Take-Home Final Exam - Answers

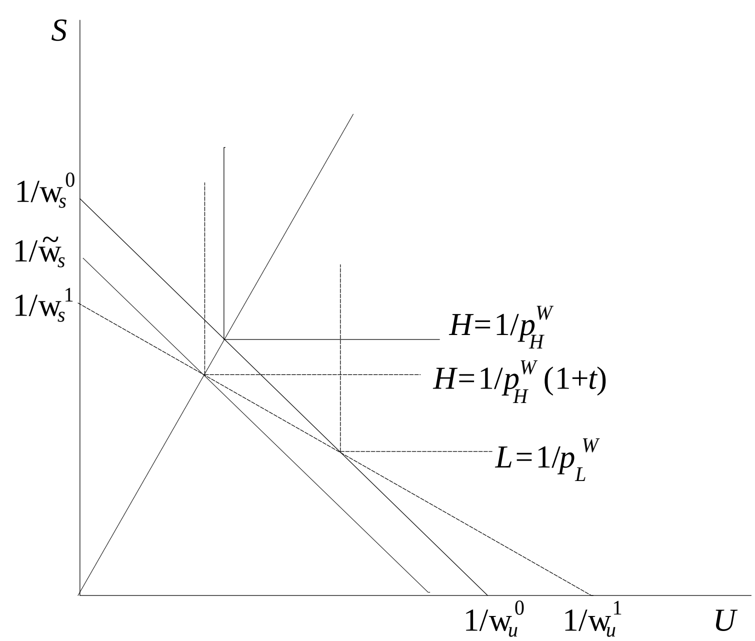
1. [25 points] Analyze the effects of a tariff on imports in a 2-good, 2-factor H-O Model with **fixed coefficient** (Leontief) technologies. Let the two factors be skilled labor, *S*, and unskilled labor, *U*, and assume a small open economy that has homothetic preferences and that produces both goods and imports the skill-intensive good under free trade. Specifically, use whatever tools you find appropriate to derive the (direction only of the) effect of the tariff on

a) [5]The real wages, w_u and w_s, of skilled and unskilled labor respectively;

Ans: Let the goods be S-intensive *H* and U-intensive *L*. World prices are fixed, since this is a small country, at p_H^W, p_L^W. The tariff on *H* raises its domestic price to $(1+t)p_H^W$. This shifts the unit value isoquant for *H* inward as shown, creating a new common tangent that is flatter than the old one.

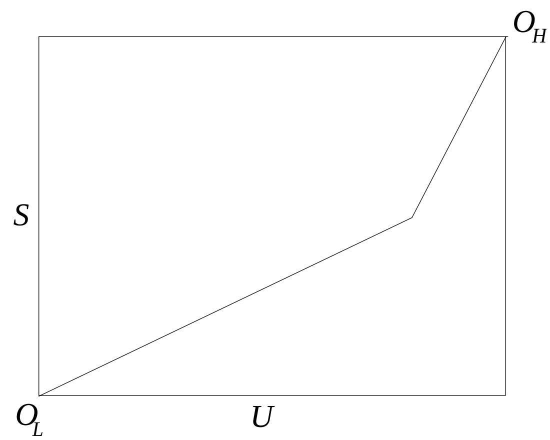
From this, we can see directly that the unskilled wage falls relative to p_L (which is fixed in the figure) and to p_H (which has increased). Thus the unskilled real wage is reduced.

The skilled wage rises in nominal terms. It also rises in real terms, as can be seen by noting that it rises above $\tilde{w}_s$, which is constructed in the figure as rising above w_s^0 by the amount that the price of *H* has risen.



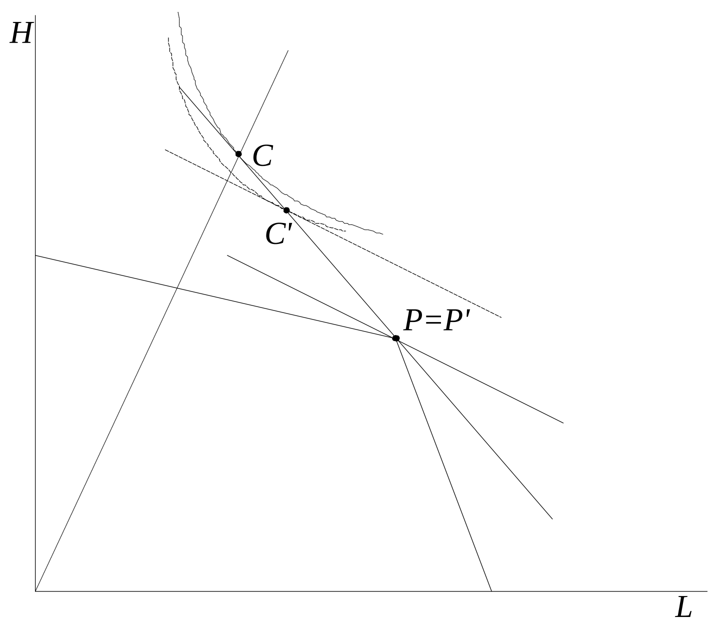
b) [5]The quantities of production, consumption, and trade of both goods;

Ans: Because of the fixed coefficient technologies, both industries continue to employ factors in the same ratios as before. There is only one way for them to be fully employed, as seen for example in the Edgeworth Box shown here. Since technologies have not changed, it follows that quantities of production do not change, even though the prices facing producers do. This is possible because the production possibility curve is kinked when technologies are fixed-coefficient, as shown in the next figure.



That is, the tariff flattens the domestic price line, but production stays at *P*. Consumption, however, moves from *C* to *C'* in response to the price change together with the redistributed tariff revenue. That is, the quantity of *H* consumed falls, while that of *L* consumed rises.

Since trade is the difference between production and consumption, the quantities of exports of *L* and imports of *H* both fall.



