A Constitutive Relation for the Viscous Flow of an Oriented Fiber Assembly

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ABSTRACT: A constitutive relation for an equivalent, homogeneous fluid is developed for the anisotropic viscous flow of an oriented assembly of discontinuous fibers suspended in a viscous fluid. The anisotropic viscous compliance matrix can be expressed in terms of three constants by assuming the equivalent fluid to be incompressible and the microstructure to have axial symmetry (transversely isotropic). By means of a micromechanics analysis, the three terms of the constitutive relation are expressed in terms of the viscosity of the matrix fluid, the fiber aspect ratio, and the fiber volume fraction. A comparison of the viscosity terms reveals that the elongational viscosity in the fiber direction varies as the square of the fiber aspect ratio and a complex function of the fiber volume fraction. Further, the ratio of the axial elongational viscosity to the transverse elongational viscosity and both axial and transverse shear viscosities was shown to be 10^4–10^6 for fiber aspect ratio of 10^2–10^3, except at extreme values of the fiber volume fraction.

INTRODUCTION

GROWTH IN THE use of composite materials depends not only on their performance relative to conventional materials, but also on the capacity for economically producing components in the required shapes. Manufacture of one of the two major groups of advanced fiber composites, the thermosetting-polymer composites, has traditionally involved manual lay-up of prepregs and autoclave cure. The other major group of advanced fiber composites consists of the thermoplastic matrix materials. Unlike their thermosetting counterparts, the thermoplastic composites may be readily thermoformed under pressure at elevated temperature. These composites also possess the advantage that the same material may be reformed and/or reconsolidated several times if required.

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Several different methods can be employed to accomplish the forming of complex geometries required. One technique that has attracted recent attention is that of diaphragm forming [1]. With this technique, the composite preform sheet is formed onto a tool surface and consolidated in a process chamber involving a sequence of applied pressure and elevated temperature. Diaphragm forming of continuous fiber composites has proven successful with several geometries, and research has been conducted to determine optimum processing parameters [2]. However, the inextensibility of continuous fiber composites in the fiber direction may limit formability. In complex geometries, the continuous fiber sheets may display wrinkling, bridging, and distortion. Flow in these materials systems has been discussed by Cogswell et al. [3–5] and Mallon et al. [6–8]. In order to enhance formability several alternative material forms have been developed. The material form examined in the present paper consists of collimated, discontinuous fibers suspended in a thermoplastic matrix to provide a sheet material with extensibility in the fiber direction [9]. While the primary mechanism for forming of a continuous fiber material is through the inplane shearing mode, the extensional deformation mode for the discontinuous fiber system allows the material to be clamped along its edges during forming. This is desirable for two reasons: first, the discontinuous composite lends itself more easily to manufacture by equipment and methods originally designed for metal sheet forming; and second, because the discontinuous material is clamped and membrane tension is present throughout forming, there should be less tendency for wrinkling and distortion in complex shapes.

It should be emphasized that the material systems described herein contain long fibers. Typical fiber length can be 50–100 mm: this is 2–3 orders of magnitude greater than the fiber lengths typical for thermoplastic injection molding. Even by the standards of the textile industry these would be regarded as *long-staple* fibers. Since fiber diameter is typically 10^{-2} mm, the fiber aspect ratio for this material is of the order of 10^3. During forming of the sheet material the deformation occurs primarily as a result of viscous flow of the molten thermoplastic matrix and rigid body displacement of the fibers. Prediction of the deformation and its relationship to the forces developed requires a constitutive relation which models the anisotropic and viscous behavior of the material.

Although there have been many studies of the flow of dilute suspensions of fibers and particles in liquids, for example by Metzner [10] and recently Rogers [11], the authors are not aware of a micromechanical analysis which treats a collimated, discontinuous fiber assembly in a viscous matrix fluid of high fiber concentration and subjected to relatively small total strains. The present paper analyzes the initial flow of such a material wherein the fiber assembly is idealized by a regular packing geometry as well as several other simplifying assumptions presented later.

