Take-Home Final Exam -Answers

1. (20 points) Determine the effects of "learning by doing" in the Dornbusch-Fischer-Samuelson Ricardian model. That is, starting from a free trade equilibrium (with zero transport costs), suppose that unit labor requirements ($a(z,t)$ at home and $a^*(z,t)$ abroad) fall over time at a constant rate per unit time, λ at home and λ^* abroad, in one or both countries, but in only those goods, z, that a country is producing. That is,

$da(z,t)/dt = -\lambda\, a(z,t)$ for all z that are being produced in the home country at time t

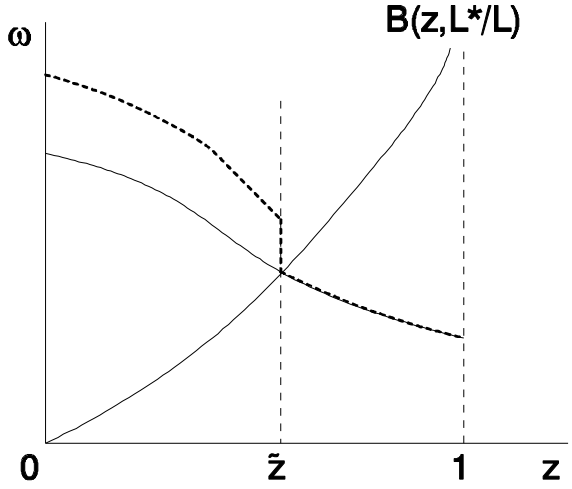
$da^*(z,t)/dt = -\lambda^*\, a^*(z,t)$ for all z that are being produced in the foreign country at time t

Specifically,

a) Suppose first that this learning takes place only the in the home country: $\lambda>0$, $\lambda^*=0$. Determine the effects over time on the range of goods produced in both countries and on their real wages.

Ans: As in DFS the equilibrium has the home country producing and exporting goods $z\in[0,\tilde{z})$ and the foreign country producing and exporting $z\in(\tilde{z},1]$ where $\tilde{z}$ is determined by the intersection of $A(z)=\frac{a^*(z)}{a(z)}$ and $B(z)=\frac{\theta(z)}{1-\theta(z)}\frac{L^*}{L}$. As learning by doing shifts portions of the $a()$ and $a^*()$ functions, we can observe the changing equilibrium from the same diagram, since the $B()$ function, depending only on preferences and labor endowments, will not change.

For part (a), with learning by doing in the home country only, $a(z)$ shrinks over time and thus $A(z)$ shifts upward over time at the rate λ, but only over the range $z\in[0,\tilde{z})$. $A(z)$ therefore acquires a discontinuity at $\tilde{z}$ as shown at the right.

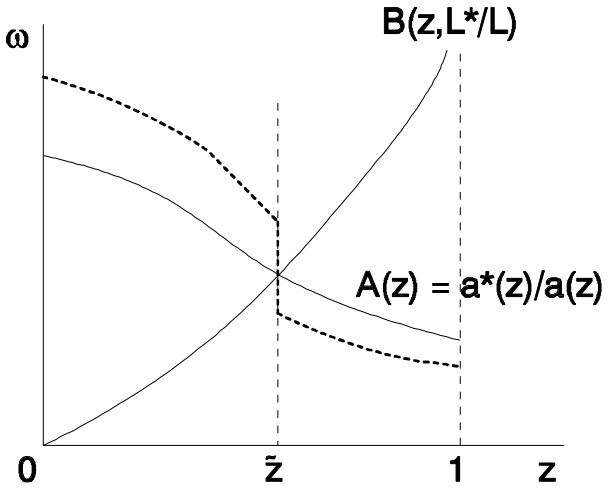


From the diagram it is clear that $\tilde{z}$ remains unchanged, so that the range of goods produced by each country remains the same, as does the ratio of their wages, $\omega = w/w^*$. Taking the foreign wage as numeraire, the domestic nominal wage is unchanged. Prices of goods in $[0, \tilde{z})$ have fallen due to the improved productivity, while prices of goods in $(\tilde{z}, 1]$ are unchanged. Therefore both real wages increase.

b) Repeat part (a) with both countries learning at the same rate: $\lambda = \lambda^* > 0$.

