=$3.75, Net Benefit= –$3.75

Token Tree Huggers: Benefit=5(2)/2=$5, Cost=$3.75, Net Benefit= +$1.25

Nature Nuts: Benefit=5(5)/2=$12.50, Cost=$3.75, Net Benefit= +$8.75

Non-Residents: Benefit=5(1)/2=$2.50, Cost=0, Net Benefit= +$2.50

At the monopoly price of \$3, consumer surplus of each group is reduced to area a, but they now bear no cost since the monopoly covers that out of its revenues. Letting B=Benefit, C=Cost, NB=Net Benefit, and ΔNB=Change in Net Benefit (you could also do this directly from changes in B and C), we get the following:

Couch Potatoes: B=0, C=0, NB=0, ΔNB= +\$3.75

Token Tree Huggers: B=(5–3)(0.8)/2=\$0.80, C=0, NB= +\$0.80, ΔNB= –\$0.45

Nature Nuts: B=(5–3)(2)/2=\$2, C=0, NB= +2, ΔNB= –\$6.75

Non-Residents: B=(5–3)(0.4)/2=\$0.40, C=0, NB= \$0.40, ΔNB= –\$2.10

At the efficient price of \$1, consumer surplus is areas (a+b). Demand is 250–50(1)=200=200,000 visits, and therefore the city's cost becomes 125+1(200)=325=\$325,000. The city's revenue is 1(200)=200=\$200,000, so its net cost is 325–200=125=\$125,000. Divided by 100,000 residents, each resident must pay 125/100=\$1.25. Therefore the benefits and costs are as follows, where ΔNB is again the change in net benefit starting from the present situation of a zero price.

Couch Potatoes: B=0, C=1.25, NB= –1.25, ΔNB= +\$2.50

Token Tree Huggers: B=(5–1)(1.6)/2=3.20, C=1.25, NB= +1.95, ΔNB= +\$0.70

Nature Nuts: B=(5–1)(4)/2=8, C=1.25, NB= +6.75, ΔNB= –\$2.00

Non-Residents: B=(5–1)(0.8)/2=1.60, C=0, NB= 1.60, ΔNB= –\$0.90

f) [6]What are the net benefits to Ann Arbor and to society as a whole of each of these options?

In the table below, the numbers from part (e) are simply multiplied by the number of people in each group, then summed for residents (all except nonresidents) and society as a whole. This gives the net benefits of having the Arb with free admission versus closing it off in the first column, and then the net benefits of moving from free admission to either privatization or efficient pricing in the other columns. Since the question was unclear about what you were to compare to here, the second table gives the net benefits of all three options compared to closing the Arb.

Table B
Group Net Benefits, in $
(Compared to Free Admission)

	Net Benefit if Free Admission (=Table A ´N)	Gain from Privatization (=Table A ´N)	Gain from Efficient Pricing (=Table A ´N)
CP (N=45)	-168,750	168,750	112,500
TTH (N=25)	31,250	-11,250	17,500
NN (N=30)	262,500	-202,500	-60,000
NR (N=50)	125,000	-105,000	-45,000
Residents	125,000	-45,000	70,000
Society as a Whole	250,000	-150,000	25,000

Table C
Group Net Benefits, in $
(Compared to Closing Down)

	Net Benefit if Free Admission (=Table B)	Gain from Privatization (=Table B, col 1+2)	Gain from Efficient Pricing (=Table B, col 1+3)
Couch Potatoes	-168,750	0	-56,250
Token Tree Huggers	31,250	20,000	48,750
Nature Nuts	262,500	60,000	202,500
Nonresidents	125,000	20,000	80,000
Residents	125,000	80,000	195,000
Society as a Whole	250,000	100,000	275,000

g) [2]Which of these options, if any, would be selected by majority voting?

The Couch Potatoes would prefer all options over the present (free admission), and would most prefer to either shut down the Arb or privatize it, but they do not have a majority (only 45,000 of 100,000 residents). The Token Tree Huggers would most prefer efficient pricing, then free admission (the status quo), then privatization, and finally closing. The Nature Nuts would most prefer the status quo of free admission, then the efficient price, then privatization, and finally closing down. Closing (if you considered it - you didn't need to) is therefore not an option, since TTH and NN will outvote CP to keep it open under any option. Among the remaining possibilities, there could be votes on any pair, with the following numbers of votes for each:

	Free vs. Private		Free vs. Efficient		Private vs. Efficient	
	Free	Private	Free	Efficient	Private	Efficient
Voters for	TTH, NN	CP	NN	CP, TTH	CP	TTH, NN
Votes (000's)	55	45	30	70	45	55
Winner	Free		Efficient		Efficient	

The Efficient Pricing option will win in majority voting, since it defeats all alternatives.

