Take-Home Final Exam
Due 5:00 PM, Monday, December 16
Answers

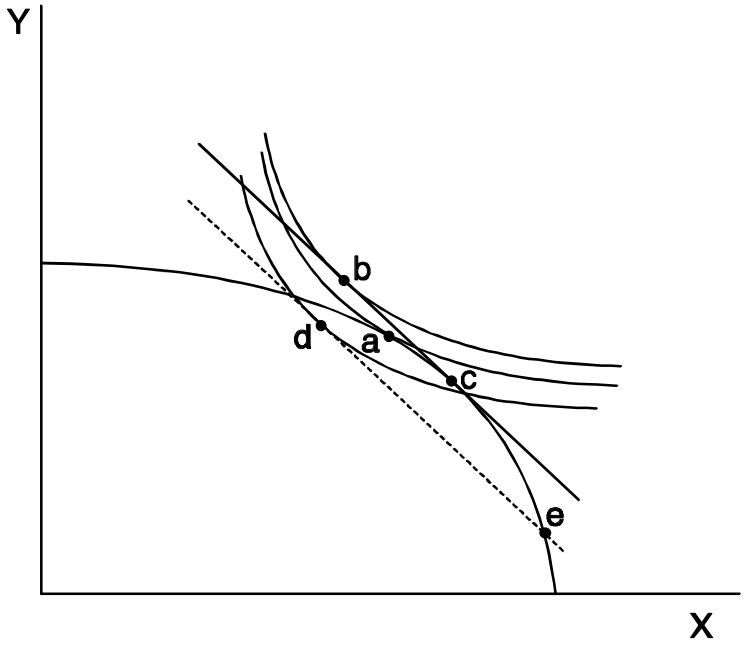
1. (20 points) **True-False Explain (Answer <u>any four</u> of the following five):** Indicate whether each of the following statements is true or false, and provide either a proof (if true) or a counter-example (if false). If the model is not stated, specify and use whatever model seems most appropriate for the purpose.

   a) If a country's trade satisfies the Law of Comparative Advantage, then it must gain from that trade relative to autarky.

   Ans: False.

   This is easier than I had intended. As written, it is false for the Ricardian case of a large and small country, where the free trade price is the large country's autarky price and it therefore neither gains nor loses from trade, even though it is exporting the good in which it has a comparative advantage. To get the sort of answer I wanted, I should have made the statement: If a country's trade satisfies the Law of Comparative Advantage, then it cannot lose from that trade relative to autarky.

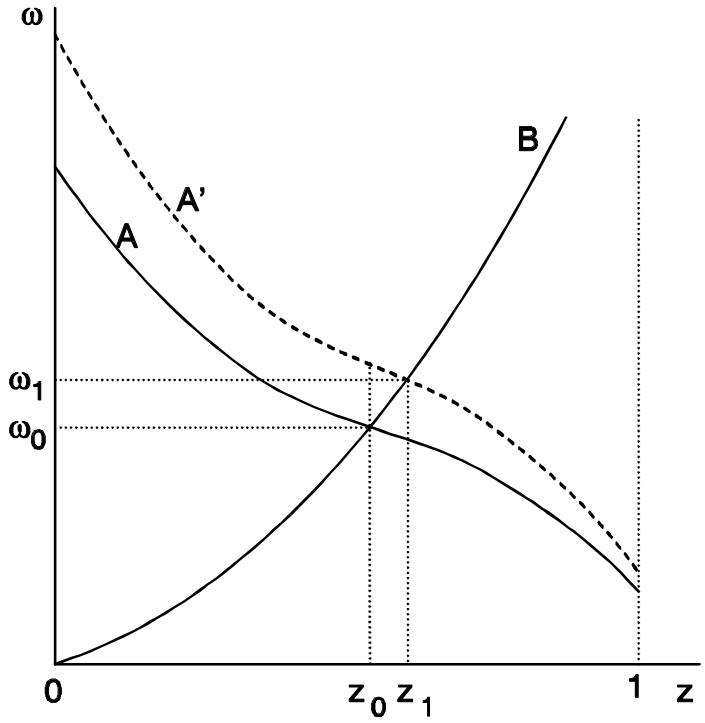
   If a country exports too much, even of the good in which it has a comparative advantage, it can make itself worse off than in autarky. Therefore, a large enough export subsidy would lower welfare. In the graph below, the country produces and consumes at a in autarky. With free trade, production moves to c and consumption to b, with a rise in welfare. By subsidizing production of X, however, production can be moved to e and the best possible consumption is then at d, with welfare below autarky.



b) In the Dornbusch-Fisher-Samuelson Ricardian continuum of goods model with free and frictionless trade, an increase in the productivity of all the workers in one country increases the real wages of workers in both countries.

Ans: True

Using the standard D-F-S diagram at the right, suppose that the increase in productivity occurs for domestic workers, reducing their unit labor requirements for all goods by some percentage, α,

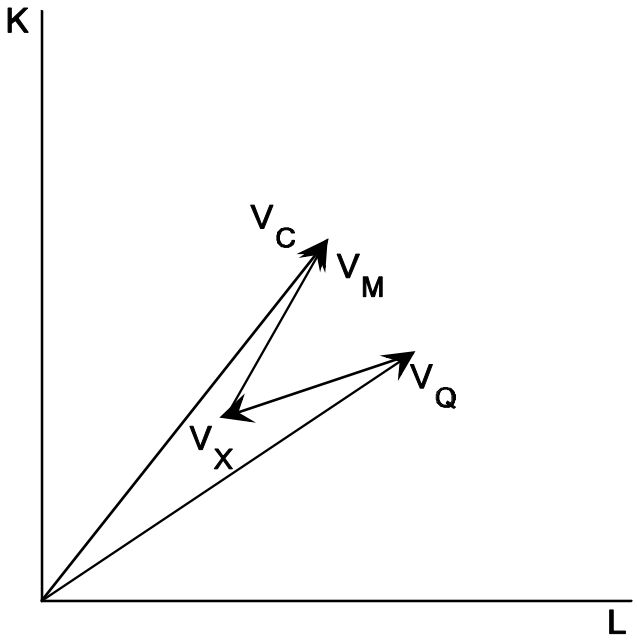


and therefore increasing relative foreign unit labor requirements, A, as shown for all goods, z. From the figure we see that the relative domestic wage, ω, rises but by less than the percentage change in α(which is given by the size of the upward shift in A).

Taking the domestic wage as numeraire, the foreign wage therefore falls by less than α, as do the costs of all goods that are still produced there, those in $(z_1,1)$. Goods previously produced domestically, in $(0,z_0)$, fall in price by α, while those that switch from production abroad to production domestically, in (z_0,z_1), do so because their prices fall by more than the foreign wage. Therefore, the domestic wage rises relative to all prices, increasing the real wage at home. The foreign wage stays constant in units of goods still produced abroad, but rises relative to all others, so that foreign workers also gain in real terms.

c) If the two-factor Heckscher-Ohlin Model with free and balanced trade had been a valid characterization of the U.S. situation at the time he wrote, Leontief would have been correct in inferring from his measurements of the factors embodied in trade that the U.S. was labor abundant.

Ans: True.



