Introduction to External Increasing Returns

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Many of the possible implications of increasing returns to scale for international trade can be understood within a simple model in which one sector has external increasing returns to scale (EIRS). As introduced to the trade literature by Ethier (1982), this means that average cost in all firms of an industry declines with increasing industry output, but that individual firms take average cost as given and not influenced by their own output. In a two-sector general equilibrium model, with constant returns to scale in the other sector, this model can be quite tractable and still display many of the properties that we associate with increasing returns to scale, such as trade patterns based on differences in country size, multiple equilibria, and the possibilities of both losses from trade and of extra gains from trade due to scale.

Consider, then, a world in which countries use one factor, labor L, to produce two goods, food F and machines M. Food is produced with constant returns to scale, its output being just a constant multiple of the labor input: $F=\beta L_F$. Machines are produced with EIRS, so that productivity, $\alpha = \alpha(L_M)$, rises with output or, more conveniently, with the amount of labor input in the country. Thus $M = \alpha(L_M)L_M$, where $d\alpha/dL_M>0$. The production possibility frontier (PPF) for this economy is shown in Figure 1. The rest of the model is conventional, with all income being spent on the two goods by consumers, who have identical homothetic preferences.

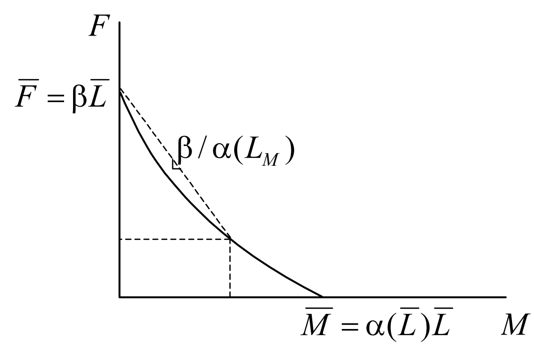

Figure 1

Autarky

Consider first the equilibrium in such an economy in autarky. Taking labor as numeraire (w=1), it will consist of an allocation of labor to the two sectors such that if both prices are equal to average costs – $p_F=1/\beta$ and $p_M=1/\alpha(L_M)$ – consumers will demand the quantities of the goods produced. A convenient way to see that this is an equilibrium, however, is first to imagine an arbitrary production point on the PPF from Figure 1, such as Q_1 in Figure 2, that is *not* an equilibrium. If production took place at this point, then in order for this combination

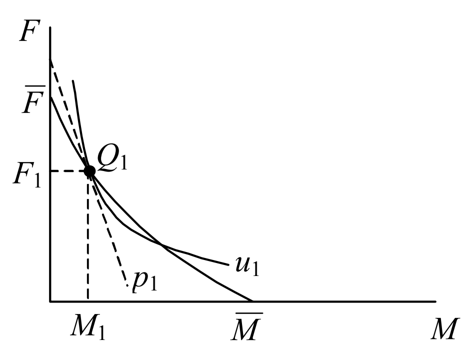

Figure 2

of the two goods to be demanded, the relative price of M would have to be p_1, the slope of the line tangent to the indifference curve through Q_1. But at that price, producers of M are getting a higher price than the average cost of production, which was shown in Figure 1 to be the slope of the line from the production point to the $\overline{F}$ intercept of the PPF. Thus, as long as the market-clearing price line for this output combination, which I'll call the demand-price line, intersects the F axis above $\overline{F}$, producers of M are making a profit and will want to expand. Conversely, if the demand-price line intersects below $\overline{F}$, they are making a loss and will contract. Autarky equilibrium therefore requires that the tangent to the indifference curve hit $\overline{F}$ exactly, as in Figure 3.

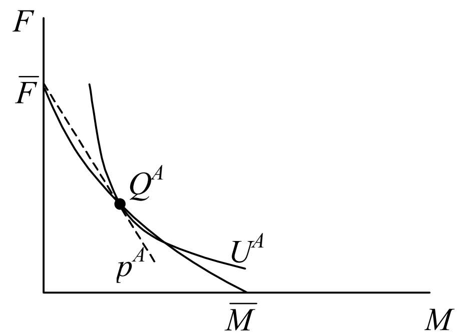

Figure 3

Another way to depict this equilibrium that will be more useful later is in terms of supply and demand. With increasing returns, we cannot depict supply in the usual way – the amount that will be supplied for any price – since that depends, because of the externality, on output itself. However we can depict average cost as a function of output, and that will turn out to serve our purpose nicely, as we will see. In figure 4 we combine this with an apparently conventional downward sloping demand curve, representing the demand price discussed above.