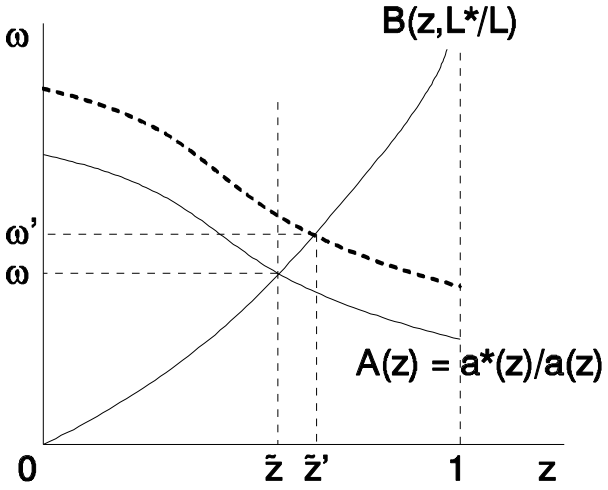
Ans: If the foreign country also learns by doing, its labor requirements also decline over time over the range where it produces, $(\tilde{z}, 1]$, and this shifts that part of the $A(z)$ curve *down*.

This makes for an even larger discontinuity, as shown. It is still true, however, that equilibrium is attained with the same $\tilde{z}$ and ω. Again taking w* as numeraire, w is constant, and prices of *all* goods fall due to the productivity improvements in both countries. Real wages again both rise, now by more than before.



c) Repeat part (a) under the assumption, admittedly perverse, that foreigners "unlearn by doing," exactly as fast as domestic residents learn: $\lambda = -\lambda^* > 0$. [Note: You do not need to determine whether it is possible for the foreign real wage to rise in this case, although you may attempt that for extra credit if you wish. I was not able to determine this.]

Ans: Now a^* rises over $(\tilde{z}, 1]$ at the same rate that a falls over $[0, \tilde{z})$, so that $A(z)$ rises uniformly over its whole range, as shown here. Now the equilibrium division between the two countries' outputs does change, $\tilde{z}$ rising and eventually approaching one. The domestic wage also rises relative to the foreign wage, as shown, but the increase is always less than the change in productivity, which is given by the vertical shift of the $A(z)$ curve.

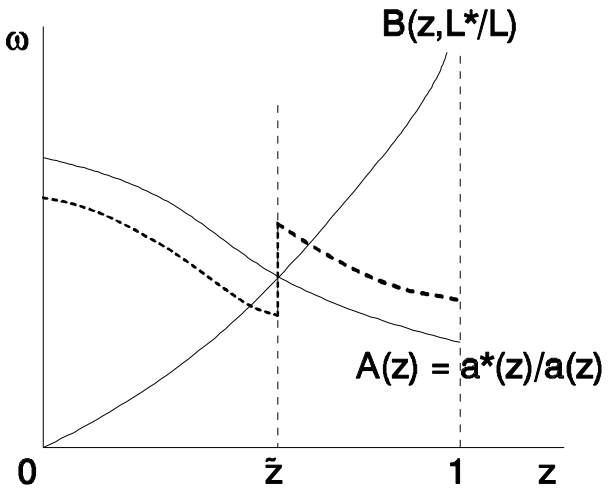


As for real wages, the home real wage is easy, although not unambiguous. Taking w* as numeraire, consider what will happen at any time t when production is divided by $\tilde{z}'$. Over time, prices of goods in $[0, \tilde{z}')$ -- call them p_H since they are produced at home -- are falling at the rate $\lambda - \hat{w}$, prices of goods in $(\tilde{z}', 1]$ -- call them p_F -- are rising at the rate λ, and the home country wage, w, is rising at a rate less than λ as indicated by the figure. Thus

$$\lambda = \hat{p}_F > \hat{w} > 0 = w^* > \hat{p}_H = \hat{w} - \lambda$$

and it follows that the home wage is falling relative to goods produced abroad and rising relative to goods produced at home. This means it is ambiguous, but we *can* say that the change in the real wage depends on the shares of the two countries in world output, and these are changing systematically. If the home country started with a sufficiently small share ($\tilde{z}$ close to zero), then it must lose initially from this combination of learning and unlearning, since it will primarily experience the increased prices of the goods produced ever more inefficiently by foreigners. But over time, its own learning leads it to take over more and more of production, its share of world output eventually approaching one, and at some point its real wage must begin to rise.

The foreign real wage is a bit harder, I think, which is why I left it as extra credit. The inequalities above tell us that the foreign wage also falls in terms of its own goods and rises in terms of home goods, so that it faces more or less the same ambiguity, dependent on the two countries' shares of production. What is not clear to me, however, is how it will fare in the long run. As $\tilde{z}'$ approaches one, the B function must become almost vertical, and the home wage must rise relative to the foreign wage at essentially the rate λ. The foreign workers are still stuck with declining productivity on a narrower and narrower range of goods, and even though this range shrinks toward zero, it never reaches it since these workers must be employed. I can't rule out that their real wage falls forever, but I can't prove it either.