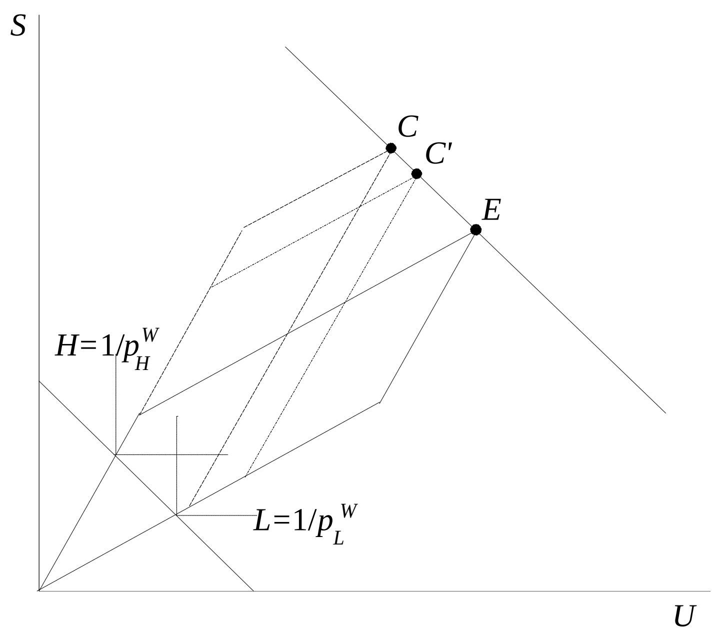
c) [5]The skilled and unskilled labor contents of total production, total consumption, and net trade.

Ans: Factor contents are unambiguous, since the per-unit factor requirements of production are fixed. In factor space, the factor content of production is just the country's endowment point.

The factor content of consumption under free trade (where the value of consumption equals the value of production and where, as always with perfect competition, the value of production equals the value of the factors employed) is on an isocost line through the endowment point and includes more *S* and less *U* than production, because consumers consume more of the S-intensive good than is produced and less of the U-intensive good. Therefore the factor content of consumption with free trade is as shown by point *C* below.

The factor content of consumption under the tariff includes less *S* and more *U* than under free trade, because consumers consume less *H* and more *L*. It is still on the same isocost line, however, since with balanced trade the value of consumption must equal the value of production at world prices. Thus it is shown by a point like *C'* below. (Note that *C'* is worth more, at the new domestic factor prices—note shown below—than the endowment, again reflecting the redistributed tariff revenue.)

The net factor content of trade is just the different between the factor content of production and the factor content of consumption. Clearly, the (positive) net exports of *S* and the (positive) net imports of *U* both fall due to the tariff.

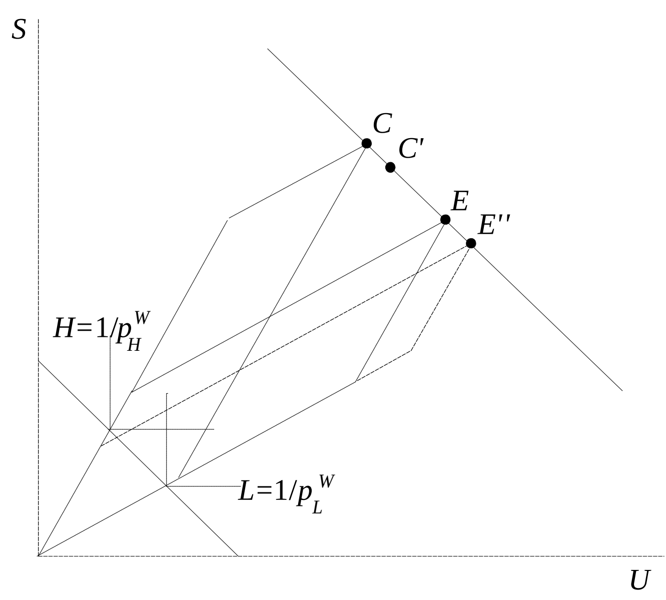


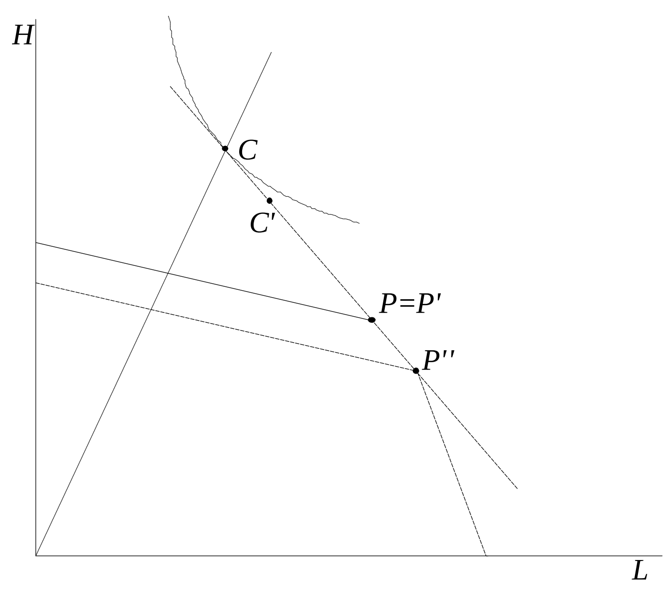
d) [5]Finally, compare your results to what would have happened if, instead of the tariff, the factor endowments of the country had been changed by the amounts of the change in the factor content of net trade that you found in part (c). That is, if the tariff caused it to import more (less) of a factor, replace the tariff with an equal increase (decrease) of that factor endowment, and vice versa for exports.

Ans: In part (c) we found that the tariff caused the country to import less *S* content and to export less *U* content. The changes both appeared as the distances from *C* to *C'* in the figure of part (c). (Since the factor content of production does not change in this case, the change in the factor content of net imports is the same as the change in the factor content of consumption, imports being consumption minus production.)

If we replace the tariff with a decrease in the endowment of *S* equal to the drop from *C* to *C'*, and also with an increase in the endowment of *U* equal to the rise from *C* to *C'*, then the endowment point will move by exactly the same amounts as the consumption point moved due to the tariff. In factor space, the endowment point moves from *E* to *E*" below, while (since there is no longer a tariff) the consumption point remains at *C*.

In output space, the production possibility frontier shifts down and to the right, its kink moving by the same amounts as the consumption point did due to the tariff. This is shown as the move from *P* to *P''*. The result, with free trade and thus fixed prices, is no change in either factor price and no change in consumption. Production of *H* falls and production of *L* rises, so that exports and imports both rise, as does the factor content of trade for both factors.





e) [5]In what sense, if any, is the change in the factor content of trade due to the tariff indicative of the factor price changes that the tariff causes?

