Problem Set #5 - Answers
Due March 27, 2000

[Numbers in brackets are the points allocated in the grading. There are 31 points total.]

1. [13] Lee is tired of sitting in the cold (but refreshing) Michigan winter drafts and is considering insulating his home. It would cost him $5000 this year to insulate (he's got a big house), and the estimated reduction in his yearly fuel costs at this year's prices would be $350. The insulation can be installed immediately, so that he saves this amount on fuel even in the current year. Assume that the nominal interest rate is 9% a year and that the rate of inflation is 2% a year on everything, including fuel unless specified otherwise. Lee plans to live in the house until he retires, 23 years from today, at which point he will sell the house and move to someplace warmer. He has already consulted a real estate agent for advice and was told that insulation will not add anything at all to the value of the house when it is sold.

   (a) [3] What is the net present value of insulating Lee's humble abode? Should Lee insulate?

   For all of the formulas in these answers, let
   B=yearly fuel cost reduction,
   C=cost to insulate,
   a=rate of real appreciation of fuel costs
   p=rate of inflation
   r=real interest rate (which is our discount rate)
   T=number of <u>future</u> benefit years
   Next, two basic points:
   - The total time span for this project is 23 years. Discounting formulas treat the first year of any span as the present, so an exponent of 0 is applied when discounting any first-year payment. This means that discounting flows in the 23rd year is done with an exponent of T=22.
   - The basic discounting formula assumes no flows in the 0 period. If a payment accrues in the 0 period, which it does here, it must be directly added to or subtracted from the net present value of the future payments in the stream to get the PV of the entire flow (it's already in PV terms so no discounting is necessary).
   - The nominal interest rate is 9% and the rate of inflation is p =2%, so the real interest rate is r=7%.

   Thus:

$$PV = B + \frac{B}{r}\left[1 - \frac{1}{(1+r)^T}\right] - C = \$350 + \frac{\$350}{0.07}\left[1 - \frac{1}{(1.07)^{22}}\right] - \$5000 = -\$779$$

Net present value is negative, so Lee should not insulate. Note that this includes the present value of a cost saving of

$$B + \frac{B}{r}\left[1 - \frac{1}{(1+r)^T}\right] = \$350 + \frac{\$350}{0.07}\left[1 - \frac{1}{(1.07)^{22}}\right] = \$4{,}221.43$$

but this is smaller than the cost of installing the insulation.

(b) [3] If fuel prices were expected to rise at a rate of 5% per year instead of the 2% rate of the general price level, what would be the net present value of Lee's insulation project? Should Lee insulate in this case?

Fuel prices are now rising three percentage points faster than other prices, which means that the real benefits of the insulation are rising at 3% a year. Use the formula for an appreciating stream of costs or benefits:

$$PV = B + \frac{B}{r-a}\left[1 - \frac{(1+a)^T}{(1+r)^T}\right] - C = \$350 + \frac{\$350}{0.07 - 0.03}\left[1 - \frac{(1.03)^{22}}{(1.07)^{22}}\right] - \$5000 = \$315.70$$

This is positive, so Lee should insulate.

(c) Suppose that the real estate agent is wrong, and that insulation does, after all, add to the value of a house when it is sold, adding the (then) present discounted value of the cost savings over all later times. The price of Lee's house today (without insulation) is $600,000. Assume as in part (a) that heating oil prices are constant in real terms.

i) [2] At today's prices, what will Lee's house be worth 23 years from today if he insulates it?

At today's prices, the cost savings from the insulation is again $350 per year. The present discounted value of that for year 23 plus every year thereafter is again $350 plus the usual formula for an constant payment per year for infinite time, discounted at the real interest rate of 7%; thus: $350+$350/0.07=$5350. Therefore the price of Lee's house 23 years from now if he insulates it will be $600,000 + $5,350 = $605,350.

ii) [2] What is the net present value today of insulating Lee's house today, allowing both for Lee's own cost saving as in part (a) and for the increased value of the house when he sells it.

The discounted value today of the $5350 increase in sale price is $5350/$(1.07)^{23}$ = $1128.57. Added to the value found in part (a) for the value

of what Lee saves himself, this is \$4221.43 + 1128.57 = \$5350. The net value, deducting the \$5000 cost of the insulation, is now positive: \$350.

iii) [1] Should Lee insulate in this case?