Although the assumptions deliberately introduced into the present treatment will cause some departure from reality, the analysis brings out the essential form of the constitutive relation in a simple and clear way and shows the dependence of the anisotropic viscosity constants on system parameters, as well as giving reasonable indication of the order of magnitude for each term. More exact predictions await more detailed theoretical analysis and experimental verification. The next section of this paper is applicable to any transversely isotropic, incompressible viscous system. It follows the well-known formulation of anisotropic elasticity and the pioneering work of Gibson [12].

ANISOTROPIC VISCOSITY

Surprisingly, prior to the paper by Gibson [12], there appears to have been no publication of the relations applicable to the anisotropic viscosity of oriented liquid systems. In addition to the analogy with anisotropic elasticity and the simplifications from symmetry, the other important factor is the assumption of incompressibility, which will almost always be a valid approximation for a fluid system. We propose to publish a fuller account of the subject elsewhere.

For the present purpose, the special case of transverse isotropy, already presented by Gibson, is all that is needed. The formulation, which we developed for application to the present problem is identical with Gibson's, but he used 3 for the fiber orientation direction whereas we used 1, and in the viscosity expressions given below, Gibson's $\lambda = \eta_{11}$, $\lambda_{\perp} = \eta_{22}$, $\eta = \eta_{12}$.

For the transversely isotropic, incompressible fluid, the viscous compliance matrix can be expressed in terms of three constants, β_{11}, β_{12}, and β_{66}:

$$\begin{bmatrix} \dot{\epsilon}_1 \\ \dot{\epsilon}_2 \\ \dot{\epsilon}_3 \\ \dot{\epsilon}_4 \\ \dot{\epsilon}_5 \\ \dot{\epsilon}_6 \end{bmatrix} = \begin{bmatrix} \beta_{11} & -\beta_{11}/2 & -\beta_{11}/2 & 0 & 0 & 0 \\ & \beta_{22} & (\beta_{11} - 2\beta_{22})/2 & 0 & 0 & 0 \\ & & \beta_{22} & 0 & 0 & 0 \\ & & & 4\beta_{22} - \beta_{11} & 0 & 0 \\ & & & & \beta_{66} & 0 \\ & & & & & \beta_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} \quad (1)$$

Unless the hydrostatic pressure is subtracted from each normal stress term, the viscous compliance matrix cannot be inverted. This is due to the assumption of incompressibility of the medium wherein hydrostatic pressure results in no deformation.

The anisotropic viscosities, η_{ij} may be expressed in terms of β_{ij} and correspond to the forms given by Gibson [12]:

$$\begin{aligned} \text{axial elongational viscosity } \eta_{11} &= \beta_{11}^{-1} \\ \text{axial shear viscosity } \eta_{12} &= \beta_{66}^{-1} \\ \text{transverse elongational viscosity } \eta_{22} &= \beta_{22}^{-1} \\ \text{transverse shear viscosity } \eta_{23} &= (4\beta_{22} - \beta_{11})^{-1} = (4\eta_{22}^{-1} - \eta_{11}^{-1})^{-1} \end{aligned} \quad (2a)$$

Note that the expression for η_{23} in terms of η_{11} and η_{22} means that any two of η_{11}, η_{22}, η_{23}, together with η_{12}, can be selected as the three independent parameters.

It is also possible to derive from Equation (1) three strain rate ratios, which are the analogues of Poisson's ratios. These are the ratios of appropriate terms in the matrix:

$$\begin{aligned} \lambda_{12} &= -(-\beta_{11}/2)/\beta_{11} = 0.5 \\ \lambda_{21} &= -(-\beta_{11}/2)/\beta_{22} = \beta_{11}/2\beta_{22} = \eta_{22}/2\eta_{11} \\ \lambda_{23} &= -(\beta_{11} - 2\beta_{22})/2\beta_{22} = 1 - \beta_{11}/2\beta_{22} = 1 - \eta_{22}/2\eta_{11} \end{aligned} \quad (2b)$$