Ans: Had the country been closed and able to consume at *C* from its own production due to having different factor endowments (also at *C*), then the same factor price changes would have occurred if its endowments were then changed by these amounts. Thus the factor price changes caused by a tariff are identical to those caused in an otherwise identical closed economy by changes in factor endowments equal to the change in the factor content of trade. That is, in this case the tariff raises the skilled wage at the same time that it reduces net imports of skilled labor in factor content terms. Thus the tariff has the same effect on the skilled wage as a reduction in the endowment of skilled labor in a suitably equivalent closed economy.

2. [40 points] This question is mostly about **fragmentation**, defined here as the splitting of a production process into two parts that can then be done in different places. The following two parts ask you to work out the effects of particular kinds of fragmentation in the context of two familiar models of international trade.

a) [20]Consider first a small, open, Ricardian economy that is initially capable of producing two (final) goods, *X* and *Y*, with unit labor requirements a_X and a_Y and labor endowment *L*. International prices of the goods, taken as given, are p_X and p_Y in terms of some numeraire.

i) [2]Write an expression for the wage of labor in this economy.

Ans: Note first that, as usual in a Ricardian economy,

$$\text{if } X > 0, \text{ then } p_X = wa_X$$

$$\text{if } Y > 0, \text{ then } p_Y = wa_Y$$

and prices are smaller than these equations indicate if the respective goods are not produced. Therefore, the wage is

$$w = \frac{p_X}{a_X} \text{ or } w = \frac{p_Y}{a_Y}$$

and is larger than these equations indicate if the respective goods are not produced. All this can be summarized by

$$w = \max\left(\frac{p_X}{a_X}, \frac{p_Y}{a_Y}\right)$$

ii) [2]Assuming that good X is exported by this economy, what restriction does that place on the prices and unit labor requirements?

Ans: Export of X requires that good X be produced and therefore that

$$\frac{p_X}{a_X} \geq \frac{p_Y}{a_Y}$$

since otherwise, from (i), the wage would be higher than p_X / a_X and good X could not be produced. This in turn implies the following restriction on unit labor requirements compared to the (given) world prices:

$$\frac{a_Y}{a_X} \geq \frac{p_Y}{p_X}$$

iii) [8]Suppose now that the technology for producing good X fragments into two parts. The first part produces a new intermediate input Z with unit labor requirement a_{X1}. The second transforms one unit of Z into one unit of X with an additional labor requirement of a_{X2}. The total labor required for both steps is the same as before: $a_{X1} + a_{X2} = a_X$. The prices of X and Y remain the same on the world market and satisfy the restriction derived in part (ii). A new world market also appears for good Z, which can be exported or imported for price p_Z. Calculate the wages that would be earned by labor in each of the following activities:

		Ans:
w_Z:	Producing Z	$w_Z = \dfrac{p_Z}{a_{X1}}$
w_{ZX}:	Producing X from Z	$w_{ZX} = \dfrac{p_X - p_Z}{a_{X2}}$
w_X:	Producing X from scratch	$w_X = \dfrac{p_X}{a_X}$
w_Y:	Producing Y	$w_Y = \dfrac{p_Y}{a_Y}$

Workers will engage in whatever activity yields the highest wage from among these choices.

iv) [2]Show that, unless workers are indifferent among occupations, they will earn more producing either only *Z* or only *X* from *Z* than in the other activities.

Ans: We already know from (ii) that $w_X \geq w_Y$. Now compare w_X to the other options, using from the result in (iii) for w_{ZX} that $p_X / a_{X2} = w_{ZX} + p_Z / a_{X2}$:

$$w_X = \frac{p_X}{a_X} = \frac{p_X}{a_{X2}} \frac{a_{X2}}{a_X} = \left(w_{ZX} + \frac{p_Z}{a_{X2}} \right) \frac{a_{X2}}{a_X} = w_{ZX} \frac{a_{X2}}{a_X} + \frac{p_Z}{a_{X1}} \frac{a_{X1}}{a_X}$$

$$= w_{ZX} \frac{a_{X2}}{a_X} + w_Z \frac{a_{X1}}{a_X}$$

Thus w_X is a weighted average of w_Z and w_{ZX}. Unless the two are equal, so that workers are indifferent, w_X must be strictly less than one of them.

v) [4]Derive and draw a graph of the quantity of trade of good *Z* as a function of its price p_Z.

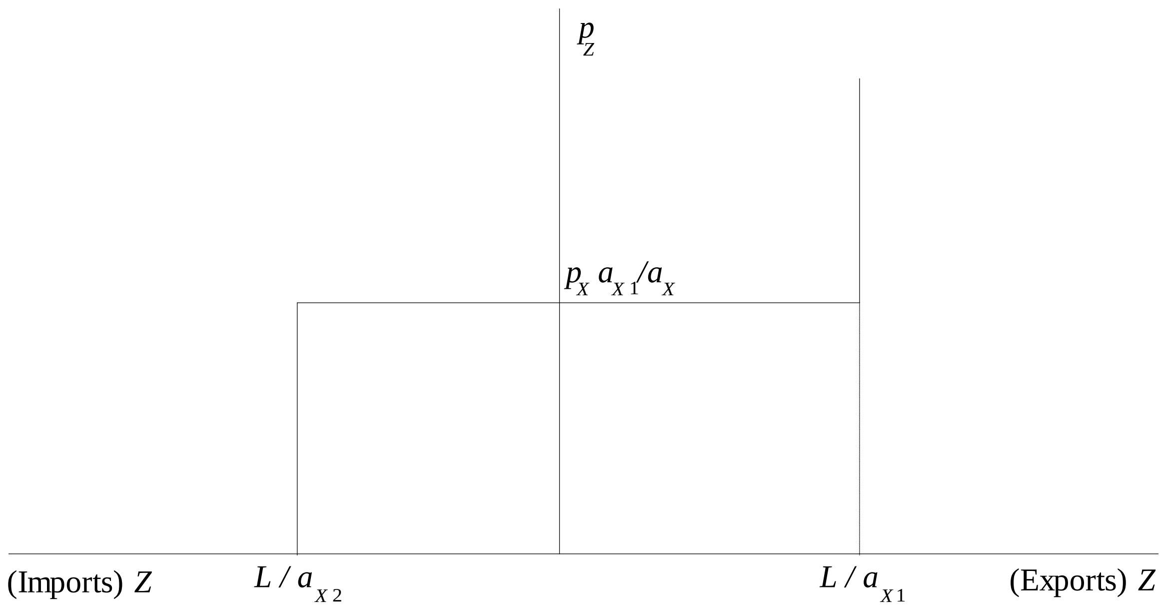
Ans: From (iv), this economy will produce only the intermediate good *Z* if $w_Z > w_{ZX}$ and it will produce only the final good *X* from the intermediate good if $w_Z < w_{ZX}$. In the former case, it must export all of the intermediate good that it produces, which is $\hat{Z} = L / a_{X2}$, since it has no use for it. In the second case, it must import all good *Z* that it needs, which is equal to the number of units of *X* that it will produce, $\breve{Z} = L / a_{X1}$. Only if the two wages are equal may it engage in a mix of activities, perhaps including producing exactly what it needs of *Z* and not trading it, and it will be indifferent between this and all the other options ranging from producing only *Z* to producing only *X* from *Z*. Therefore, using the wages derived in (iii), the country will:

Produce	Trade *Z*	If
Only Z	export $\hat{Z} = L / a_{X2}$	$\frac{p_Z}{a_{X1}} > \frac{p_X - p_Z}{a_{X2}}$
Only X from Z	import $\breve{Z} = L / a_{X1}$	$\frac{p_Z}{a_{X1}} < \frac{p_X - p_Z}{a_{X2}}$

The division between these two cases, where $w_Z = w_{ZX}$, is

$$\frac{p_Z}{a_{X1}} = \frac{p_X}{a_{X2}} - \frac{p_Z}{a_{X2}}$$
$$p_Z a_{X2} = p_X a_{X1} - p_Z a_{X1}$$
$$p_Z a_X = p_X a_{X1}$$
$$p_Z = p_X \frac{a_{X1}}{a_X}$$

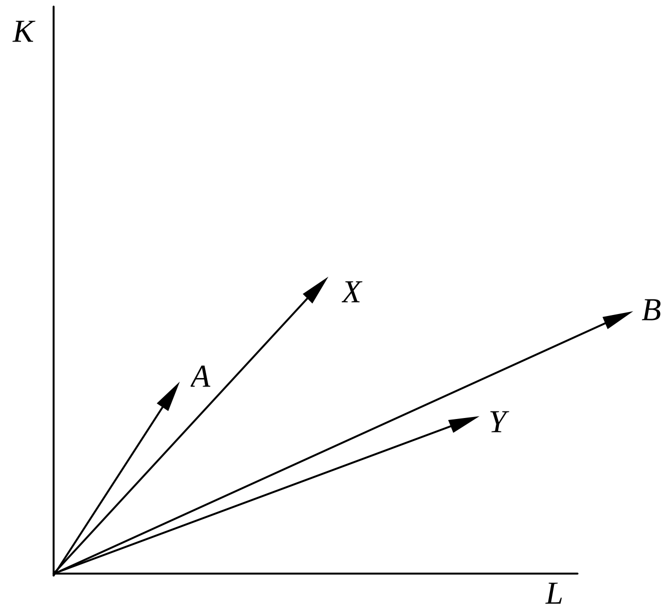
Therefore, the graph of exports is:



vi) [2]Under what circumstances, if any, does this country gain from this possibility of fragmentation, and in what sense?

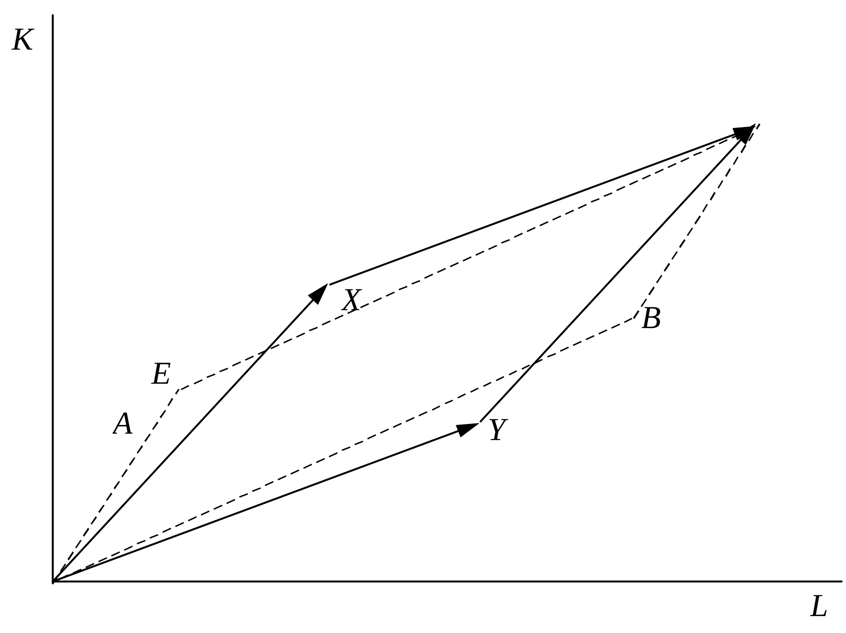
Ans: As long as $p_Z \neq p_X \frac{a_{X1}}{a_X}$, w is above $\frac{p_X}{a_X}$ and the entire population gains from fragmentation, since their (real, since prices are fixed) wages rise. At worst, if $p_Z = p_X \frac{a_{X1}}{a_X}$, then there is neither gain nor loss.

b) [20]The diagram at the right shows, as vectors *X* and *Y*, the quantities of capital and labor needed for production of two (final) goods in a 2×2 Heckscher-Ohlin (H-O) economy. The quantities of *X* and *Y* themselves are those demanded in an integrated world economy. Suppose instead, however, that the world is not integrated, but rather is divided into two countries whose factor endowments are immobile internationally and are given by the vectors *A* and *B* in the figure (where *A*+*B*=*X*+*Y*). Assume for simplicity that both industries have fixed coefficients, the ratios of capital to labor used in both industries being fixed independently of factor prices.



i) [8]What can you say about the patterns of production and trade, and also about factor prices in the two countries, in a world equilibrium of these countries with free and frictionless trade? That is, who will produce and export what, and how will the factor prices compare across countries?