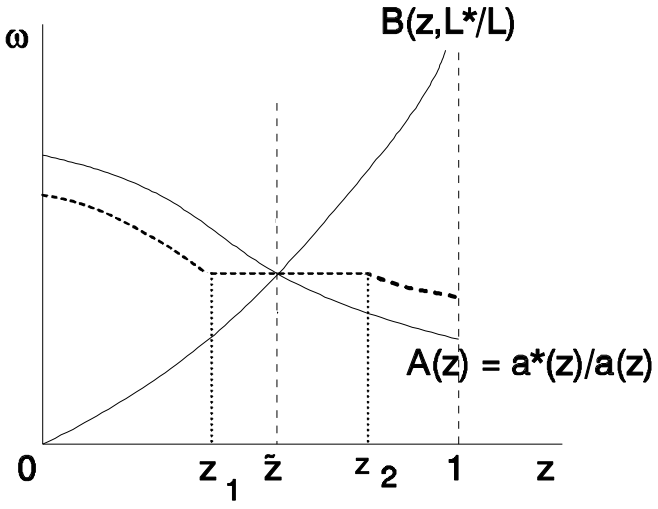


d) [Also optional, for extra credit only.] What would you expect to happen if both countries were to unlearn by doing, that is, $\lambda = \lambda^* < 0$?

Ans: If both countries were to

unlearn by doing, then it would seem that the two ends of the *A* function would shift in directions opposite to what they did in part (b), as shown here.

However, as soon as even a tiny bit of the left portion fell below a tiny bit of the right portion, production would shift to the other country, where productivity has not yet fallen. If time were discrete, this suggests that we might observe production oscillating between one country and the other, each producing just long enough to "unlearn" its productivity advantage. But in continuous time (as the time derivatives in the problem imply), and even as a possible equilibrium in discrete time, I think, I would expect to find both countries producing an increasing range of goods, for which their productivity would fall at the same rate. That is, starting from the initial equilibrium in which only good $\tilde{z}$ could be produced in both countries (with its production in each indeterminate), as the unlearning begins we will find productivities declining only to the point that the other country can compete, and from then on both will produce and unlearn together. After some time elapses, the picture would look like that on the right:



Both countries would produce all goods in $[z_1, z_2]$, and their outputs in that range would be indeterminate. Over time that range would expand, eventually encompassing the whole unit interval.

One thing that I don't know about this, however, is whether the relative wage would stay at its initial level, as I've drawn it, or might instead rise or fall over time. This matters, of course, only for how the two groups of workers do relative to each other. In real terms both are losing.

2. (20 points) Consider the two-sector, two-factor, two-country, Heckscher-Ohlin Model, with the two countries having identical technologies and the home country initially exporting the capital-intensive good. There is now an improvement in technology in both countries, making it possible to produce 25% more of the labor-intensive good with any given combination of factor inputs. Determine the effects of this change on i) quantities of production and trade, ii) prices of goods, and iii) real returns to factors in both countries.

Assume the following:

- The countries start and remain in free trade.
- Preferences are identical and Cobb-Douglas. (This will make the problem easier. If it doesn't, then you are not getting it right.)

Consider each of the initial patterns of specialization listed below, examining in each case the possibilities that these patterns of specialization will and will not change due to this change in technology. Hint: Look first at the world production possibility frontier and the equilibria in the world market.

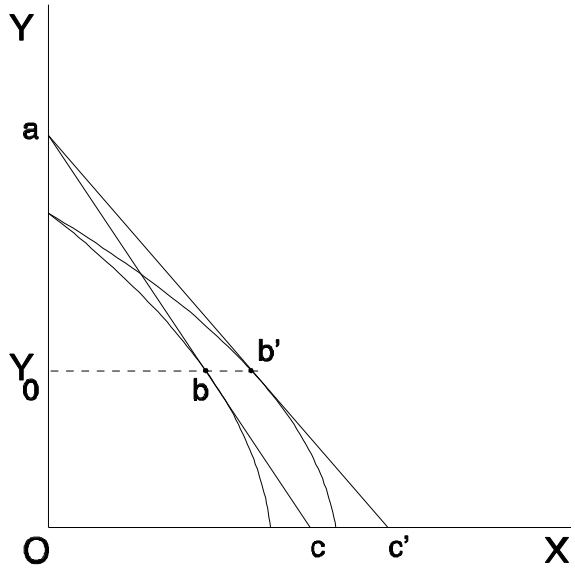
a) Both countries initially produce both goods.

b) The home country initially produces both goods but the foreign country initially specializes in only one.