Leontief measured the capital and labor contents of representative equal-value bundles of U.S. exports and imports, and he found that the capital-labor ratio embodied in exports was smaller than that embodied in imports. Leamer showed that if there is unbalanced trade and/or if there are additional factors, then the Heckscher-Ohlin model does *not* imply anything about the relative factor ratios embodied in trade. However if, as stated, the assumptions of two factors and balanced trade were both valid for

the U.S. economy Leotief was examining, these objections would not arise.

This is implicit in Baldwin's diagram, above, for illustrating what goes wrong when trade is *not* balanced. Let V_Q, V_C, V_X, and V_M be the factors embodied in production (hence endowments), consumption, exports, and imports respectively, so that by definition, $V_C = V_Q - V_X + V_M$. Then it is clear from the figure that if all of these vectors must be strictly positive and if there is balanced trade (ie., at some positive factor prices w, $w(V_X - V_M)=w(V_Q - V_C)=0$), then if V_X is flatter than V_M, it must be true that V_Q is also flatter than V_C. Since with identical homothetic preferences, V_C is proportional to world factor endowments, it follows that the relative abundance of capital in the country's endowments (the slope of V_Q) is smaller than the relative abundance of capital in the world (the slope of V_C).

More formally:

$$k_X < k_M \;\Rightarrow\; \frac{K_X}{L_X} < \frac{K_M}{L_M}$$

$$\Rightarrow\; \frac{K_X}{L_X} + \frac{w}{r} < \frac{K_M}{L_M} + \frac{w}{r}$$

$$\Rightarrow\; \frac{rK_X + wL_X}{rL_X} < \frac{rK_M + wL_M}{rL_M}$$

$$\Rightarrow\; L_M < L_X \;\text{ by balanced trade}$$

$$\Rightarrow\; L_Q = L_C - L_M + L_X > L_C$$

Similarly, $L_M<L_X$, together with balanced trade, yields

$$r(K_X - K_M) = w(L_M - L_X) < 0 \;\Rightarrow\; K_X < K_M$$

$$\Rightarrow\; K_Q = K_C - K_M + K_X < K_C$$

Together, $L_Q>L_C$ and $K_Q<K_C$ imply

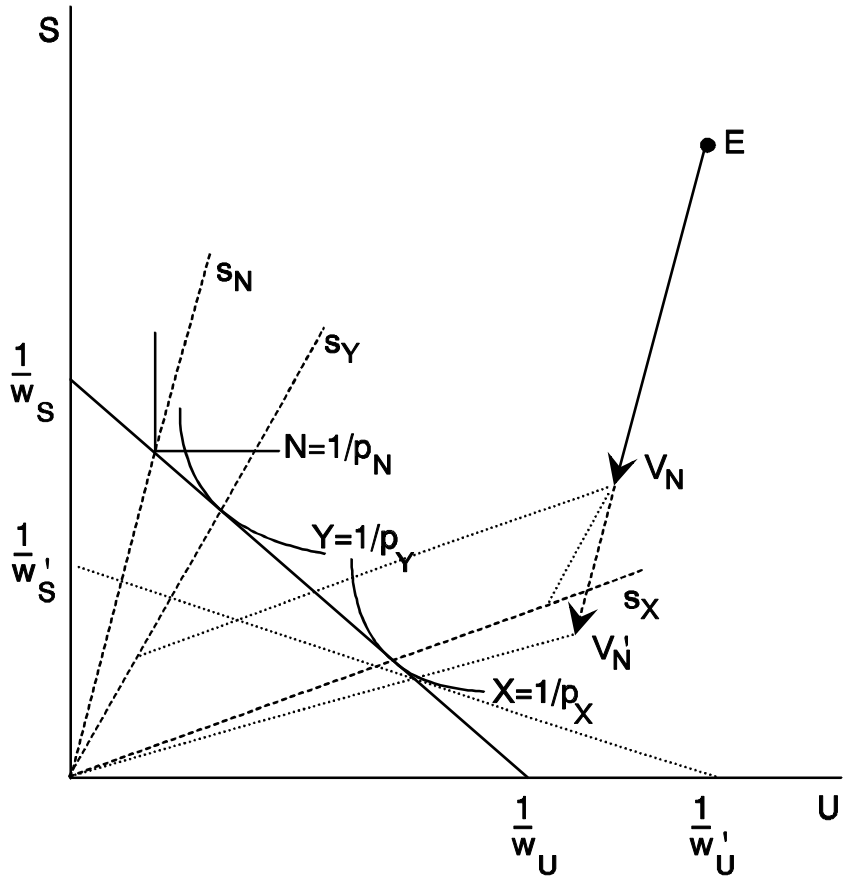
$$\frac{K_Q}{L_Q} < \frac{K_C}{L_C} = \frac{K_W}{L_W}$$

d) The rise in the wage of skilled labor relative to unskilled labor in the United States in the early 1980s could have been caused by the increase in the U.S. trade deficit during that period.

Ans: True.

A rise in the trade deficit means an increase in expenditure relative to income. This can have an effect on production and relative prices, especially if production of nontraded goods, which must expand with an increase in expenditure, uses factors of production in proportions different from endowments.

In the example shown below, goods X and Y are produced and traded freely at fixed world prices, using ratios of skilled to unskilled labor shown by the rays s_X and s_Y. As long as both are produced, factor prices are determined as w_S and w_U, and these in turn determine the price of the nontraded good, N, which is produced using the factor ratio s_N. (The Leontief technology for sector N is not necessary, but it simplifies things below.)



The factor endowment is shown at point E. Given an initial level of expenditure, some amount of the nontraded good will be demanded, production of which will occupy the vector of factors V_N, with slope s_N. Subtracting this from E, the remaining factors are allocated across X and Y in the usual way, determining outputs of both.

An increase in expenditure increases the demand for the nontraded good, expanding the vector of factors needed to produce it to V_N'. As drawn, because the nontraded good is quite skill-intensive, this withdraws so much skilled labor from the factors available to industries X and Y that it is no longer possible to produce them both at prevailing factor prices. The country instead specializes in good X (in addition to N), and factor prices change to those which will induce sector X to demand just the factors now available to it. That is, the wages change to w_S' and w_U', the former rising and the latter falling. (Actually, this is not quite right, unless the demand for the nontraded good is implausibly completely price inelastic. The new factor prices will change the price of good N and thus the demand for it away from the demand assumed for the vector V_N', the slope of which would also have changed if we had allowed factor substitution in N.)

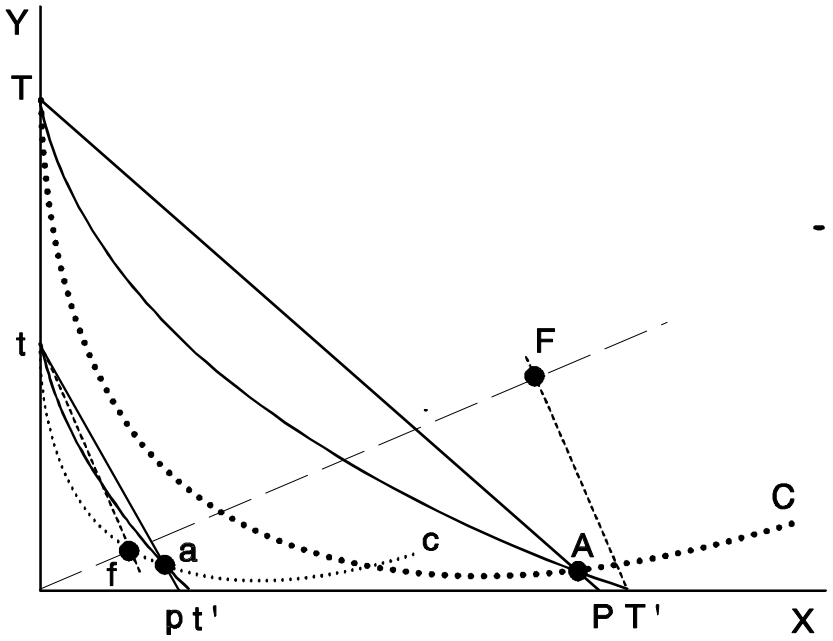
e) A large country has less to gain from trade than a small country.