That is, now taking good F as numeraire, and measuring the country's output of good M along the horizontal axis, curve AC shows the average cost of M. This declines as M rises, due to the increasing returns. Curve D is the demand price shown in Figure 2. That is, for any output of M, such as M_1, and the output F_1 corresponding to it along the PPF, the height of D is the relative price of M at which the ratio F_1/M_1 would be demanded, and thus the price at which the market would clear if M_1 were produced.

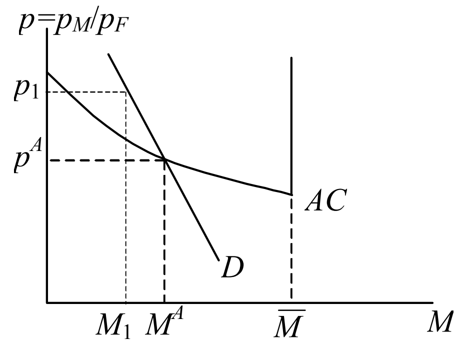

Figure 4

Autarky equilibrium is at the intersection of these curves, at machine output M^A and relative price of machines p^A. This could in principle occur more than once if the demand curve is flat enough and the AC curve steep enough, although I will assume that this does not happen. The dynamics of adjustment to this market equilibrium are as follows. Suppose output is initially, say, M_1, less than M^A. Then the price that clears the market for that output will be p_1, from the demand curve, which is higher than average cost, and machine producers will make a profit. Over time they will therefore expand output moving toward the equilibrium. If output starts above M^A the opposite will occur.

Now consider two such economies, Home and Foreign (with the common convention that Foreign is marked with an asterisk, *), and assume that Home has a larger labor force than Foreign, $\overline{L} > \overline{L}^*$. Assuming that they share the same technologies, their AC curves will be the same, except that the Foreign one will stop short of the end of the Home one, due to its smaller labor force.

However, their demand curves will be different, in spite of their identical preferences. Each output of M will correspond to a smaller output of F in Foreign than in Home, because Foreign will have less labor left over to employ in the F industry. With therefore a larger relative output of M to sell to consumers, Foreign's relative price of M must be lower, for any given output, M. Thus the D^* curve is below (and thus to the left of) D, as shown.

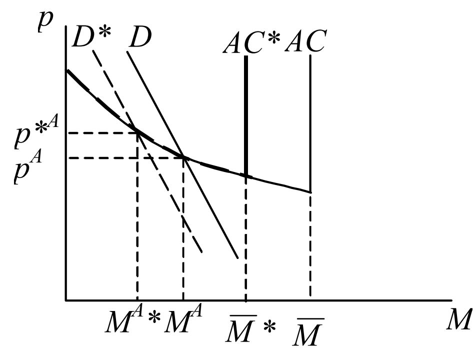

Figure 5

This gives us Figure 5, where Foreign produces less M in autarky than Home, at a higher relative price and average cost. Thus we get one not very surprising result: with (external) increasing returns to scale in one industry and not the other, the larger country has a lower autarky price of the EIRS good.

Free Trade

Now what happens if these two countries open to free trade? Assuming that outputs do not initially change, the world price will settle somewhere between the two autarky prices, since the world ratio of M to F output will be between the autarky ratios. But with a world price above p^A and below p^{*A}, Home producers of M make a profit while Foreign producers make a loss. Therefore the machine industry expands at home and contracts abroad. Where this process will end we cannot tell from Figure 5, but this points us in the direction of the large country, Home, exporting the EIRS good once we get to a free trade equilibrium.

To find the free trade equilibrium we now construct a world supply-and-demand diagram analogous to Figures 4 and 5. This starts with an average cost curve that records what the average cost of machines will be for the marginal producer as we increase world output from zero to the maximum possible. That, however, depends on who is doing the producing. I will assume, based on the conclusion just reached that the Home country is likely to get to the market first, that only Home produces M unless world output exceeds the maximum, $\overline{M}$, that Home is able to produce. Thus the marginal machine producer is in Home for M up to $\overline{M}$, and in Foreign beyond that. Thus the World AC curve in Figure 6 consists of the Home AC curve followed by the Foreign one, AC^*.