Ans: The above vectors can be used to construct the factor use lens (solid lines) and the factor endowment lens (dashed lines), which appear as follows:



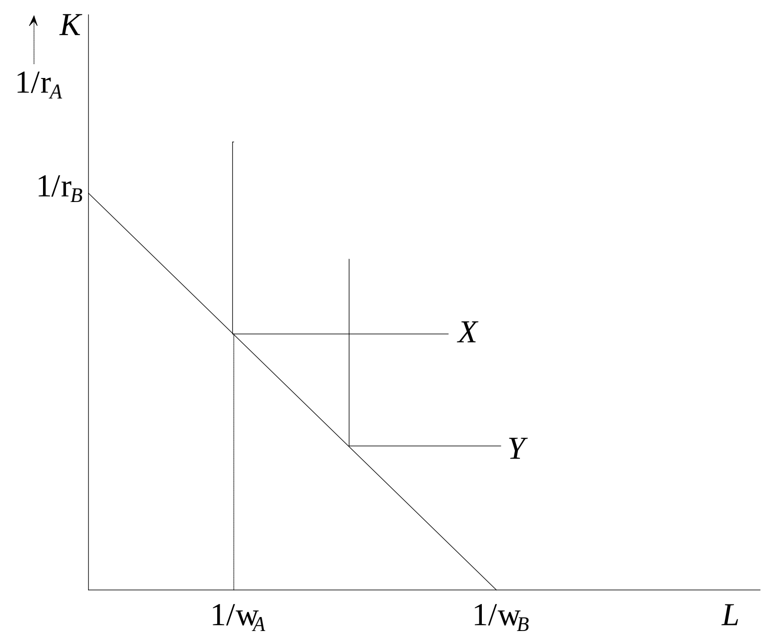
Since the endowment point E is outside the use lens, it follows that factor price equalization is *not* possible here. Either country A or country B must specialize.

Given the fixed coefficient technologies, country A cannot fully employ its capital, since it does not have enough labor to operate all of it in either industry. A will therefore specialize completely in producing good X, which leaves the smallest amount of its capital unemployed. The rate of return to capital in country A will be zero, while the wage will be labor's average product in producing X.

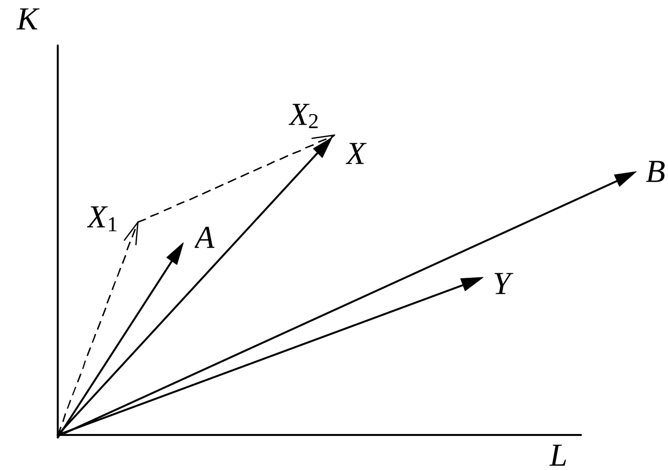
Country B's endowment lies between the two factor use vectors, and they can be fully employed only by producing both goods. Thus B will produce both X and Y, exporting Y in exchange for X from country A. Both factor prices will be positive in country B, consistent with the usual Lerner-Pearce diagram but with L-shaped unit value isoquants as shown below. Their exact values depend on goods prices, however, which depend also on demands in the two countries.

Nonetheless, we can infer immediately that

$$w_A > w_B, \quad r_B > r_A = 0$$



ii) [8]Suppose now that the technology for producing good *X* becomes fragmented as shown in the figure below: The same vector *X* is split into the two dashed vectors, X_1 and X_2 that add up to *X*. X_1 produces an intermediate input, *Z*, that together with the inputs X_2 will now produce the same quantity of *X* as before. (All other vectors are the same.) Once again, what can you say about the patterns of production, trade, and factor prices after this fragmentation of technology?

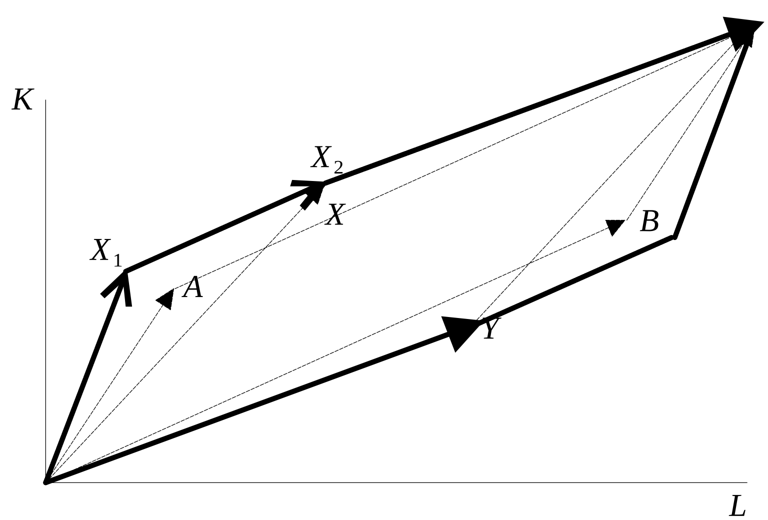


Ans: Now there are three goods being produced, *X*, *Y*, and *Z*, if we think of good *X* as only being produced via technology X_2 together with *Z*. The factor use lens is now the heavy solid line shown below. The endowment lens, which is unchanged from before, is now inside it.

Factor price equalization is therefore possible and will occur. Because there are three production activities now, and still only two factors, we cannot say exactly what will be the pattern of production and trade. Country A will necessarily produce at least some of the intermediate input *Z* using the factor vector X_1 since that is the only way it can fully employ its capital. It must also produce something else in addition, but either final *X* or *Y* will do. It will export intermediate good *Z* in exchange for *Z* and/or *Y*.

iii) [4]Is anyone made better off by this fragmentation?