Ans: This could be true or false depending on what model you use to explain trade. Since there does exist a plausible model in which the statement is false, that is probably the best answer, but it is worthwhile also noting the models in which the statement is true. There are also models in which the statement does not make much sense, since it is unclear how one should compare the sizes of gains from trade across countries if both gains are positive.

In the simple two-good, two-country Ricardian model, if one country is large enough compared to the other, then its autarky prices become the world prices with free trade, and it gains nothing from trade, while the small country does gain. In this case, then, the statement is true.

In the two-good, two-country Heckscher-Ohlin model with smooth technologies, both countries normally gain from trade, although there is again a sense in which large countries gain less. The larger a country is, the closer the equilibrium world price with free trade will be to that country's autarky price, and hence the less it will gain from trade.

If there are increasing returns to scale (IRS), however, one can construct a case in which, starting from autarky where the large country has the lower relative price of the IRS good due to its size, the large country gains while the small one loses from a move to free trade. This, then, is a counterexample to the statement:

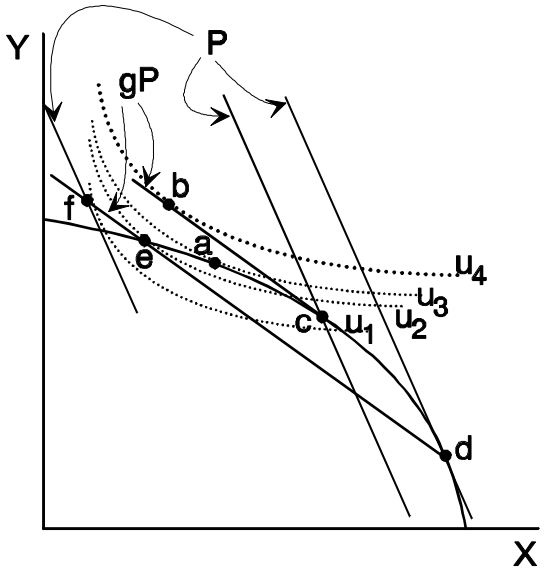


In the example shown, two goods are produced with only labor, good X having increasing returns external to firms and good Y having constant returns. A small country has production possibilities tt', while a large country, with twice the labor and the same technology, has production possibilities TT'. Using proportionally identical price-consumption curves through t and T, autarky equilibria are a and A, with autarky price lines tap and TAP respectively. By construction, the larger country must have the lower autarky price of the increasing returns good, X.

With free trade and any initial prices between the two autarky prices, there will be exit from the X industry in the small country and entry in the large, taking both to complete specialization at t and T' respectively. The free trade price lines are tf and FT' (parallel and of equal length), the small country exporting Y and consuming at f, the large country exporting X and consuming at F. It is evident from the figure that the small country has lost from trade, while the large country has gained.

2. (10 points) Suppose that a country starts from an equilibrium in which it faces real transport costs in its international trade and that it initially has no government intervention (tax or subsidy) in trade. Its government then learns of the purported advantages of "free" international trade, and it misunderstands that term to mean the net absence of *all* barriers to trade. It therefore institutes a system of trade subsidies that exactly offset all transport costs in international trade. Compare the resulting equilibrium to the equilibrium without this policy and also to autarky, both in terms of the welfare of the country as a whole and of different groups within it. You may assume, if it is helpful for your analysis, any standard model of trade that you find convenient, and also that transport costs take Samuelson's iceberg form.

Ans: Consider a small open 2x2 Heckscher-Ohlin economy with a comparative advantage in good X. The world relative price of good X in terms of Y is P, but with transport costs that cause only a fraction, g, of what is exported to reach the world market, the price received by exporters is only gP.



This assumes transport costs only on exports. If exports of X and imports of Y both "melt" to fractions g_x and g_y, respectively, then $g=g_x g_y$, since an exporter also loses part of its revenues of Y while transporting them back home. In any case, I assume gP to be greater than the country's autarky price, so that it still trades even without a subsidy.

In autarky, the country produces and consumes at point a in the figure, reaching utility level u_3. With free trade, producers face the price gP and produce at c, while consumers also face that price and consume at b. Utility is u_4.

If the government were to subsidize all production of good X, not just exports, then production would move to point d. Trade would take place at price gP, consumers facing the world price net of transport cost and consuming at e, for a utility of u_2. However a subsidy to exports only, while it would also move production to d, would confront consumers with the higher world price of X, since that is the price that the export subsidy has caused to prevail on the domestic market. Consumption is at f, utility at u_1.

As drawn, then, the export subsidy has reduced welfare of the country as a whole below free trade and also below autarky. The former is necessarily true, but the latter is not. Had the transport cost and hence the subsidy been sufficiently small, both e and f could have been above the u_3 indifference curve.

Effects on factor owners depend on the prices faced by producers. The price of X was already made higher in free trade than in autarky, and the subsidy makes it higher still. Thus the owners of the factor used intensively in the production of good X (the abundant factor according to the H-O theorem) receive a higher real wage due to the subsidy than they would have gotten in both free trade and, even more so, in autarky. Similarly, owners of the other factor suffer a decline in real wage due to the subsidy, relative to both free trade and autarky. Of course the subsidy must be financed somehow, and if lump-sum taxes come from owners of the abundant factor, they too may lose.

3. (35 points) Use a Heckscher-Ohlin Model with two factors, labor and land, to determine how each of the following variables behaves over time as the country's labor force grows from just above zero to very large. Assume that the country is small and faces fixed world prices, that it has free trade and perfect competition, and that preferences are homothetic.