Yes.

d) [2] Calculate the present value of insulating Lee's house if he were to decide never to sell it at all, but rather to keep it for himself and for his descendants forever. How does your answer compare to your answer in part (c-ii), and why?

This is just the present value of an infinite stream of \$350 forever, starting in the current year:

$$PV = B + \frac{B}{r} - C = \$350 + \frac{\$350}{0.07} - \$5000 = \$350$$

This is the same as the answer to part (c-iii), because it really does not matter whether we impute the services of the insulation to Lee himself, as here, or to subsequent owners of the house and build it into the price they pay him. The answer should be the same.

2. [18] You are currently 40 years old (imagine that!) and you expect to work another 25 years before retirement. In your job as a public policy analyst, you are earning \$40,000 a year and, unless you take steps to change that, you expect to continue earning that same amount in real terms for the rest of your working career. This is a problem, because while you do have a little savings put aside – just \$5,000 in a savings account earning 5% a year nominal interest – you also owe \$23,000 on credit cards (you have great credit, as a policy analyst) on which you are paying 18% a year nominal interest. The rate of inflation is just 2%.

a) Your boss suggests the following option: take a course in benefit-cost analysis at the SPP (you didn't learn it the first time), and she will raise your pay to \$45,000 a year. The course will cost you \$5,000 tuition up front, plus you will have to miss work without pay for 6 months starting 6 months from now. Your salary would go up starting a year from now.

i) [4] What is the present value to you of the increased pay you will get from this?

The formula for this is simply that of a constant repeated every year for a finite number of years. To apply it, you need to know the number of years and the discount rate. You intend to work only 25 more years, and the raise doesn't start until a year from now. So you'll get the raise for years 1 to 24.
As for the discount rate, the choices seem to be 3%, the real interest rate you are earning on savings, or 16%, the real interest rate you are paying on the credit

cards. The appropriate one to use is the one that applies to your marginal transactions. Even if you use the savings to pay the tuition, the cost of living during the six months of unemployment will force you to borrow more, presumably at the 16% real rate. Therefore the appropriate discount rate is 16%. [I'll give the answers also for 3%, so that you'll see the difference.]
The size of the raise is \$45,000–40,000=\$5,000. Therefore the present value of the raise is:

$$\frac{\$5{,}000}{0.16}\left[1-\frac{1}{(1+0.16)^{24}}\right]=\$30{,}363$$

[or, at 3% interest, $\frac{\$5{,}000}{0.03}\left[1-\frac{1}{(1+0.03)^{24}}\right]=\$84{,}678$].

ii) [2] What is the present value of the cost to you of taking the course?

The cost includes two parts: tuition and foregone earnings. Both occur in the current year, and therefore need not be discounted. Tuition is \$5000. Earnings for the half year of work that will be missed is \$20,000. So present value of cost is \$25,000.

iii) [2] What, then, is the present value of taking the course *and* getting the raise?

Just (i) minus (ii): \$5,363 [or \$59,678 at 3%].

iv) [2] Should you take the course or not? Write a one-paragraph, non-technical explanation of your decision for your spouse, who is not an economist.

Yes. First try, "Honey, trust me. We'll be fine." If that doesn't work, tell him or her that the extra earnings over 24 years, 24 times \$5000 = \$120,000, are considerably more than the cost of \$25,000. Your spouse, not an economist, may accept this, even though you know it is irrelevant. On the other hand, he or she may point out that you don't have the money to afford this, and that you would have to go further into debt. You then explain that you have calculated that even if you have to borrow still more from the credit cards to finance both tuition and your consumption during the course, you will be able to pay back the debt from the extra earnings and still come out ahead. In fact, that is exactly what you have done by using the credit-card real interest rate to discount the future earnings, but you don't need to say that. Explain instead that, for example, you can borrow up to \$25,000, then pay back, say, \$5000 each year to the credit card until that additional loan is paid off, and you'll then have some years at the end with extra pay. You might also mention how much you'll enjoy taking a course at SPP.

b) [4] Your doctor just told you that you have an incurable disease and have only 5 years to live. During this time the disease will not interfere with your performance as a policy analyst, but at the end of 5 years exactly you will surely die. Which of your answers in part (a) are changed by this news?