3. [24 (3 each)]Calculate the present discounted value of the projects listed in the table below, which reports for each of four projects, a, b, c, and d, [this should have said "eight projects, a,…,h"] the relevant interest rate, r, and the benefits (positive) and costs (negative) in the present (t=0), and each of t years from the present.

Project	Interest rate	Benefits (+) and Costs (–) in present (0) and future years, t= 0	1	2	3	4…9	10	11	12…∞
a)	5%	–700	300	400					
b)	3%	5	–5	–5	–5	–5	–5		
c)	7%	–200	14	14	14	14	14	14	14
d)	10%							100	100
e)	6%	–50	–50	–50	6	2*t	20	75	
f)	–2%	–1000	100	100	100	100	100		
g) x=1.03	4%		10x	$10*x^2$	$10*x^3$	$10*x^t$	$10*x^{10}$	$10*x^{11}$	$10*x^t$
h)	1%		10	10	10	10	10	10	10

a) $PV(a) = -700 + \frac{300}{1.05} + \frac{400}{(1.05)^2} = -700 + 285.7 + 362.8 = -51.5$

Uses general formula, $PV = \sum_{t=0}^{T} \frac{X_t}{(1+r)^t}$

b) $PV(b) = 5 + \frac{-5}{0.03}\left[1 - \frac{1}{(1.03)^{10}}\right] = 5 - 166.7[1 - 0.744] = -37.651$

Uses formula for constant X_t, t=1,…,T, $PV = \frac{X}{r}\left[1 - \frac{1}{(1+r)^T}\right]$

c) $PV(c) = -200 + \frac{14}{0.07} = -200 + 200 = 0$

Uses formula for constant X_t, t=1,…,4, $PV = \frac{X}{r}$

d) $PV(d) = \frac{100}{0.1} - \frac{100}{.01}\left[1 - \frac{1}{(1.1)^{10}}\right] = 1000 - 1000[1 - .3855] = 385.5$

Uses both of the formulas in (b) and (c) by evaluating the infinite sum, then subtracting the missing finite sum. This could also be done more directly by realizing that in year 10, this will be a constant amount each year from the next year on, and therefore will have the value in year 10 of X/r=100/.1=1000. Then just use the general formula to discount this back to the present, $1000/(1.1)^{10}$.

e)

$$PV(e) = -50 + \frac{-50}{1.06} + \frac{-50}{(1.06)^2} + \frac{6}{(1.06)^3} + \frac{8}{(1.06)^4} + \frac{10}{(1.06)^5} + \frac{12}{(1.06)^6}$$
$$+ \frac{14}{(1.06)^7} + \frac{16}{(1.06)^8} + \frac{18}{(1.06)^9} + \frac{20}{(1.06)^{10}} + \frac{75}{(1.06)^{11}} = -33.6825$$

Uses general formula, $PV = \sum_{t=0}^{T} \frac{X_t}{(1+r)^t}$. Most easily evaluated in a spread sheet.

f) $PV(f) = -1000 + \frac{100}{-0.02}\left[1 - \frac{1}{(0.98)^{10}}\right] = -1000 + \frac{100}{0.02}\left[\frac{1}{(0.98)^{10}} - 1\right] = 119.4057$

Uses formula for constant X_t, t=1,…,T, $PV = \frac{X}{r}\left[1 - \frac{1}{(1+r)^T}\right]$. Note that first bracketed expression is now negative, since 0.98 is less than one, so this does still come out positive.

g) $PV(g) = \frac{10.3}{0.04 - 0.03} = \frac{10.3}{0.01} = 1030$

Uses formula for appreciating $X_t = X(1+a)^{t-1}$, t=1,…,4, $PV = \frac{X}{r-a}$. Note that formula has appreciation starting only in period 2, but problem has it in period 1, so the X in the formula is 10(1.03)=10.3, not just 10.

h) $PV(h) = \frac{10}{0.01} = 1000$

Uses formula for constant X_t, t=1,…,4, $PV = \frac{X}{r}$. You might want to note the similarity between this and part (g). If the quantity appreciates at a constant rate, this is almost like reducing the interest rate by that amount.