i) Output of each good

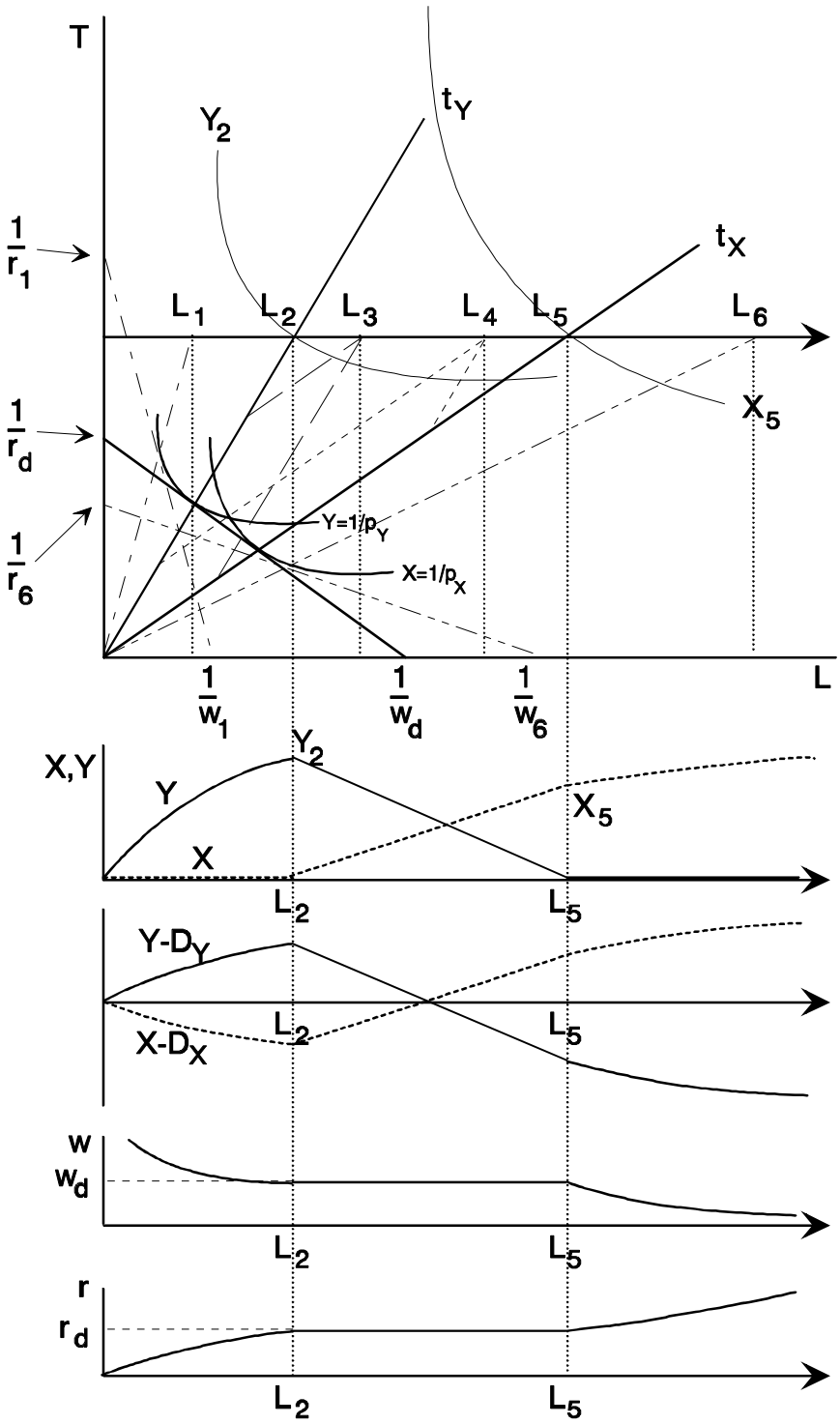
ii) Net exports of each good

iii) Real price of each factor

a) Assume first that there are two goods, that there are no factor intensity reversals, and that transport costs are zero. You may use whatever analytical tool or tools you like, but give your answers by means of graphs of each of the above variables as functions of the labor force, and be sure to show how you get your results.

Ans: The Lerner-Pearce diagram, in the top panel below, shows how factor allocations, outputs, and factor prices vary as the labor endowment is raised along the horizontal arrow representing a fixed endowment of land. Output of Y rises to a maximum of Y_2 at $L=L_2$, falls linearly to zero at L_5, and remains zero from there on. Output of

X stays at zero until L_2, rises linearly to X_5 at L_5, then continues to rise at a decreasing rate thereafter (due to diminishing returns). Both are graphed in the next panel. Demand is not given in the Lerner-Pearce diagram, but assuming it to remain proportional to national income, the patterns of net trade are shown in the panel below. Factor prices are also indicated. The latter are nominal, but since the prices of goods are fixed, they are also real.

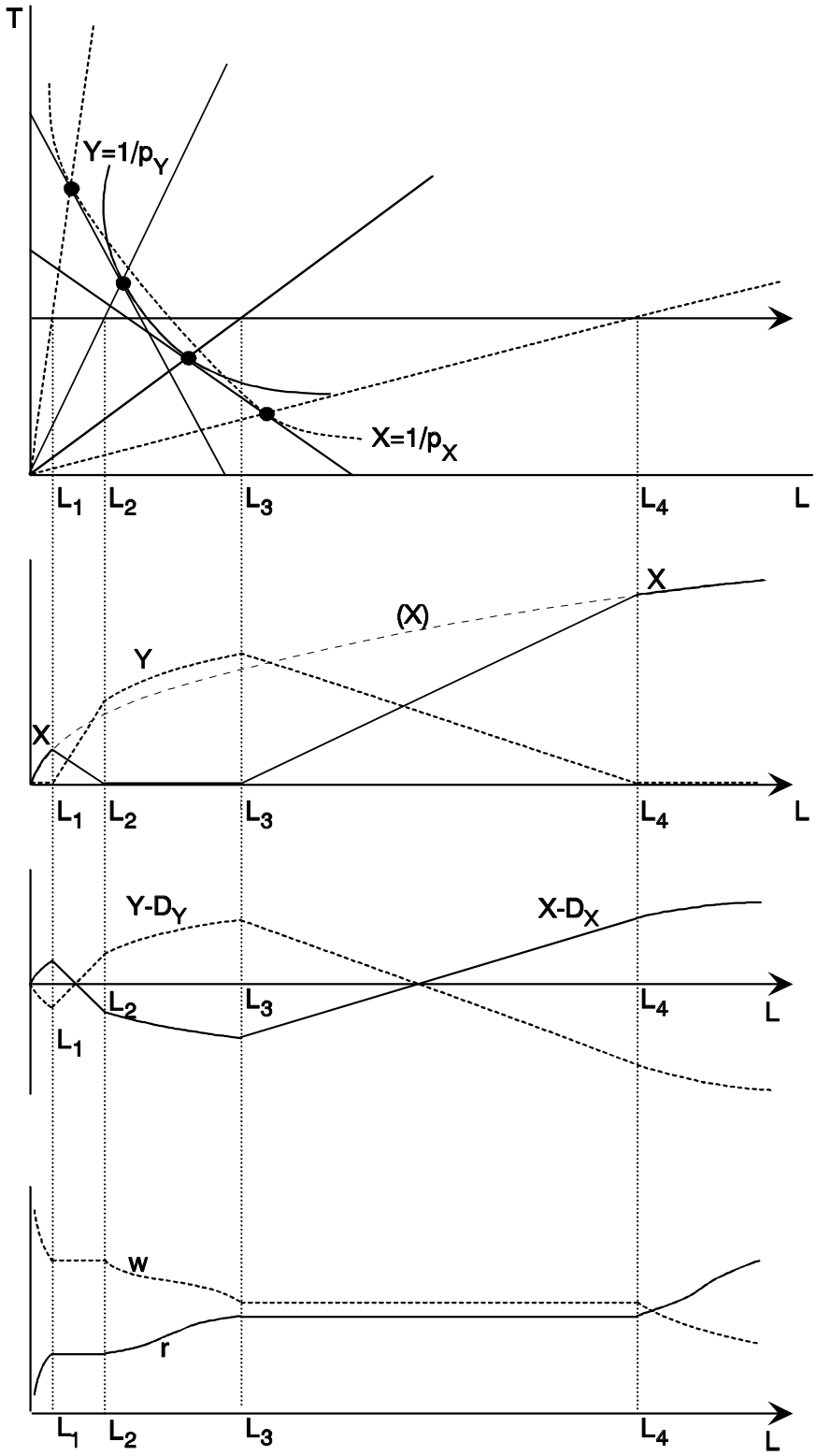


b) Indicate briefly how your answers to part (a) would be changed if there were each of the following changes (one at a time) in the above assumptions. For this you do not need to derive your results, and you need only provide enough graphs to indicate which variables behave differently from part (a) and the nature of that difference.

i) There is a single factor-intensity reversal.