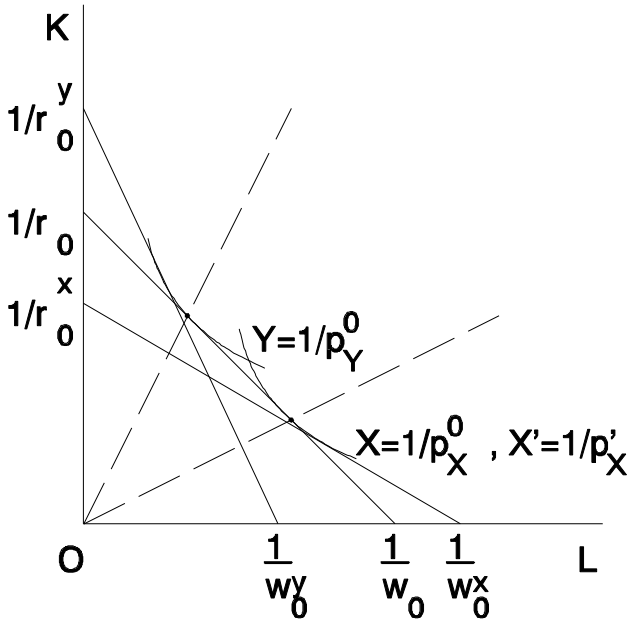
c) The home country initially specializes in one good, but the foreign country initially produces both.

d) Both countries initially specialize.

Ans: The change in technology increases the output of the labor-intensive good -- call it X -- by 25% for any combination of inputs. This therefore can be thought of as just a renumbering of the X-isoquants. If an Edgeworth production box is used to derive the PPF, this means that the efficiency locus will be unchanged, and that each point on it will now map to the same output of Y and to 25% more of X on the new PPF. That is, the PPF of any country is expanded by 25% in the X direction. The world PPF is just the envelope one gets by sliding one country's PPF around the other, and it therefore also expands by 25% in the X direction, as shown.



Suppose that the world equilibrium was initially at point b, with relative price of X given by the slope of the price line, abc. If we consider the same 25% expansion of this price line in the X direction, it will have the same Y-intercept at point a and it will be tangent to the new PPF at the same level of Y, Y_0, at a 25% higher level of X. Noting that the fraction of expenditure on good X is Y_0a/Oa in both cases, it follows that with Cobb-Douglas preferences (for which expenditure shares are constant) point b' must be the new world equilibrium. That is, recalling how the PPF is derived from the underlying isoquants, the new world equilibrium has both countries allocating the same factors to each industry at they did before, producing, consuming, and trading the same quantities of Y as before but quantities of X that are 25% higher. Thus, while we are told to consider all possible patterns of specialization in the initial equilibrium and the possibility that these patterns might change in the new equilibrium, it turns out that these statements about quantities fully describe what happens regardless of specialization, and it is not possible that patterns of specialization will in fact change.



As for prices, it is clear from the PPF diagram above that the price of X falls by just enough to keep the value of world output of X equal to what it was before, even though the quantity has risen by 25%. This means that in the Lerner-Pearce diagram for determining factor prices, the unit value isoquant of X is unchanged if we take good Y as numeraire. Therefore, regardless of patterns of specialization, (producing both goods with factor prices w_0, r_0 in the figure, producing only X with factor prices w_0^x, r_0^x, or producing only Y with factor prices w_0^y, r_0^y) factor prices will remain fixed in units of Y. Since the price of X falls, this means that all factor prices rise in real terms, in both countries, and regardless of the initial pattern of specialization.

3. (10 points) In the Heckscher-Ohlin Model with arbitrary numbers of goods and factors, suppose that prices change by various amounts, $\hat{p}_i$, ranging from a low of ρ_{min} to a high of ρ_{max}, with $\rho_{min} < \rho_{max}$. Assuming that all factors are employed in positive quantities in all industries, does there necessarily exist a factor whose real return rises unambiguously due to this collection of price changes? Does there necessarily exist a factor whose real return falls unambiguously?

a: Yes, to both, if one assumes that all price changes are marginal and that all industries are producing both before and after the price changes (which may be implied in the problem, but is unclear).

Using the Jones notation, we know that for all i

$$\hat{p}_i = \sum_j \theta_{ji}\hat{w}_j$$

regardless of the numbers of goods i and factors j. Let i_{max} be the good for which $\hat{p}_{i_{max}}=\rho_{max}$. Then

$$\hat{p}_{i_{max}} = \sum_j \theta_{ji_{max}}\hat{w}_j = \rho_{max}$$

implies

$$\exists j_F \ni \hat{w}_{j_F} \geq \rho_{max} > \rho_{min} \qquad (1)$$

Let i_{min} be the good for which $\hat{p}_{i_{min}}=\rho_{min}$. Then

$$\hat{p}_{i_{min}} = \sum_j \theta_{ji_{min}}\hat{w}_j = \rho_{min}.$$

Together with (1) this implies

$$\exists j_E \ni \hat{w}_{j_E} < \rho_{min}$$

Finally, $\hat{w}_{j_E}<\rho_{min}<\rho_{max}$ together with (1) implies $\hat{w}_{j_F}>\rho_{max}$. Thus the price of factor j_F rises relative to all goods and the price of factor j_E falls relative to all goods.