MICROMECHANICS ANALYSIS

The primary objective of the micromechanics analysis is to develop relationships between the primary anisotropic viscosities, η_{ij} and the properties of the oriented fiber assembly and matrix fluid. Consider an aligned fiber assembly wherein long discontinuous and rigid fibers are arranged in a regular cross-sectional geometry (hexagonal or square array) and suspended in a Newtonian fluid. At the interface between the fibers and matrix fluid, a no slip condition is assumed. The geometry of the fiber assembly is shown in Figure 1. In addition, there are several primary assumptions in the proposed model which may be described as follows:

- The discontinuous fibers are straight and collimated with ends touching so that their direction coincides with the "1" direction as shown in Figure 1.
- In the transverse plane (2-3) the fibers are arranged in a hexagonal or square array consistent with a given fiber volume fraction, f.
- It is assumed that neighboring fibers can be treated as if they are arranged, so that fiber ends in one row are next to fiber centers in adjacent rows. This ge-

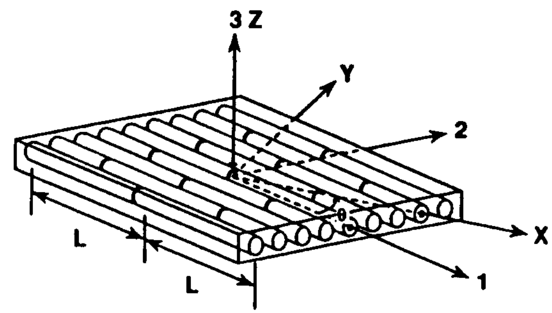

Figure 1. Oriented fiber assembly.

ometry is possible for a square array, but not for a hexagonal array, where the assumption must be regarded as a simplifying approximation.

- The kinematic assumption in the model specifies that a velocity field of linear variation be imposed upon the fiber assembly.

FIBER ARRAY GEOMETRIC RELATIONS

Consider the geometric arrangements of fibers shown in Figure 2. For fibers of diameter, D and arranged in a fixed pattern where the spacing between fibers, S, then the fiber volume fraction, f is given as:

$$f = \frac{1}{B^2}\left(\frac{D}{S}\right)^2 \tag{3}$$

$$B^2 = \frac{2\sqrt{3}}{\pi} \quad \text{(hexagonal array)}$$

$$B^2 = \frac{4}{\pi} \quad \text{(square array)}$$

ELONGATIONAL VISCOSITY

If a linear variation in velocity in the direction of the fibers is imposed upon the oriented fiber assembly, the relative velocity of adjacent fibers may be determined by assuming that the fibers travel at the velocity of their centroids (Figure 3). Hence, the relative velocity of two adjacent fibers of length, L is given as:

$$\Delta \dot{u} = \dot{\epsilon}_1 L/2 \tag{4}$$

where $\dot{\epsilon}_1$ is the extensional strain rate of the fiber assemblage. Therefore the apparent shear strain rate in the fluid contained between the nearest points of two adjacent fibers is given as

$$\dot{\gamma} = \dot{\epsilon}_1 L/[2(S - D)] \tag{5}$$

The induced shearing strain rate, $\dot{\gamma}$ generates a shearing stress, τ on the fiber surfaces equal to the product of the fluid shear viscosity, η and the strain rate, $\eta\dot{\gamma}$. At a cross section through the fiber midpoints, as shown in Figure 4, one-half the fibers will carry the total load, and so the fiber tensile stress at the midpoint will be $2\sigma/f$, where σ is the average stress on the system. The force equilibrium, indicated in Figure 4, implies that the tensile force at the fiber midpoint must equal the total surface shear force over a length $L/2$. This leads to the equation:

$$\tau = \frac{\sigma}{f}(D/L) \tag{6}$$

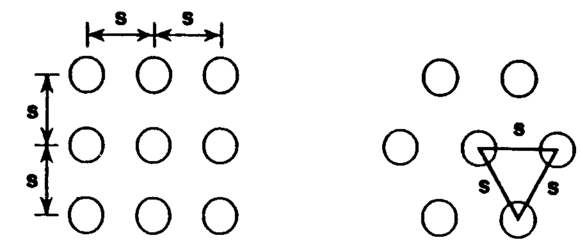

Figure 2. Fiber array geometries.