Ans: With a single factor intensity reversal, the two unit-value isoquants of the Lerner-Pearce diagram cross twice, creating two common tangencies as shown in the top panel on the next page. Given that, and the implied two cones of diversification, we can trace out the factor allocations, levels of output, trade, and factor prices as above. The results appear in the lower three panels of the graph on the next page.

Note that the output of good X, which is positive both at very low and at very high levels of labor endowment, in both cases follows the single production function indicated by the thin dotted curve labelled (X).


T
Y=1/p_Y
X=1/p_X
L1
L2
L3
L4
L
X
(X)
Y
X
L1
L2
L3
L4
L
Y-D_Y
X-D_X
L2
L3
L4
L
L1
w
r
L1
L2
L3
L4
L

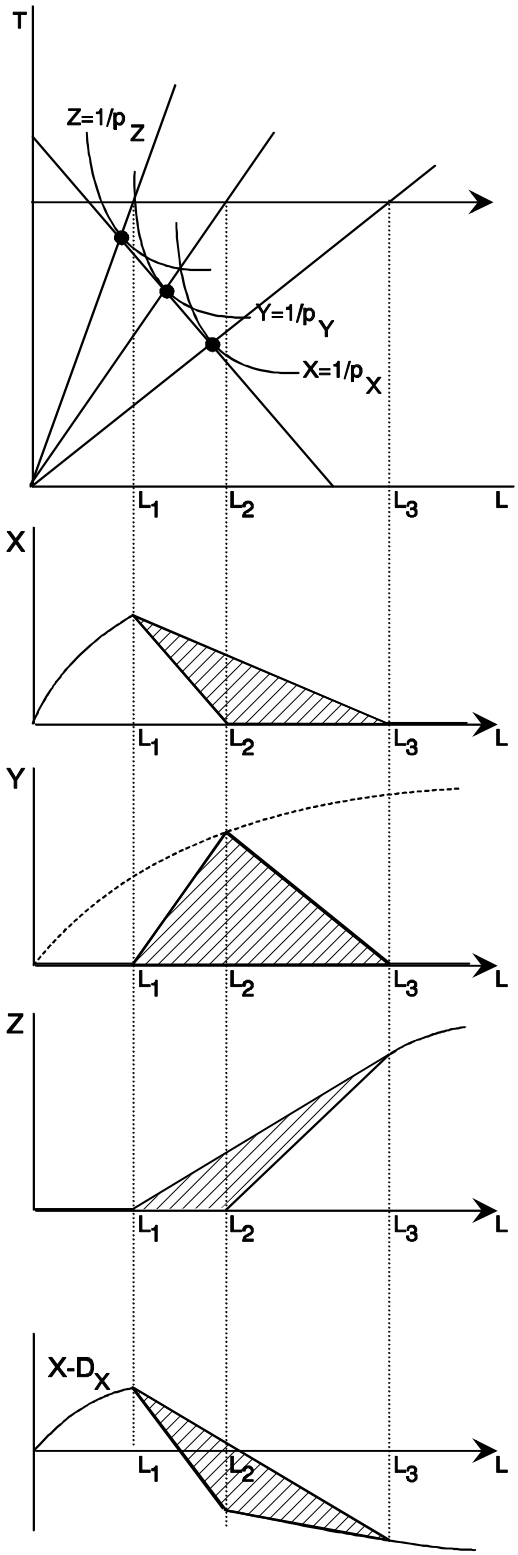
ii) There are three goods, and world prices **do** permit production of all three at common factor prices.

Ans: The main difference here is that outputs, and thus levels of trade, are indeterminant. The three unit-value isoquants all are tangent to a single straight line (which is why production of all three is possible), but the result in the interior of the diversification cone is that many different combinations of the three goods will fully employ both the factors.

In these graphs, three separate panels now show the outputs of the three goods, with shaded areas showing the range of outputs possible for each at each level of labor endowment, L.

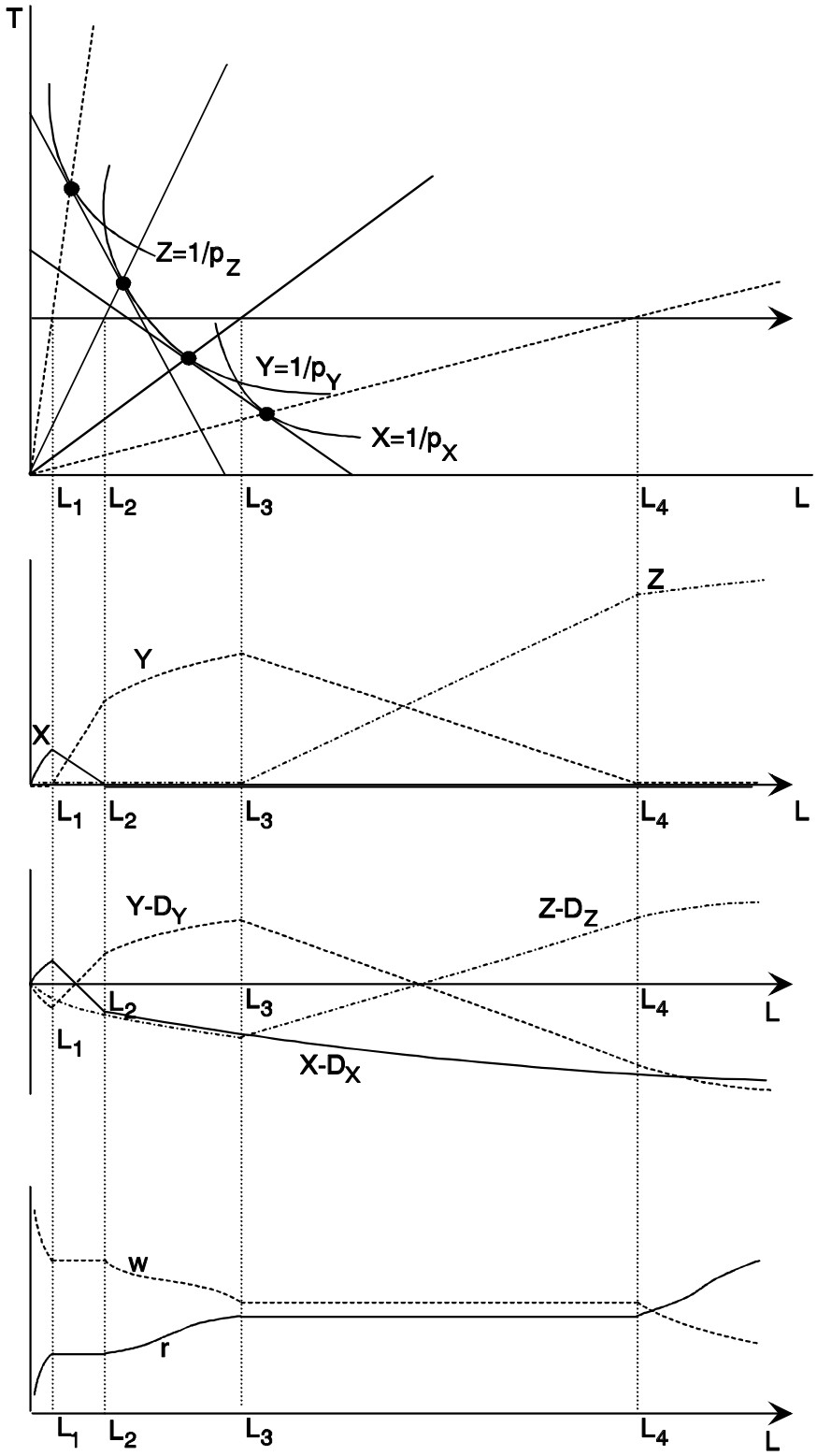
Trade is shown only for good X, the graphs for Y and Z being similarly inferred from their outputs. Again, there is a range of possibilities throughout the diversification cone, including a sizable range of L's for which the direction of trade of good X is unknown.