4. (15 points) Explain the differences between Trefler (1993) "International Factor Price Differences: Leontief Was Right," *JPE,* and Trefler (1995) "The Case of the Missing Trade and Other Mysteries," *AER*. Also discuss whether it would have made sense to publish Trefler (1993) if Trefler (1995) had been published first.

Ans: Trefler (1993) (T93)examines the factor price equalization (FPE) and Heckscher-Ohlin-Vanek (HOV) theorems, noting that both perform poorly empirically, and he also asks whether an assumption of factor-augmenting technology differences can reconcile both with reality. To do this he calculates a set of such differences that would make the HOV theorem hold exactly and then notes that the factor prices that would be implied by these differences correspond surprisingly closely to observed factor prices, at least for labor. This is not a formal test of any theory, largely because he allows such a large number of technology differences (one per factor and country) that HOV can be made to hold exactly. However, he shows theoretically that for completely arbitrary data this calculation need not have yielded economically meaningful results -- that is, the calculated technology multipliers, π, could have been negative -- yet in his results all are positive. Secondly, there is no reason in general for these calculated multipliers to correspond to observed relative factor prices, yet in fact they correspond remarkably well. He therefore takes these results as evidence in favor of such technology differences in the real world.

Trefler (1995) (T95) does not address FPE at all, but rather tries to do a more formal test of HOV against alternatives suggested by the technology differences of T93 and also by an alternative to the identical preferences assumption of HOV that preferences display instead a bias toward home produced goods. First, however, he motivates these alternatives very compellingly by displaying the empirical departures from HOV in two suggestive ways. The first shows that there is far less trade among countries than HOV would predict. The second shows that factor endowment data show poor countries to be abundantly endowed with all or most factors, while rich countries are abundantly endowed with hardly any. The first empirical observation motivates his alternative hypothesis of home bias in consumption, while the second motivates (as does T93) various alternatives involving technological differences. He then does a series of formal hypothesis tests, and these do, not surprisingly at this point, reject HOV in favor of these alternatives and combinations of them.

Differences between the two papers are therefore as follows:

1) T93 deals with both HOV and FPE, T95 only with HOV.

2) T93 has no formal hypothesis test, while T95 does.

3) T93 in a sense makes its point by one compelling manipulation and display of data, while T95 motivates both its formal analysis and its conclusions by two quite different displays of the data.

4) Finally, there is one more technical difference that should be mentioned: T93 assumes such a general form for technological differences that they permit modified HOV to hold exactly. It is therefore such a general specification that it could not be rejected by the data. T95, therefore, uses two more restricted specifications of technology differences, one that requires them to be (Hicks-) neutral and another that permits non-neutral differences between groups of rich and poor countries.

As for the importance of T93 after T95 is known, it is undeniable that T93 loses some of its punch after reading T95. Once formal tests of HOV and alternatives have been performed in T95, the purely suggestive evidence in T93 is not so surprising. However, by dealing with FPE and showing that a variant of one of these alternatives does well in explaining factor price differences as well as trade, T93 does make an independent and important contribution.

5. (20 points) Suppose that we are concerned about the low wage being paid to unskilled labor, especially in comparison to the wage going to skilled labor. Use the two-factor, two-good Heckscher-Ohlin Model of a small open economy, with the two factors being skilled and unskilled labor, to evaluate ***and compare*** the policies listed below in terms of

i) their ability to raise the unskilled wage to some given real level,

ii) the effect of doing so on the skilled wage, and

iii) the welfare cost to the economy as a whole of doing so.