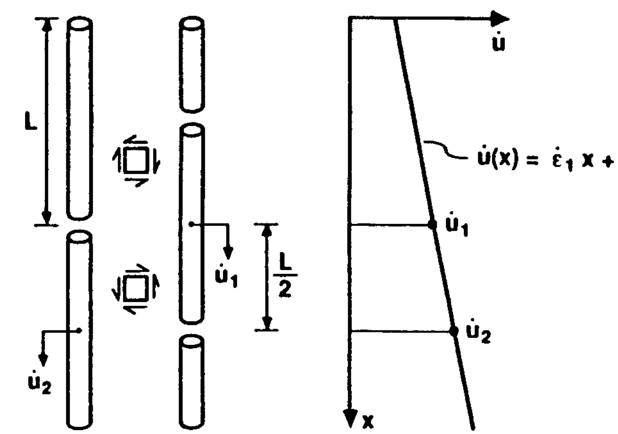

Figure 3. Relative fiber velocity.

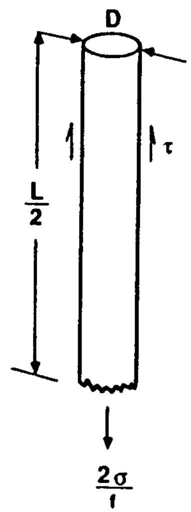

Figure 4. Fiber force balance.

Combining Equations (3), (5) and (6) yields the expression for the elongational viscosity, η_{11}

$$\eta_{11} = \frac{\eta f}{2}\left[\frac{B\sqrt{f}}{1 - B\sqrt{f}}\right](L/D)^2 \tag{7}$$

Batchelor [13] treated the dilute or semidilute suspension case earlier and developed the following relation for elongational viscosity:

$$\eta_{11} = \eta\left[3 + \frac{4f(L/D)^2}{3\ln(\tau/f)}\right] \tag{8}$$

The author assumed the solution valid for the condition

$$L \gg S \gg D \tag{9}$$

and hence fiber volume fractions of 1 percent or less. Figure 5 shows a comparison of the present theory to that of Batchelor, where reasonable agreement is shown for the range $0 \leq f \leq 0.2$ if 3η is added to Equation (7). However, the Batchelor theory predicts a finite viscosity for the maximum fiber packing fraction, while the present theory is unbonded for $f = 1/B^2 = F$.

Goddard [14] extended the Batchelor theory to non-Newtonian fluids and investigated the influence of shear thinning (power-law fluids) upon the fluid stress

field. More recently Acrivos and Shaqfeh [15] developed a relationship similar to the Batchelor formula employing quite distinct physical arguments. However, these new results were also restricted to the semidilute range of fiber volume fraction.

AXIAL SHEAR VISCOSITY

To determine the axial shear viscosity, η_{12}, a velocity field corresponding to pure shear deformation in the 1-2 plane must be imposed upon the fiber assemblage, $\dot{\gamma}_{12}$. From the relative motion of the fibers it is possible to determine the shearing strain rate in the matrix fluid, $\dot{\gamma}$ as follows:

$$\dot{\gamma} = \dot{\gamma}_{12}[S/(S - D) + 1] \tag{10}$$

The imposed shearing stress may be assumed to be equal to that in the matrix fluid. Hence the apparent axial shear viscosity for the fiber assemblage and matrix fluid may be given by:

$$\eta_{12} = \eta[S/(S - D) + 1] \tag{11}$$

Combining Equations (3), (10) and (11) yields the influence of fiber volume fraction on the axial shear viscosity.

$$\eta_{12} = \frac{\eta}{2}\left[\frac{2 - B\sqrt{f}}{1 - B\sqrt{f}}\right] \tag{12}$$

TRANSVERSE SHEAR AND ELONGATIONAL VISCOSITIES

The transverse shear viscosity for a parallel fiber assembly suspended in a viscous matrix fluid has been studied by Cogswell [16] and Balasubramanyam et al.