A graph for factor prices is not provided, since it would look the same as the graphs in case (a), with L_1 and L_3 playing the roles of L_2 and L_5 there respectively.



iii) There are three goods and world prices **do not** permit production of all three at common factor prices.

Ans: This case looks exactly like the case of a single factor-intensity reversal of part (i) except that the two ends of the single X isoquant there are now parts of two different isoquants, for goods X and Z. This changes the corresponding graphs of outputs and trade, as shown in the graphs on the next page, but otherwise makes no difference. In particular, the graphs of the two factor prices are the same as in part (i).

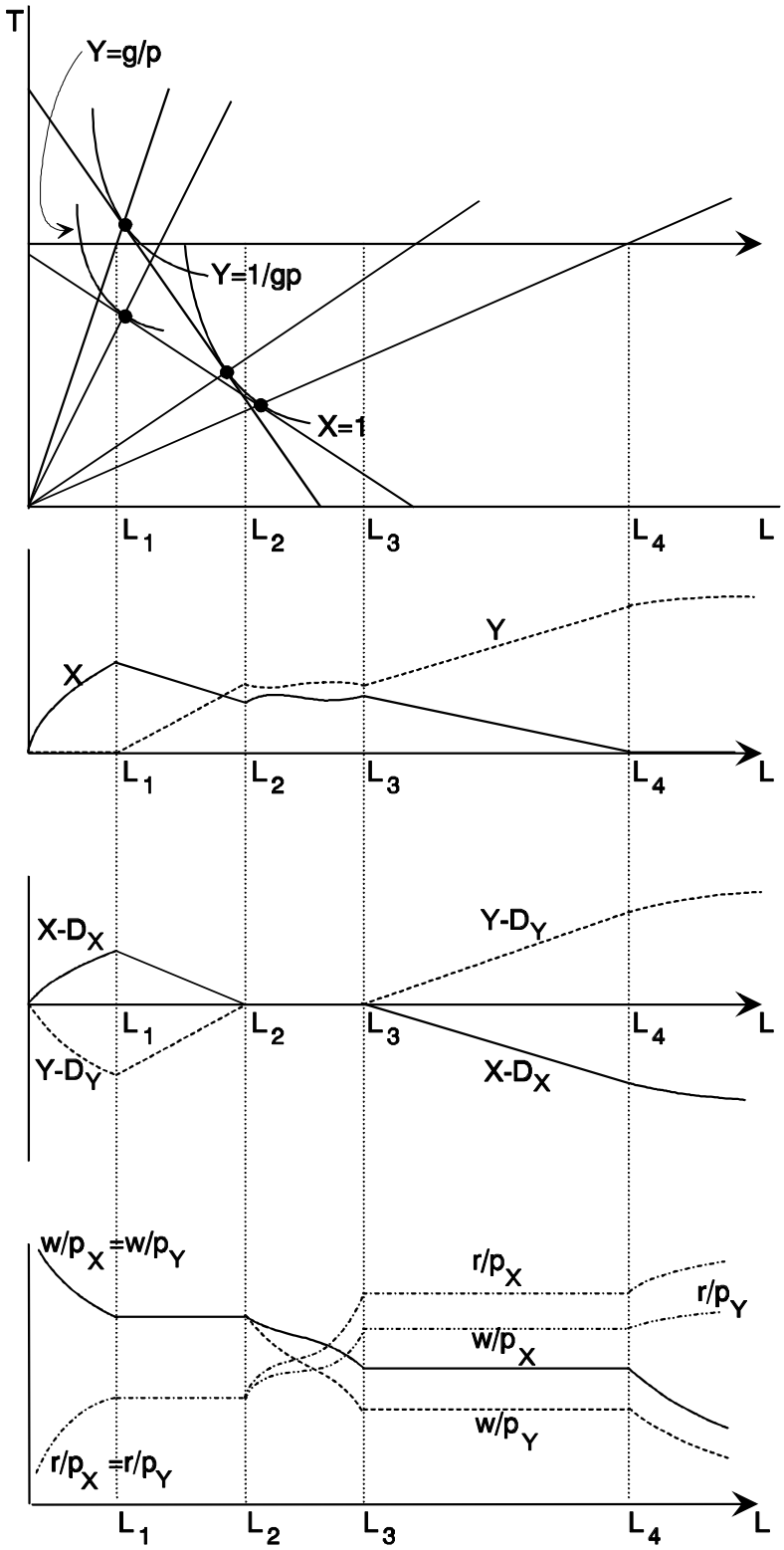

T
Z=1/p_Z
Y=1/p_Y
X=1/p_X
L1
L2
L3
L4
L
Z
Y
X
Y-D_Y
Z-D_Z
X-D_X
w
r

iv) There are positive transport costs.

Ans: The presence of positive transport costs means that, even though world prices are fixed, the implied domestic prices are not. Returning to the two good model with X as the labor-intensive good, if L is small so that the country exports good Y, then the relative price of good Y will be below the world price by the amount of the transport cost. When L become large and the country exports X, then the relative price of good Y will be correspondingly above the world price. Letting X be the numeraire, the two prices for Y define to unit-value isoquants for Y, and therefore two corresponding diversification cones, as shown in the Lerner-Pearce diagram on the next page.

As the labor endowment rises from a low level, the economy first specializes in good Y as before, facing the lower price for Y and thus the upper diversification cone. When L reaches L_1, production of X begins, and output of Y declines linearly just as it did in part (a). However, at some point inside the cone, the output of Y just equals the level of domestic demand (which cannot be found in this diagram), and trade ceases. This occurs at some level of labor endowment, assumed to be L_2 in the figure. At this point trade, in the next panel, is zero. As L rises further, the domestic price of Y begins to rise so as to clear the domestic market, and the unit-value isoquant for Y contracts inward. This continues until the price reaches world price of Y plus transport cost, P/g, at which point the economy begins to import Y at L_3. From here on, prices are again fixed, and outputs change linearly, continuing the fall in output of Y and the rise in output of X. Note that between L_2 and L_3, where the economy is in autarky, the responses of the two outputs to growth in L are ambiguous.