Ans: Yes. World output of *X* and/or *Y* goes up, since all capital can now be used productively, just as if it had been able to move internationally. The owners of capital in country *A* are certainly better off, since their return rises from zero to something positive.



3. [15 points] Consider the following "friends and enemies" version of the Rybczynski Theorem:

> Assume a H-O Model with arbitrary numbers of factors and goods, all of which are initially produced using strictly positive amounts of every factor. Then if prices of all goods are held fixed, an increase in the endowment of any one factor, holding all other factor endowments constant, will cause the output of one good (the "friend") to expand more than in proportion to the increased endowment, and it will also cause the output of some other good (the "enemy") to contract.

a) [8]For each of the following cases, either prove this Theorem or provide a counterexample. (Your counterexample may be part of your answer to part (b).)

i) More goods than factors.

Ans: If there is only one factor, as in the Ricardian model, then the Theorem is False. If, as the theorem assumes, a Ricardian economy is initially producing both goods, then an increase in the labor endowment can lead to an increase in the output of both, so that there is no enemy, and these increases could both be equal in percentage terms to the increased endowment, so that there is no friend either. Output is indeterminate here, but that does not help. I did not think of this case when I wrote and first answered this question, and I found this counterexample in one of your answers. I therefore give full credit to those of you who, like me, did not catch this.

If there are at least two factors (and thus at least three goods), then the Theorem is True. Proof: With more goods than factors and all goods produced, the factor price equalization theorem tells us that if prices of all goods are held fixed, then prices of factors will also be fixed. Therefore, in Jones's notation, $\hat{p} = 0$, implying $\hat{w} = 0$, and the equation relating factor endowment changes to changes in outputs simplifies to

$$\hat{V}_i = \sum_{j=1}^{n} \lambda_{ij} \hat{X}_j$$

for any factor *i*, where $\sum_{j=1}^{n} \lambda_{ij} = 1$. That is, $\hat{V}_i$ is a weighted average of the changes in outputs.

Let factor 1 be the one whose endowment increases. Then $\hat{V}_1 > 0$ and

$\hat{V}_2 = \ldots = \hat{V}_m = 0$. With all $\lambda_{ij} > 0$, $\hat{V}_1 > 0$ implies $\sum_{j=1}^{n} \lambda_{1j} \hat{X}_j > 0$, which in turn implies that $\exists\ j_0 \ni \hat{X}_{j_0} > 0$. Likewise, $\hat{V}_2 = 0$ implies $\sum_{j=1}^{n} \lambda_{2j} \hat{X}_j = 0$, which together with $\hat{X}_{j_0} > 0$ implies that $\exists\ j_E \ni \hat{X}_{j_E} < 0$. This is the "enemy." Finally, $\sum_{j=1}^{n} \lambda_{1j} \hat{X}_j = \hat{V}_1 > 0$, together with $\hat{X}_{j_E} < 0$, implies $\exists\ j_F \ni \hat{X}_{j_F} > \hat{V}_1$. This is the "friend."

(Note how this proof fails in the one-factor case. There is then no second factor with $\hat{V}_2 = 0$.)

ii) More factors than goods.

Ans: The Theorem is not true in this case. Instead, the specific factors model provides a counterexample. See part (b).

Actually (and again I realized this only from reading your answers), this is not quite right. The specific factors model in part (b) can be interpreted either as a two factor model with one factor immobile between sectors, or as a three factor model with two factors not being used in one sector. Either way it violates the assumptions of the Theorem. I believe that you could easily modify the specific factors model so as to satisfy the assumptions and still approximate its behavior. Just have each specific factor used in a tiny but essential amount in the other sector.

b) [7]Illustrate *one* (not both) of your results using *either* (not both) of the following two special cases of the general H-O Model. That is, for just one of the models listed below, either identify the friend and the enemy of each factor, and show that they respond as described in the Theorem, or show that a friend or enemy does not exist for one of the factors.

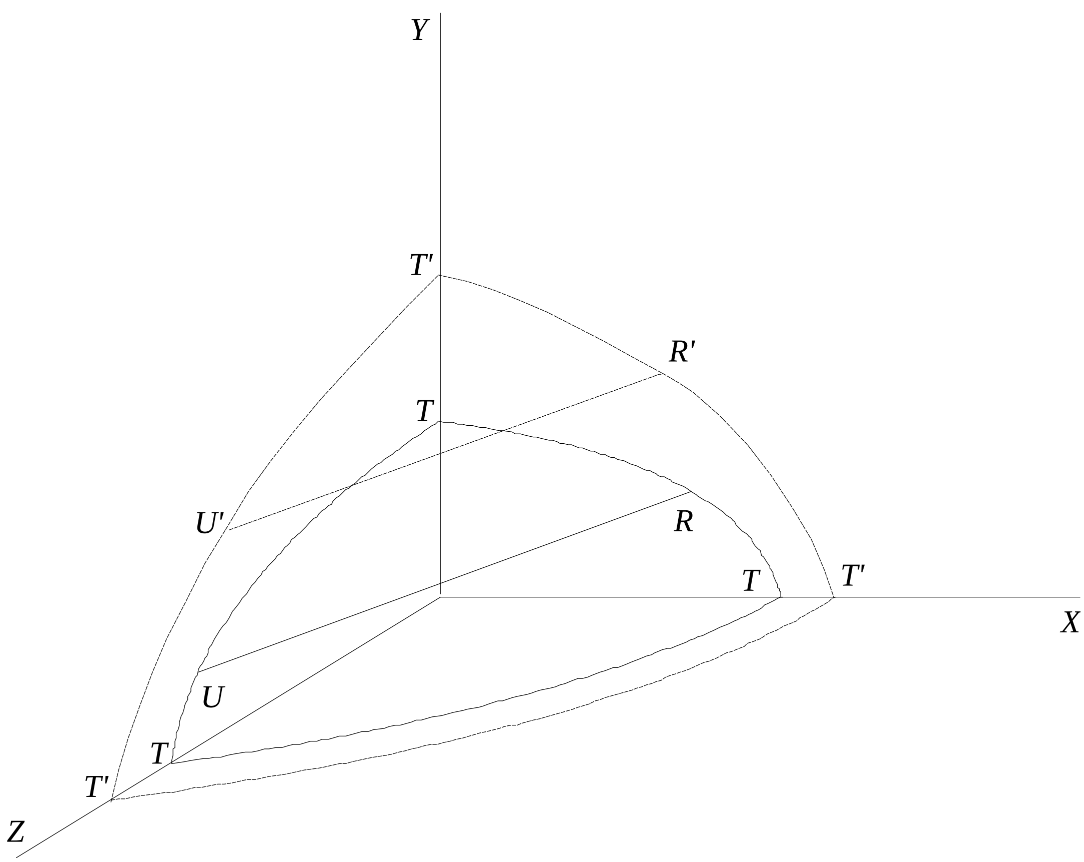
i) The 3-good, 2-factor H-O Model of Melvin.