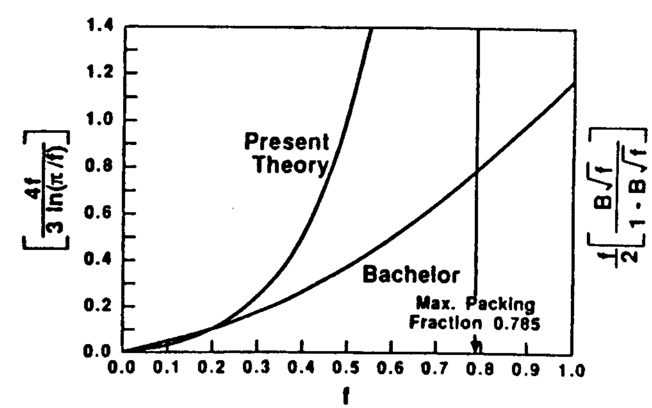

Figure 5. Present theory and Batchelor theory.

[17]. The following relationship has been proposed:

$$\eta_{23} = \eta[1 - f/F]^{-2} \quad (13)$$

where F is the maximum possible volume fraction and is equal to B^{-2}. For the two cross-sectional geometries examined in the present work, namely hexagonal and square arrays the values of F are $\pi/2\sqrt{3}$ and $\pi/4$, respectively.

It should be noted that the transverse shear viscosity is related to the two elongational viscosities in the present model.

$$\eta_{23} = (4/\eta_{22} - 1/\eta_{11})^{-1} \quad (14)$$

For $\eta_{22}/\eta_{11} \ll$ Relation (14) reduces to

$$\eta_{22} = 4\eta_{23} = 4\eta[1 - B^2 f]^{-2} \quad (15)$$

It is convenient to display the terms of the anisotropic viscosity matrix in Table 1 to show the influence of the fiber aspect ratio, volume fraction, and cross-sectional geometry upon each term.

STRAIN RATIOS

For the long fiber system, in which $\eta_{11} \gg \eta_{22}$, the strain ratios, given in Equations (2b), reduce to:

$$\lambda_{12} = 0.5$$

$$\lambda_{21} = 0$$

$$\lambda_{23} = 1$$

Table 1. Anisotropic viscosity terms.

Viscosity	Viscous Compliance	General Form
η_{11}/η	$(\beta_{11}\eta)^{-1}$	$\frac{f}{2}\left[\frac{B\sqrt{f}}{1 - B\sqrt{f}}\right](L/D)^2$
η_{12}/η	$(\beta_{66}\eta)^{-1}$	$\frac{1}{2}\left[\frac{2 - B\sqrt{f}}{1 - B\sqrt{f}}\right]$
η_{22}/η	$(\beta_{22}\eta)^{-1}$	$4[1 - B^2 f]^{-2}$
Term	**Hexagonal**	**Square**
B	$(2\sqrt{3}/\pi)^{1/2}$	$2/\sqrt{\pi}$
F	$\pi/2\sqrt{3}$	$\pi/4$

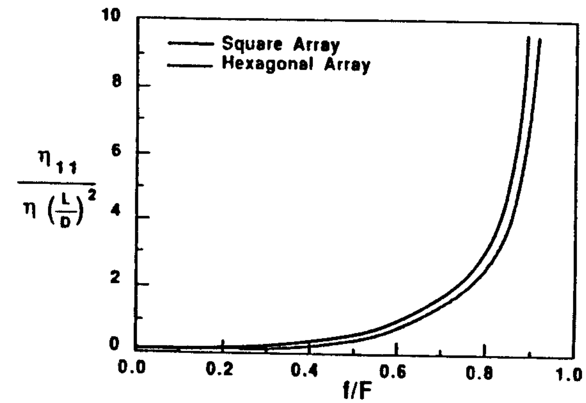

Figure 6. *Elongational viscosity versus fiber volume fraction.*

DISCUSSION

The influence of normalized fiber volume fraction, f/F, upon the composite axial elongational viscosity η_{11}, normalized by $\eta(L/D)^2$, is shown in Figure 6. The elongational viscosity is seen to increase rapidly with fiber volume fraction towards infinity as f/F approaches 1. The range in normalized volume fraction where the developed analysis should be deemed to accurately represent the composite physics should be restricted to $0.25 < f/F < 0.83$. The linear approximation for fluid shear strain rate in Equation (5) is appropriate in the range $1.1 \leq S/D \leq 2.0$. It is interesting to note that in this region the apparent viscosity of the oriented fiber assembly/matrix fluid composite is not greater than 5 times the viscosity of the matrix fluid for a hypothetical fiber aspect ratio of unity.