Factor prices are shown in the bottom panel. For low L, goods prices are fixed and the factor prices follow the same sorts of paths as in part (a). This is true regardless of which good is numeraire, and the picture assumes that X is numeraire, while units of goods are such that the price of Y is also one in this range. At L_2, however, the relative price of good Y begins to rise in autarky. Therefore, while the factor prices change in a determinant way in terms of X (w falling and r rising , since L is getting larger), both fall faster in terms of Y (or rise less rapidly) in autarky as the price of Y rises. Nonetheless, we can be sure that the real wage falls and the real rental on land rises in this range, simply applying the Stolper-Samuelson theorem.


T
Y=g/p
Y=1/gp
X=1
L1
L2
L3
L4
L
Y
X
L1
L2
L3
L4
L
X-DX
Y-DY
L1
L2
L3
L4
L
Y-DY
X-DX
w/pX =w/pY
r/pX
r/pY
w/pX
w/pY
r/pX =r/pY
L1
L2
L3
L4
L

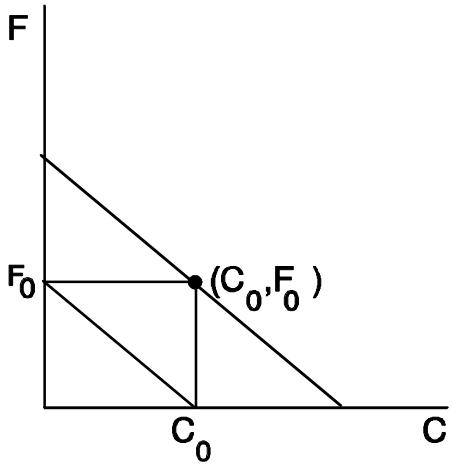
4. (15 points) Consider a two-factor (labor and land), two-sector (clothing and food) economy with the following unusual features: production of clothing requires only labor, while production of food requires only land; workers derive utility from only food, while land-owners derive utility from only clothing (that is, each wants only what the *other* produces). Otherwise the economy has the usual features: constant returns to scale, perfect competition, absence of distortions, etc.

a) Determine the autarky relative price and the free trade offer curve for this economy.

Ans: With constant returns to scale and only a single factor used in producing each good, the amount of that factor needed per unit will be a constant, and fixed endowments of the factors imply fixed outputs of the respective goods, so that it is as though the economy was endowed with the goods directly. Selecting units so that a unit of each good requires one unit of the corresponding factor, endowments of L labor and T land then imply outputs of clothing C_0=L and of food F_0=T. The production possibility frontier is just the point (C_0,F_0), or with free disposal the rectangle defined by the origin, the axes, and this point.

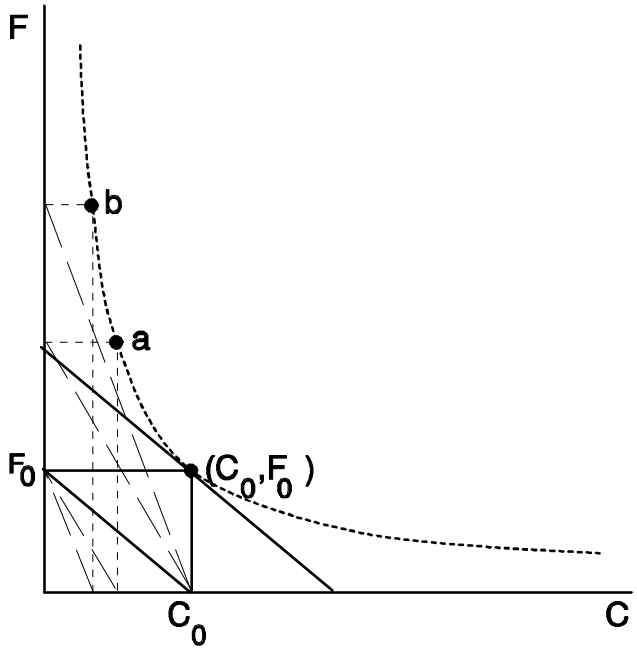
In autarky, workers will sell all of their clothing to landowners in exchange for all of the latter's food, so that the autarky relative price of clothing must be

$$p^a = \frac{p_C^a}{p_F^a} = \frac{F}{C}$$



The slope of the autarky price line is therefore found as the slope of the downward sloping diagonal of the production possibility rectangle, as shown here.

With free trade and an arbitrary price, workers and landowners will again each sell all of their output, and we can construct the price-consumption locus of the economy by just graphing what they buy at various prices, as shown below. This price-consumption locus is also the country's offer curve, if viewed relative to the production point as an origin.



More formally and precisely, for any relative price $p=p_F/p_C$ of food, demands for the two goods are $F=C_0/p$ and $C=pF_0$. Exports of clothing are then $X_C=C_0-C=C_0-pF_0$, implying $p=(C_0-X_C)/F_0$. Imports of food can then be expressed as

$$M_F = F-F_0 = \frac{C_0}{p} - F_0$$

or

$$M_F = \frac{F_0C_0}{C_0-X_C} - F_0$$

This is the equation of the country's offer curve, which is a rectangular hyperbola bounded by asymptotes at the maximum exports, $M_F=-F_0$ and $X_C=C_0$.

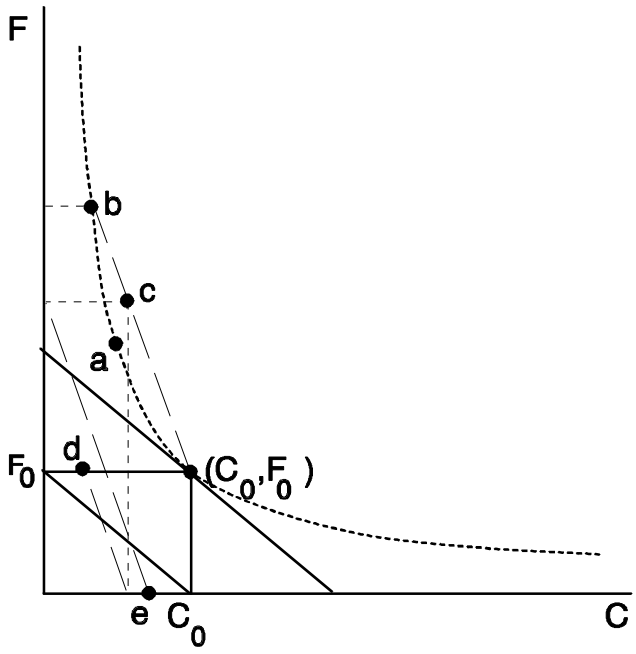
b) Starting from a free trade equilibrium in which the country exports clothing, work out the effects on quantities produced, consumed and traded of an increase in the world price of clothing.

Ans: This is just a movement along the offer curve, or price consumption curve, found above, such as from point a to point b. Clearly, quantities produced are unchanged, while the economy as a whole

consumes more food and less clothing than before the price increase. Quantities traded of both goods increase.

c) In what sense, if any, does the country gain from this "improvement" in its terms of trade?