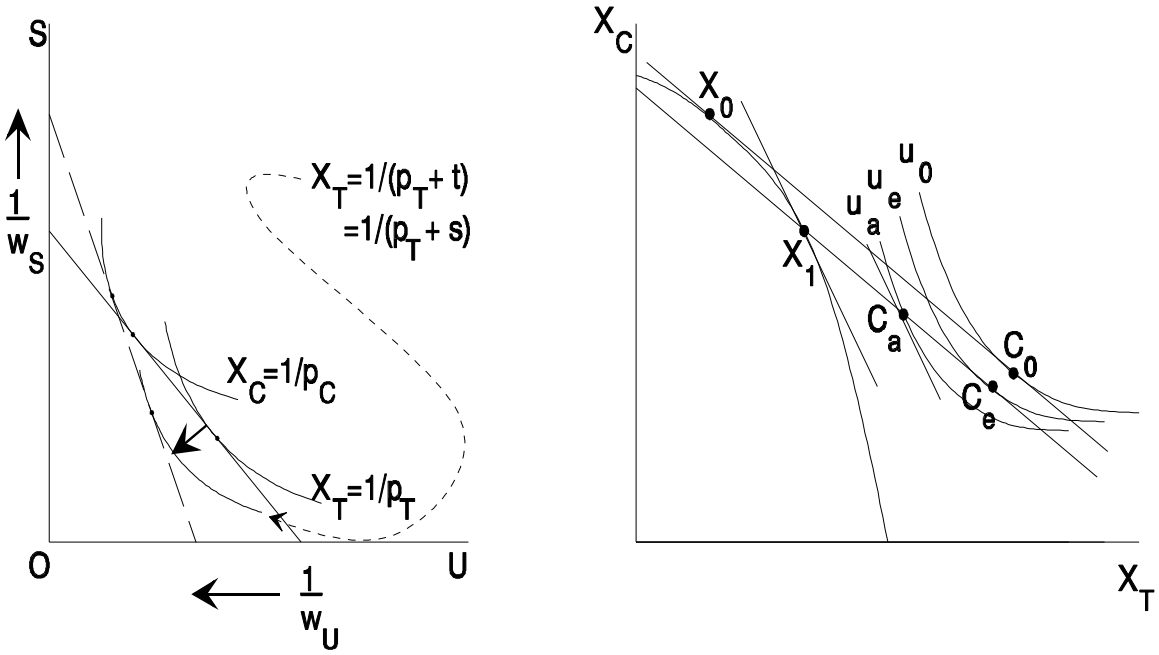
Assume that the country in question is abundantly endowed with skilled labor, compared to the rest of the world. Also, assume for concreteness that the two goods are unskilled-labor-intensive textiles and skilled-labor-intensive computers.

a) A tariff on imports of textiles

Ans: The effects of a tariff on imports of textiles are shown in the two diagrams on the next page. Because the country is small, world prices, p_C and p_T, are given and determine the initial unit value isoquants in the Lerner-Pearce diagram, as well as the production and consumption points in the PPF diagram. A tariff, t, raises the domestic price of textiles to p_T+t, shifts the unit value isoquant for T inwards, and raises the unskilled wage and lowers the skilled wage as in the Stolper-Samuelson Theorem. In the PPF diagram, the production point moves to X_1, with more output of T, and consumption moves to C_a, which is on the world price line where an indifference curve is tangent to a parallel domestic price line. Country welfare therefore falls to u_a.

Thus a tariff *can* raise the unskilled wage, while at the same time lowering the skilled wage and lowering the welfare of the economy as a whole. I will

examine the other suggested policies in terms of their abilities to accomplish the same, or a better, result.



b) A tariff on imports of computers

Ans: Computers are not imported, so a tariff on imports of them will have no effect at all. (A tax on exports of computers, on the other hand, would be completely equivalent to the tariff on imports of textiles.)

c) A tax or subsidy on consumption of textiles

Ans: A tax or subsidy on consumption of textiles has no effect on factor prices, and therefore cannot achieve the desired result. If such a policy were used, it would lower welfare by moving consumption along the world price line, but there is no point in using it. (A subsidy could, it would seem, lower prices to consumers and thus raise both real wages. However, unless the subsidy is financed by systematically overtaxing the owners of one factor, the welfare of both must fall along with that of the country.)

d) A tax or subsidy on consumption of computers

Ans: Same as (c).

e) A tax or subsidy on production of textiles

Ans: A subsidy on textile production will have the same effect on producers' prices as a tariff (a tax would have the opposite effect, and therefore would move factor prices the wrong way). Thus the same Lerner-Pearce diagram can show its effects. In the PPF diagram, however, the textile subsidy of s changes producers' prices but leaves consumers facing world prices. Thus consumption moves only to C_e and welfare to u_e, which is higher than u_a.

Therefore a production subsidy can achieve the same increase in the unskilled wage at lower welfare cost.