Ans: This one's tricky, which is why I decided to let you (and me) do just one of these. With the Melvin indeterminacy of production, you cannot say for sure which goods are friends and enemies, even though they must exist. And showing that they do exist, regardless of how the indeterminacy is resolved, more or less repeats the proof in part (a).

It is possible to illustrate, however, as below. Goods *X*, *Y*, and *Z* are produced with capital and labor, *X* being most labor intensive, *Y* most capital intensive, and *Z* in between. Production possibilities are initially *TTT*, and prices are such that the country can produce anywhere along the rule, *RU*.

Now the capital stock increases. This causes the production possibilities to expand as shown to *T'T'T'*, the intersections of that surface with the three axial planes shifting out in Rybczynski fashion, biased towards the more capital-intensive good. Thus the new rule, *R'U'*, on which the country must produce with unchanged prices, is parallel to the old. It intersects the *XY* plane above and to the left of *R* and the *YZ* plane above and to the right (behind, really) *U*.

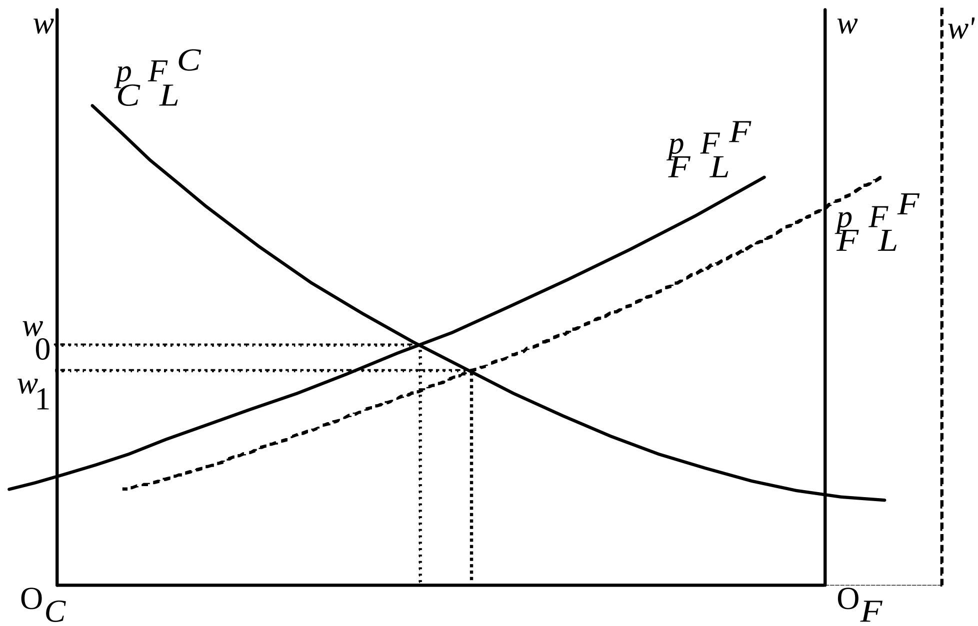
Thus the country's production must move from somewhere on *RU* to somewhere on *R'U'*, and we don't know where in either case. However, by trying all possibilities, you can verify that the output of one good will always fall and the output of another will always rise more than in proportion to the capital stock.



ii) The Specific Factors Model with two goods, two specific factors, and one mobile factor

Ans: We need to consider each factor separately.

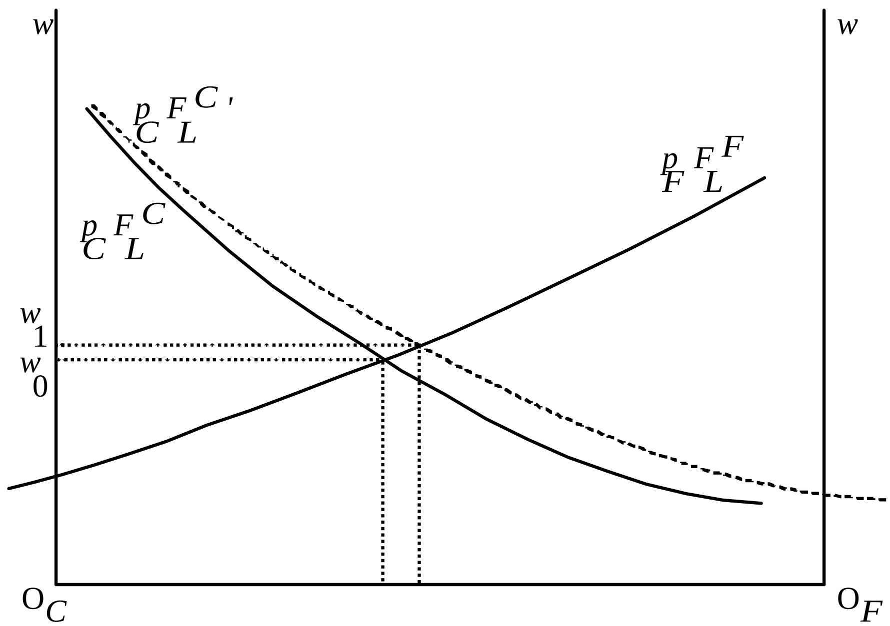
Labor: Letting the mobile factor be labor, an expansion of the endowment of labor expands the box diagram, as shown below. This leads to an increase in the outputs of both goods (since with fixed specific factors, each employs more labor), but there is no assurance that either expands more than in



proportion to the expansion in the labor force (if each expands employment in the same proportion, then diminishing returns assures that both outputs expand by less). Therefore, labor has no enemy, in the sense of the Theorem, and it may not have a friend.