Part (i) is certainly changed, since you now get the raise for fewer years, years 1 to 4, dying in five years at the end of year 4. The present value of the raise at 16% real interest is now much smaller:

$$\frac{\$5{,}000}{0.16}\left[1-\frac{1}{(1+0.16)^4}\right]=\$13{,}991$$

[The present value at 3% is also smaller, $18,585.]

The cost in (ii) is unchanged, and since it is now greater than the present value of the benefit, the present value of the course and raise together in (iii) is negative: –$11,009 [or –$6,415 at 3%]. Your decision in part (iv) should therefore be **not** to take the course. The explanation to your spouse should not be difficult, except for telling about the bad news from the doctor.

c) [2] Your lawyer just told you that your doctor faked his diploma from medical school, and you decide that there is a 70% chance that the doctor is wrong and you will live past retirement after all. What is now the present discounted *expected value* of taking the course and getting the raise?

This is just the sum of the different present values times their probabilities. That is, 0.7($5,363)+0.3(–$11,009) = $451

d) [1] Doing your best to imagine yourself in the situation of part (c), tell us what you think would be the certainty equivalent for you personally of the course/raise. This is subjective, and there is no *correct* answer, but you should explain why your answer is higher, lower, or equal to the expected value from part (c).

Most people are probably risk averse, and that means that when an outcome is uncertain like this, with only a small positive expected value and a chance of losing quite a bit, they won't do it. In this case, of course, the bad outcome of the course not having been worth the cost also goes along with the doctor being right, and your being dead.

So which is worse? Living to a ripe old age knowing that you could have had a higher income if only you'd ignored the doctor and taken the course? Or discovering that you are about to die and that you are leaving your spouse in even greater debt than you needed to?

For me, it doesn't seem worth it, and I wouldn't do it. To find my certainty equivalent, I need to think how much the current cost would have to be reduced for me to do it after all. If tuition was zero, so that the only cast was the $20,000 lost earnings, would that be good enough? I don't think so. The 30% chance of leaving my spouse even $20,000 more in debt still seems high for an extra $5000 a year if I live, especially considering the debt that we already have. Maybe, if I could get the cost down to $15,000, I'd do it. And if so, that means the certainty equivalent of this opportunity is, for me, –$10,000.

e) [1] Terribly sorry, but your boss just learned about the disease and withdrew the offer. (Don't hold it against her – she just did it in order to get you back to the $40,000 salary as a starting point for this next part of the question.) But not to worry: you've also just learned from your brother-in-law about a drug that he can get for you that will help, called Extendo. If you take it every day from now on, it will stop the progression of the disease. That is, if you do have the disease, then each day that you take Extendo will delay your death from the disease by one day. How much would you be willing to pay each year for this drug, assuming that you believe that it will work? Again, there is no correct answer to this; it is subjective. But see if you can provide some explanation for the number that you give.

What you are being asked is how much you'd be willing to pay each year to reduce your probability of dying by 30% – the probability that the doctor is right. (This is not quite right, I know. I should say "to reduce by 30% your probability of dying five years from now." So perhaps you should be discounting this in some fashion. I'm not going to do that, but you are welcome to.) Any number will do. I personally can't imagine spending so much that it would really alter my life, so I'd spend considerably less than my income. I'll say that $1000 a year is my maximum.

In case you are wondering what this was all about, when I first used this problem there was one more part. I asked next what the answer to part (e) implicitly meant about the value that you place on your life. I took that part out, this year, because we hadn't dealt with valuing life yet in the course. But now, I presume, we have. So her is my answer to that, in case you are interested:

Let your answer to part (e) be X. You've said that avoiding a 30% chance of dying is worth X per year. That suggests that X is 30% of the value, V_1, that you assign to extending your life by one year, so that V_1 is X/0.3. In my case, then, the value of extending my life by one year is apparently $3,333. That sort of answer is, I think, acceptable. However, you could go further. To extend life indefinitely (with respect to this particular cause of death) you must pay V_1 every year forever, which has a present value of V_1 divided by the real interest rate, or X/0.3/r. Thus the value of my life in this sense is $3,333/0.16 = $20,833. Still pretty cheap, aren't I?