Take-Home Final Exam
Due 5:00 PM, Wednesday, Dec. 17

Instructions:

The exam is open book and open notes. That is, you are free to look at anything that was written by others before you got this -- books, articles, working papers, lecture notes, etc. Do not, however, talk to each other about these questions. If you are uncertain about how to interpret any of the questions, ask me, not other members of the class. I will be available by e-mail, and you should check your own e-mail periodically in case I have sent clarifying comments to the course group.

You should answer all questions, being guided by the points in brackets to suggest how much effort you should put into each (there are 100 points total).

You may give me your answers in blue books, if you wish, or on either handwritten or printed 8🗎 by 11 sheets. In either case, be sure that the pages are attached to each other. Put your name **only** on the cover or cover sheet, starting your answers on the page after that, so that I can fold back the cover and read the exams without knowing who you are.

1. [25 points] Analyze the effects of a tariff on imports in a 2-good, 2-factor H-O Model with **fixed coefficient** (Leontief) technologies. Let the two factors be skilled labor, *S*, and unskilled labor, *U*, and assume a small open economy that has homothetic preferences and that produces both goods and imports the skill-intensive good under free trade. Specifically, use whatever tools you find appropriate to derive the (direction only of the) effect of the tariff on

   a) The real wages, w_u and w_s, of skilled and unskilled labor respectively;

   b) The quantities of production, consumption, and trade of both goods; and

   c) The skilled and unskilled labor contents of total production, total consumption, and net trade.

   d) Finally, compare your results to what would have happened if, instead of the tariff, the factor endowments of the country had been changed by the amounts of the change in the factor content of net trade that you found in part (c). That is, if the tariff caused it to import more (less) of a factor, replace the tariff with an equal increase (decrease) of that factor endowment, and vice versa for exports.

   e) In what sense, if any, is the change in the factor content of trade due to the tariff

indicative of the factor price changes that the tariff causes?

2. [40 points] This question is mostly about **fragmentation**, defined here as the splitting of a production process into two parts that can then be done in different places. The following two parts ask you to work out the effects of particular kinds of fragmentation in the context of two familiar models of international trade.

   a) Consider first a small, open, Ricardian economy that is initially capable of producing two (final) goods, X and Y, with unit labor requirements a_X and a_Y and labor endowment L. International prices of the goods, taken as given, are p_X and p_Y in terms of some numeraire.

      i) Write an expression for the wage of labor in this economy.

      ii) Assuming that good X is exported by this economy, what restriction does that place on the prices and unit labor requirements?

      iii) Suppose now that the technology for producing good X fragments into two parts. The first part produces a new intermediate input Z with unit labor requirement a_{X1}. The second transforms one unit of Z into one unit of X with an additional labor requirement of a_{X2}. The total labor required for both steps is the same as before: $a_{X1} + a_{X2} = a_X$. The prices of X and Y remain the same on the world market and satisfy the restriction derived in part (ii). A new market also appears for good Z, which can be exported or imported for price p_Z. Calculate the wages that would be earned by labor in each of the following activities:

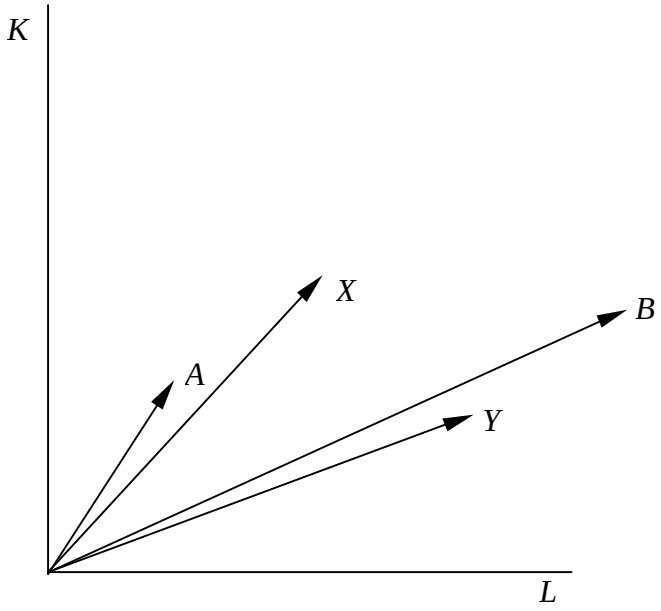
w_Z:	Producing Z
w_{ZX}:	Producing X from Z
w_X:	Producing X from scratch
w_Y:	Producing Y

      iv) Show that, unless workers are indifferent among occupations, they will earn more producing either only Z or only X from Z than in the other activities.

      v) Derive and draw a graph of the quantity of trade of good Z as a function of its price p_Z.