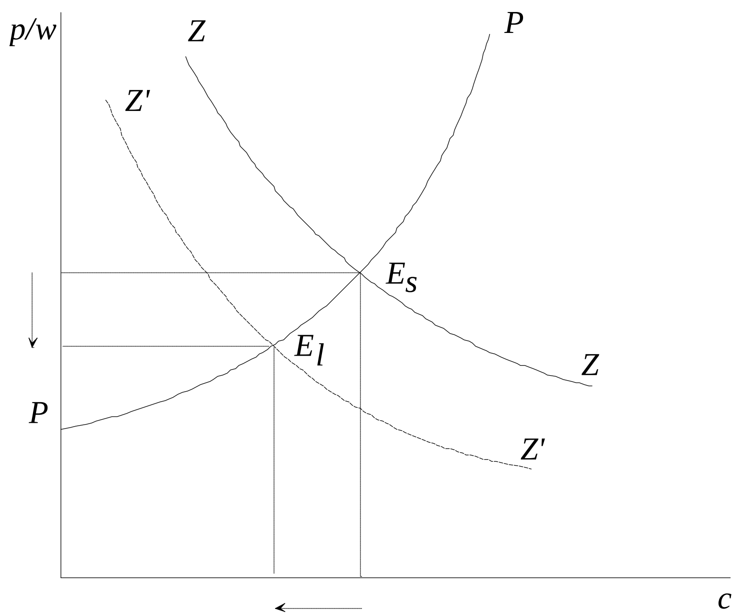
Specific Factor: (Either will do.) The figure below shows the effect of increasing the endowment of the specific factor in the *C* industry – call it capital. Assuming constant returns to scale, the marginal products of both factors will remain constant if both factors expand in the same proportion. Therefore, for any level of the wage, the labor needed to justify it will now be expanded by the same percentage as the increase in the capital stock. In other words, the marginal product of labor curve, as well as the value of it when multiplied by a fixed price, expands outward from the vertical axis by a constant percentage equal to the increase in capital.

As can be seen, output of good *C* expands while output of good *F* contracts. Therefore a specific factor *does* have an enemy in the sense of the Theorem: the output in the other sector. It does not have a friend, however, since output in its own sector necessarily expands by less than the percentage increase in the specific factor. We know this, since to expand by that amount, we would have to move horizontally to the new marginal product curve, and instead we move by less than that.



4. [15 points] Suppose a world of many countries, each like the countries modeled in Krugman (1979) but with labor endowments, L_j, that may be different. That is, consumers in country j maximize a utility function $U^j = \sum_{i=1}^{n_j} v(c_i^j)$ where c_i^j is per capita consumption of the i^{th} variety in country j, while n_j firms, one for each variety i, produce a quantity $x_i^j = L_j c_i^j$ incurring a labor cost of $l_i^j = \alpha + \beta x_i^j$ with $\alpha, \beta > 0$. In equilibrium, firms charge prices p_i^j that maximize their profits given outputs of other firms; the wage, w^j, is determined to clear the labor market ($L_j = \sum_{i-1}^{n} l_i^j$); and free entry of firms drives profits to zero: $\pi_i^j = p_i^j x_i^j - w^j l_i^j = 0$.

   a) Following Krugman, suppose first that the v functions are such that $v'>0$, $v''<0$, and $d\varepsilon / dc_i^j < 0$ where ε is the (positive) elasticity of demand facing any firm as a function of its consumers' per capita demand. Holding constant the world's population, $L^W = \sum_j L_j$, determine how the following variables depend on an individual country's population, L_j.



Ans: For an individual country, and also for the world, this is exactly the model of Krugman (1979), and we can analyze it using his diagram, reproduced at the right. The figure shows the "effect" of an increase in the population, as valid for a comparison of two countries in autarky as for a single country (or world) that experiences an increase in population. As in Krugman, such an increase causes a fall in each consumer's consumption of each variety, c, as they also consume a larger number of varieties. The ratio of price to wage falls, implying an increase in the real wage.

      i) The ratio of the wage to any good's price, w^j / p^j, in autarky.

      Ans: As long as the countries are not trading, then each will have its variables determined in a diagram like this, and countries with larger populations will correspond to the equilibrium E_l while smaller countries will be at E_s. Thus the larger is a country's population, the larger will be the ratio of its wage to the price of a typical variety.

ii) The ratio of the wage to any good's price, w^j / p^j, in free trade.

Ans: If all countries are trading freely, then wages and prices are determined in the world market, a single wage and price prevailing in all countries. In that case, the ratio of wage to price is the same in all.

iii) The gain in consumer utility of going from autarky to free trade.

Ans: All consumers gain going from autarky to free trade, since their only income is wages and the real wage rises in every country. However, since consumers in small countries start out with lower wages, and consumers everywhere end up the same, the size of the gain from trade is larger for consumers in small countries. Consumers also gain in all countries from access to more varieties of the differentiated product, and since small countries have fewer varieties in autarky than large countries, consumers in small countries gain more from variety as well as from the increase in their wage relative to price.

b) Now suppose instead that $d\varepsilon / dc_i^j = 0$. Which of your answers in part (a) are altered, and why?

Ans: The sign of $d\varepsilon / dc_i^j$ does not matter for the *ZZ* curve. It does matter, however, for the *PP* curve, which reflects monopoly markup prices for the producers of the differentiated good. If the demand elasticity that they face does not fall as consumption levels rise, then the markup will remain constant and the price charged by firms will also be constant. That is, the *PP* curve will be horizontal. Thus the ratio of wage to price does *not* after all depend on population. The answers to part (a) are changed as follows:

i) Country size will no longer matter for the ratio of wage to price in autarky. All countries will have the same ratio.

ii) Country size still does not matter for the ratio of wage to price in free trade (thus no change).

iii) Country size will no longer matter for the change in the wage-price ratio going from autarky to free trade. However, it does still matter for the size of gains from trade, since this includes the variety effect that is still present.