[There are a couple of complications that this simple answer ignores. First, a subsidy *s* equal to the tariff, *t*, will actually leave unskilled workers somewhat better off, since with a subsidy they will face world prices but with a tariff they will face the higher domestic price of T caused by the tariff. Therefore a somewhat smaller subsidy than *t* could achieve the same change in the unskilled real wage. Second, the subsidy must be financed somehow, just as the tariff revenue must be spent. Our usual assumption of lump-sum taxes and subsidies to handle financing does not settle the issue here, because we need to know how skilled and unskilled workers share the burdens and benefits of these redistributions. Where possible, a neutral assumption would be to always combine taxes and subsidies that have the same price-distorting effects, adjusting their levels so that the revenues from one finance the other. This is not possible with a tariff, since there is no corresponding subsidy on another behavior that will have the same effect. However the subsidy on production of T here could actually be a combined subsidy on T and tax on C, adding to *t* (or somewhat less, as noted), that breaks even.]

f) A tax or subsidy on production of computers

Ans: A tax on production of computers will have the same effects as a subsidy to production of textiles. [And can be combined with a subsidy to eliminate revenue consequences, as just noted.]

g) A tax or subsidy on employment of unskilled labor

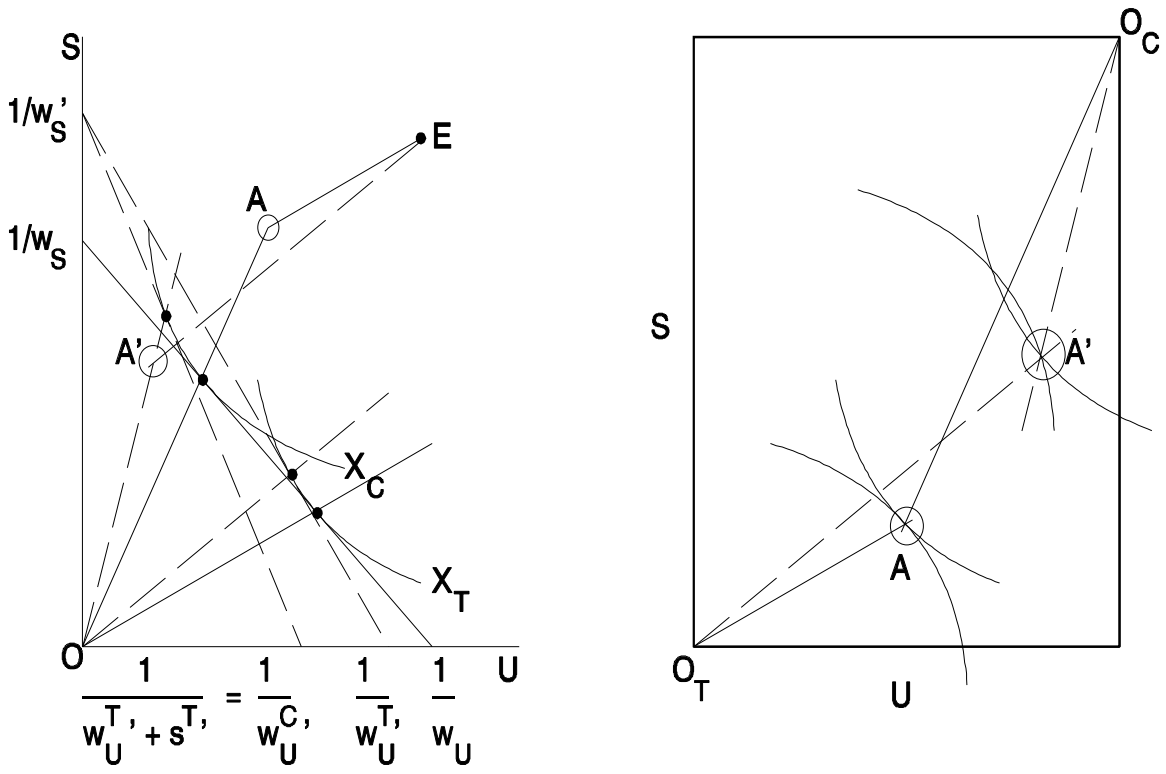
h) A tax or subsidy on employment of skilled labor

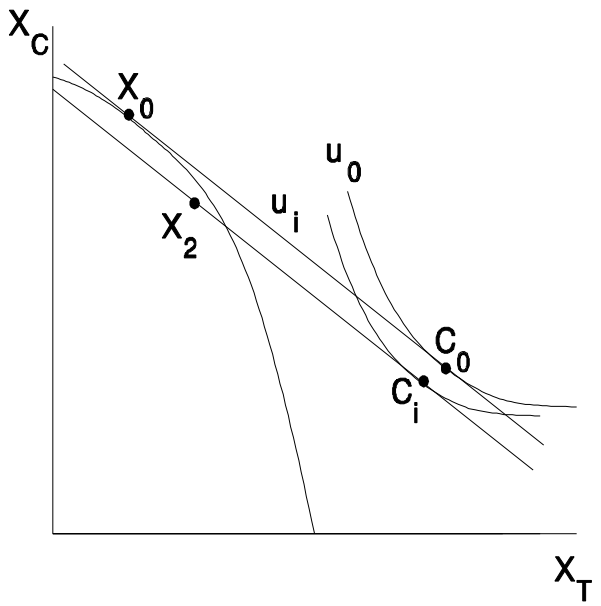
Ans: Taxes and subsidies on factors regardless of industry leave both industries paying the same price for each, so that the equilibrium allocations are all the same as with free trade, with the factors bearing the entire incidence of the taxes/subsidies. This would be true even in a closed economy since the factors are completely inelastically supplied, but it is true in this non-specialized small open economy even if factor supplies can vary, since the prices producers pay for factors are determined by FPE. Thus a tax or subsidy on S has no effect on the wages paid by producers, which remain at their free trade levels, w_U and w_S. Of course the wages received by the workers do change, by the amounts of the taxes/subsidies. Therefore, a subsidy to unskilled workers will raise the after-subsidy wage of U without causing any distortions, leaving the economy at welfare level u0. This is the best policy yet for achieving the desired result. [Again, in order to avoid budget problems, this policy could consist of a subsidy to U combined with a tax on S.]