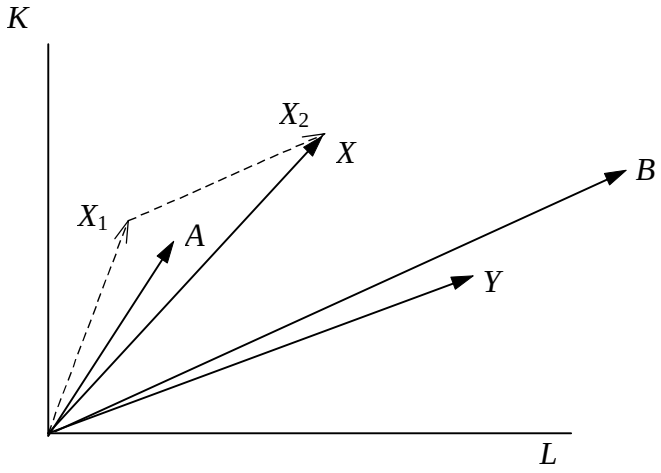
      vi) Under what circumstances, if any, does this country gain from this possibility of fragmentation, and in what sense?

b) The diagram at the right shows, as vectors *X* and *Y*, the quantities of capital and labor needed for production of two (final) goods in a 2×2 Heckscher-Ohlin (H-O) economy. The quantities of *X* and *Y* themselves are those demanded in an integrated world economy. Suppose instead, however, that the world is not integrated, but rather is divided into two countries whose factor endowments are immobile internationally and are given by the vectors *A* and *B* in the figure (where $A+B=X+Y$). Assume for simplicity that both industries have fixed coefficients, the ratios of capital to labor used in both industries being fixed independently of factor prices.



i) What can you say about the patterns of production and trade, and also about factor prices in the two countries, in a world equilibrium of these countries with free and frictionless trade? That is, who will produce and export what, and how will the factor prices compare across countries?

ii) Suppose now that the technology for producing good *X* becomes fragmented as shown in the figure below: The same vector *X* is split into the two dashed vectors, X_1 and X_2 that add up to *X*. X_1 produces an intermediate input, *Z*, that together with the inputs X_2 will now produce the same quantity of *X* as before. (All other vectors are the same.) Once again, what can you say about the patterns of production, trade, and factor prices after this fragmentation of technology?



iii) Is anyone made better off by this fragmentation?

3. [15 points] Consider the following "friends and enemies" version of the Rybczynski Theorem:

> Assume a H-O Model with arbitrary numbers of factors and goods, all of which are initially produced using strictly positive amounts of every factor. Then if prices of all goods are held fixed, an increase in the endowment of any one factor, holding all other factor endowments constant, will cause the output of one good (the "friend") to expand more than in proportion to the increased endowment, and it will also cause the output of some other good (the "enemy") to contract.

a) For each of the following cases, either prove this Theorem or provide a counterexample. (Your counterexample may be part of your answer to part (b).)

   i) More goods than factors.

   ii) More factors than goods.

b) Illustrate *one* (not both) of your results using *either* (not both) of the following two special cases of the general H-O Model. That is, for just one of the models listed below, either identify the friend and the enemy of each factor, and show that they respond as described in the Theorem, or show that a friend or enemy does not exist for one of the factors.

   i) The 3-good, 2-factor H-O Model of Melvin.

   ii) The Specific Factors Model with two goods, two specific factors, and one mobile factor

4. [15 points] Suppose a world of many countries, each like the countries modeled in Krugman (1979) but with labor endowments, L_j, that may be different. That is, consumers in country j maximize a utility function $U^j = \sum_{i=1}^{n_j} v(c_i^j)$ where c_i^j is per capita consumption of the i^{th} variety in country j, while n_j firms, one for each variety i, produce a quantity $x_i^j = L_j c_i^j$ incurring a labor cost of $l_i^j = \alpha + \beta x_i^j$ with $\alpha, \beta > 0$. In equilibrium, firms charge prices p_i^j that maximize their profits given outputs of other firms; the wage, w^j, is determined to clear the labor market ($L_j = \sum_{i-1}^{n} l_i^j$); and free entry of firms drives profits to zero: $\pi_i^j = p_i^j x_i^j - w^j l_i^j = 0$.

   a) Following Krugman, suppose first that the v functions are such that $v' > 0$, $v'' < 0$, and $d\varepsilon / dc_i^j < 0$ where ε is the (positive) elasticity of demand facing any firm as a function of its consumers' per capita demand. Holding constant the world's population, $L^W = \sum_j L_j$, determine how the following variables depend on an individual country's population, L_j.

      i) The ratio of the wage to any good's price, w^j / p^j, in autarky.

      ii) The ratio of the wage to any good's price, w^j / p^j, in free trade.

      iii) The gain in consumer utility of going from autarky to free trade.

   b) Now suppose instead that $d\varepsilon / dc_i^j = 0$. Which of your answers in part (a) are altered, and why?

5. [5 points] This is free, I hope: Go back and read the instructions at the start of the exam, and take a moment, before you hand it in, to make sure you are following them.