The demand curve for the world market, D^W, is defined as it was for the countries in autarky. It is not simply the horizontal sum of D and D^*, but it is still well defined and downward sloping: as world output of M increases world output of F declines, and their ratio requires a decreasing relative price. Its position depends on preferences for M relative to F, as always, but here these preferences matter a great deal for the kind of equilibrium that will arise. For the particular preferences underlying the demand

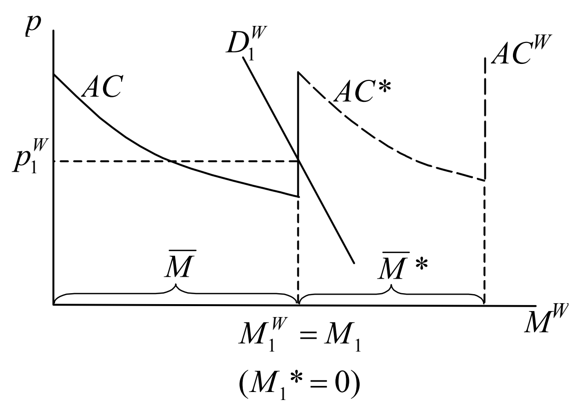

Figure 6

curve D_1^W in Figure 6, this equilibrium has both countries completely specialized, with Home producing only *M*, and with a world price, p_1^W, that is above the average cost of producing *M* in Home while below the average cost of producing the first unit of good *M* in Foreign.

This is only one of several possibilities. Two others are shown in Figure 7, for demand curves D_2^W and D_3^W. If demand for *M* is somewhat low, then Home produces both goods, while Foreign produces only *F*. If demand for *M* is rather high, then Foreign produces both goods while Home produces only *M*. In this respect, the model is very like the Ricardian model with constant costs for both goods.

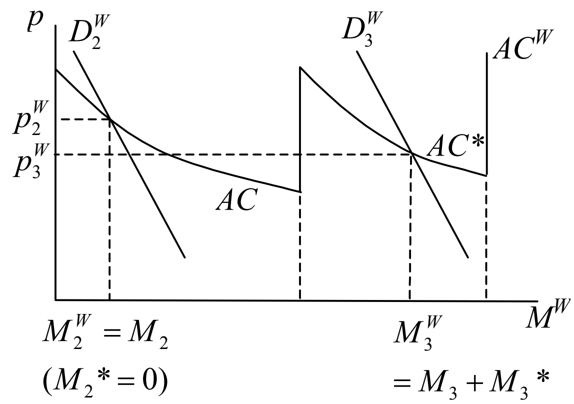

Figure 7

However, the behavior of prices is rather different. As in the Ricardian Model, when both countries specialize completely, then the prices depend entirely on demand and on the maximum outputs that the countries are able to produce with their labor forces, price does not depend at all on costs. But if either country produces both goods, then price depends on that country's average cost. And this average costs depends in turn on how much that country produces.

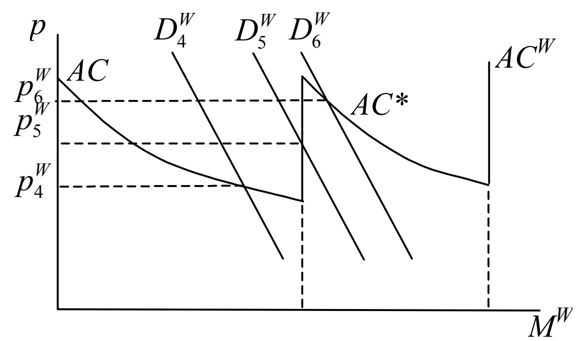

Figure 8

In the case shown in Figure 7, the equilibrium with higher demand for *M* has a lower price for it, just because it happens to be large enough to take greater advantage of scale economies abroad than was possible at Home in the equilibrium for D_2^W. This possibility of greater demand causing a lower price is not unusual, however, in the presence of scale economies, and it would happen also if either of these equilibria were perturbed slightly by shifting the demand curves a bit to the right.

What is therefore perhaps more surprising is that shifts of preferences toward the EIRS good can cause its relative price to rise, in spite of the scale economies, as it does in Figure 8 moving left to right among demand curves D_4^W, D_5^W, and D_6^W. Here the shift in demand raises price because it exhausts the Home country's ability to produce the good.