i) [Optional, for extra credit only.] How would a tax or subsidy on employment of unskilled labor in the textile industry only work here?

Ans: A tax or subsidy on unskilled labor in the textile industry only will cause the two industries to face different factor prices, moving them away from the efficiency locus in the production Edgeworth Box and inside the PPF. This is illustrated in the figures below.

In the Lerner-Pearce diagram, the two industries now face different isocost lines, since the computer industry must pay a higher wage to U than the textile industry in order to equal the textile wage plus subsidy and attract





workers. Thus the two isocost lines have the same S-intercept (where they pay the same wage), but different U-intercepts, as shown. Otherwise the equilibrium requires the usual tangencies, and this raises the ratio of skilled to unskilled workers employed in both industries, but by different amounts. As a result (and for reasons similar to the workings of the Stolper-Samuelson Theorem), both factors are reallocated from computers into textiles, the allocations moving from the circled points *A* to *A'* in both the Lerner-Pearce diagram and the Edgeworth Box.

In output space, production moves toward more textiles, but it also moves inside the PPF. From there, with free trade, consumption occurs optimally on the world price line. This result is clearly worse than the optimal policy of subsidizing unskilled workers in both industries, since it achieves the lower welfare u_i. How it compares to a tariff and a production subsidy is hard to tell, however, since they involve different distortions, not just more or larger ones.

6. (10 points) Does the Stolper-Samuelson Theorem hold in the Helpman-Krugman modification of the two-country Heckscher-Ohlin Model in which one sector, X, is monopolistically competitive? You may assume, as Stolper and Samuelson did, that there are positive outputs in both industries, both before and after the price change. You may also assume, as Helpman and Krugman did, that preferences for the differentiated product are of the Spence-Dixit-Stiglitz type, that there are many producers with free entry, and that the production function for each firm producing X is $X = F(K_x,L_x) - F_0$ where F( ) is a conventional neoclassical production function with constant returns to scale, and F_0 is a fixed cost. [Suggestion: What you are looking for here are relationships between prices of goods and prices of factors that either do or do not have the properties needed to derive the Stolper-Samuelson Theorem.]

Ans: S-D-S preferences with many producers imply that the elasticity of demand faced by each firm is a constant, σ, which is also the elasticity of substitution among varieties. Facing a constant elasticity of demand, these producers will charge a constant markup over marginal cost:

$$p = \frac{\sigma}{\sigma - 1}\, c = \lambda c$$

Let $c(w,r)$ be the minimum unit cost function corresponding to the function $F(K_x,L_x)$, ($c(w,r) = \min \{wL + rK \mid F(K,L) \geq 1\}$). Then total cost for one firm, *i*, is

$$C(X_i) = F_0 c(w,r) + c(w,r) X_i$$

and free entry implies zero profit and therefore

$$0 = PX_i - C = \lambda c X_i - cF_0 - cX_i$$

or

$$(\lambda-1)cX_i = cF_0$$

and hence

$$X_i = \frac{F_0}{\lambda-1}$$

Thus industry output, X, is produced by many firms, each producing $X_i=F_0/(\lambda-1)$. The number of these firms can be found by comparing X and X_i as

$$n = \frac{X}{X_i} = \frac{(\lambda-1)}{F_0}X$$

Industry cost is therefore

$$C(X) = n\left(F_0 + \frac{F_0}{\lambda-1}\right)c(w,r)$$

$$= \frac{\lambda-1}{F_0} X \frac{\lambda F_0}{\lambda-1} c(w,r)$$

$$= \lambda c(w,r) X$$

Thus the industry behaves just like a perfectly-competitive, constant-returns-to-scale industry with unit cost function $\lambda c(w,r)$. Any theorems that just depend on the properties of this function, including the Stolper-Samuelson Theorem, must hold here.