Ans: If nothing else is done, then workers in this economy gain (they get to consume more food) while landowners lose (they can consume less clothing) due to this price change. One could not claim that to be an improvement for the country as a whole. However, the rise in price is an "improvement" in the terms of trade in the sense that, if it were accompanied by a suitable redistribution of income, it would make possible greater consumption by both groups. Specifically, as shown below, the higher price of clothing makes it possible for the country to trade to point c, which yields more of both goods and thus greater welfare for both groups. To attain this different outcome, it is necessary to accompany the price change with a redistribution from workers to landowners. A redistribution that serves the purpose is shown below, transferring some of the clothing output of the workers in lump-sum fashion to landowners, so that the endowment of the landowners is at point d while that of the workers is at point e. Drawing in their budget lines at the new price, one can see that landowners get more clothing, the transfer more than compensating them for the higher price they have to pay, while workers also get more food, the higher price of their endowment more than compensating them for the tax.



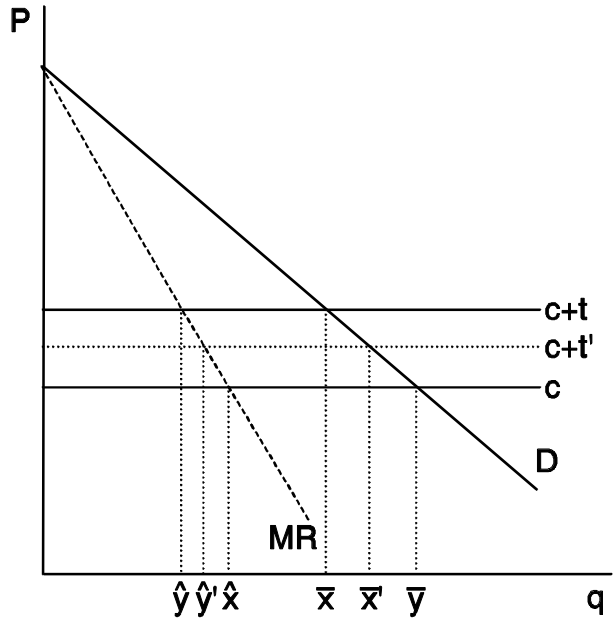
5. (15 points) Consider a two-country, international Cournot duopoly in which, initially, two identical firms with constant marginal cost, one from each country, sell a homogenous product both into their own and into each other's markets, where there is identical linear demand. Derive the effects of a fall in the constant transport cost on the volume of trade and on consumer welfare and profits of each firm in both countries. Can you tell what happens to total profits?

Ans: The geometry of this situation is shown in the diagrams on the next page. From the symmetry of the problem, we can look at just one of the two markets. Using the demand and marginal revenue curves, together with the constant marginal cost of supplying the market from at home c and abroad before (c+t) and after (c+t') the fall in transport costs, we can identify the end points of the two reaction curves. $\hat{x}$ is the domestic firm's sales if foreign sales are zero, while $\bar{y}$ is the level of foreign sales that would drive domestic sales to zero. Similarly, $\hat{y}$ and $\hat{y}'$ are the foreign firm's sales as a monopolist (x=0) before and after the drop in transport costs, while $\bar{x}$ and $\bar{x}'$ are the levels of domestic sales that would drive the foreign firm out of the market in the two cases. From the implied reaction-curve diagram below, one can see immediately that the drop in transport cost causes the domestic firm's sales, x, to fall, the foreign firm's sales (in the domestic market, and hence imports) to rise, and (noting the downward sloping 45° lines representing constant $x+y$) total sales to consumers to rise. The latter means that we have moved down the demand curve, raising consumer surplus. The rise in y implies that the profits of the domestic firm fall, since the demand curve it faces shifts to the left. In contrast, the profits of the foreign firm must rise, since it experiences a drop in cost and could have therefore increased its profit by just holding output fixed, and must do even better by changing y. The only qualitative result that I can't see how to get from the diagram is the effect on total profits of the two firms together.

For more quantitative results, and to determine total profits, look at the equations of this model. Letting the inverse demand curve be $p=a-bq$, the two profit functions for sales in the domestic market (total profits would be twice this, minus any fixed costs) are:

$$\pi = x(a-b(x+y)) - cx$$

$$\pi^* = y(a-b(x+y)) - (c+t)y$$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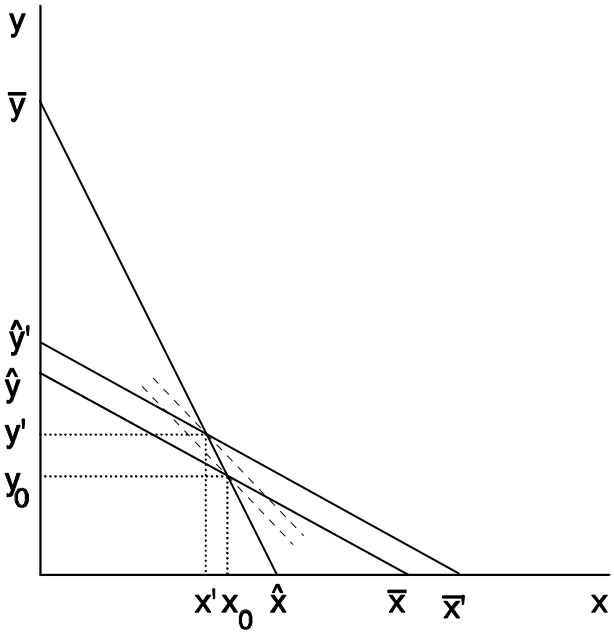
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Maximizing the first with respect to x and the second with respect to y, one gets:

$$x = \frac{a-c+t}{3b}$$

$$y = \frac{a-c-2t}{3b}$$

from which the drop in x and the rise in y as t falls are immediate. Adding these to get $q=x+y$ and substituting into the profit functions yields

$$q = x+y = \frac{2a-2c-t}{3b}$$

$$\pi = \frac{(a-c+t)^2}{9b}$$

$$\pi^* = \frac{(a-c-2t)^2}{9b}$$

From this we confirm that total output rises while profits of the domestic firm fall and profits of the foreign firm rise as t falls. Consumer welfare is given by consumer surplus as follows:

$$CS = \frac{[a-(a-bq)]q}{2} = \frac{bq^2}{2}$$

This increases with q and thus with the fall in transport cost.

Finally, total profit is

$$\pi + \pi^* = \frac{1}{9b}[2(a-c)^2 - 2t(a-c) + 5t^2]$$

the derivative with respect to t of which is

$$\frac{d(\pi+\pi^*)}{dt} = \frac{1}{9b}[10t-2(a-c)] \gtreqless 0 \quad as \quad t \gtreqless \frac{a-c}{5}$$

Thus, for t sufficiently close to zero, a fall in transport cost increases total profit of the two firms, but for larger t profits actually fall. The latter is easily understood, as a fall in transport costs that are initially prohibitive subjects both firms to erosion of their monopoly positions. It is perhaps more surprising that once these costs become lower, further reductions raise the combined profits of the firms.

6. (5 points) This is a freebie, or should be: Go back and read the instructions at the start of the exam, and take a moment, before you hand it in, to make sure you are following them.