Alternate Equilibria

These equilibria simply *assumed* that the larger Home country would be the first to produce the EIRS good. This assumption was motivated by the good's lower autarky price in Figure 5, but the dynamics of getting from autarky to free trade is not intrinsic to the model, and in any case could be altered by policy interventions along the way. An equally valid equilibrium would have Foreign producing *M* first, and the prices in that equilibrium would sometimes be different from those above. For example, if we interchange *AC* and *AC** from Figure 6, we get Figure 9. Here, comparing to Figure 6 for demand curve D_1^W, the equilibrium price $p_1^{W}{}'$ is higher, and the pattern of specialization is different. A similar interchange in Figure 7 leaves prices unchanged, but reverses the roles of the countries.

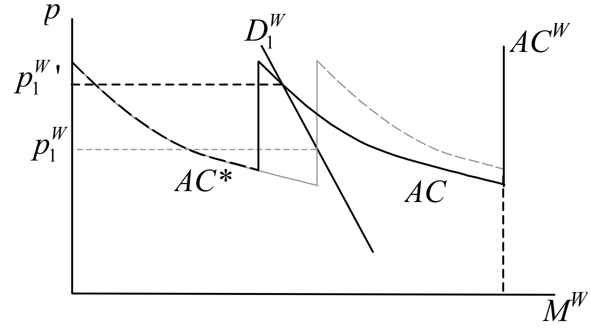

Figure 9

Thus even with demand sufficiently inelastic that it crosses a given AC^W curve only once, as assumed here, there still are in fact multiple equilibria. And the differences between them matter.

To see this, consider the simplest case, ignored until now, in which the two countries are identical in size. Here reversing the order will not matter at all for world equilibrium prices, but it will reverse who produces *M* and who produces *F*. Suppose that demand is such that we fall just short of having a second country produce *M*, as on the left in Figure 10 below. Then the price of *M* is almost as high as it can be, and the trading equilibrium in *M,F* space appears as on the right below. That is, while the countries share the same PPF and face the same world price, one specializes in *F* and consumes at C_1, while the other specializes in *M* and consumes at C_2. Not only is the latter better off than the former, the former is worse off than in autarky. (The autarky equilibrium is not shown in Figure 10, but with homothetic preferences it must lie on the PPF below the ray through C_1, and thus on a higher indifference curve.)

Thus not only can a country lose from trade when there are EIRS, the determination of which country gains more, and which loses if that in fact happens, depends on which country happens to specialize in the EIRS good. With both countries identical as in Figure 10, both have a strong incentive to try to influence the move to the free-trade equilibrium by using policy, perhaps subsidizing production or export of *M* during the transition. The outcome cannot be known, but the potential for conflict is obvious. This is only one special case, but the message is valid more generally.

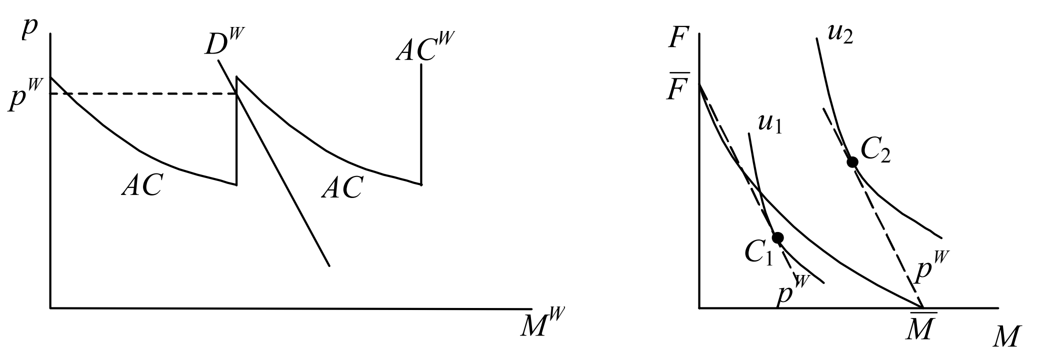

Figure 10

References

Ethier, Wilfred J. 1982 "Decreasing Costs in International Trade and Frank Graham's Argument for Protection," *Econometrica* 50, September, pp. 1243-68.