When the influence of fiber aspect ratio (L/D) upon elongational viscosity is isolated, the results are shown in Figure 7. Here an aspect ratio of 10^2 corresponds to an increase in viscosity of 10^4 and so on. Clearly the fiber aspect ratio is the dominant term in Equation (7). Hence, the ratio of the apparent elongational viscosity of the oriented fiber assembly to the viscosity of the matrix fluid should be expected to be 10^4 to 10^6 for aspect ratios of 10^2 to 10^3 and for normalized volume fractions of 0.25 to 0.83.

Figure 8 shows the influence of normalized fiber volume fraction upon the axial shear viscosity. These results reveal that the axial shear viscosity, which is independent of L/D, is only one order of magnitude greater than that of the matrix viscosity for a normalized volume fraction of 0.83. In Figure 9 the dependence of the transverse elongational viscosity, also independent of L/D, upon normalized fiber volume fraction is shown. Here the maximum ratio of apparent viscosity to that of the matrix fluid viscosity exceeds 100.

It is also instructive to examine the degree of anisotropy in the apparent viscosities for the oriented fiber assembly suspended in a viscous fluid matrix. Comparing the results shown in Figures 6–9, it is clear that the dependence of the elon-

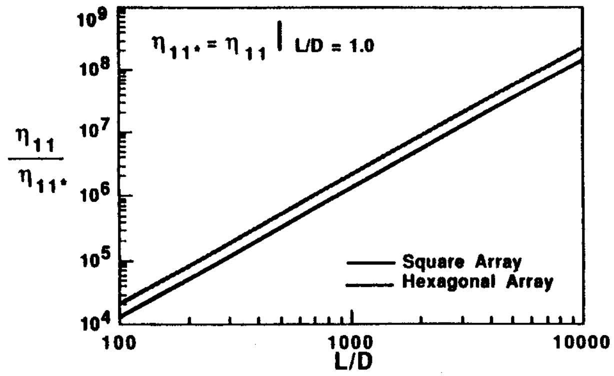

Figure 7. Elongational viscosity versus fiber aspect ratio.

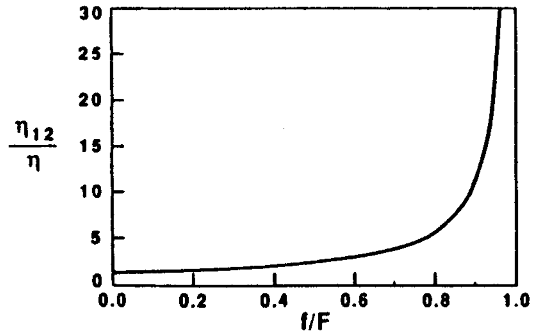

Figure 8. Axial shear viscosity versus fiber volume fraction.

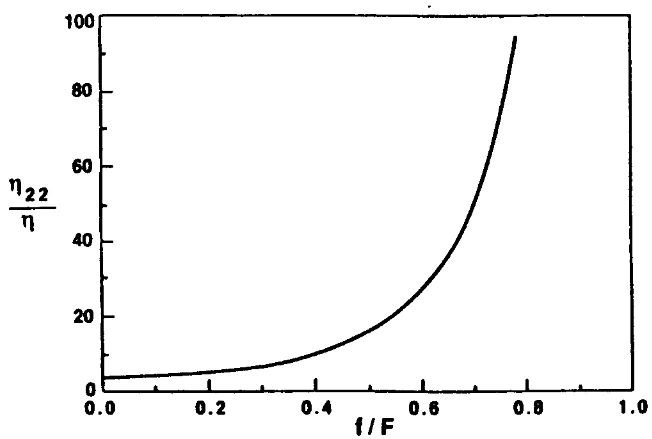

Figure 9. Transverse elongational viscosity versus fiber volume fraction.

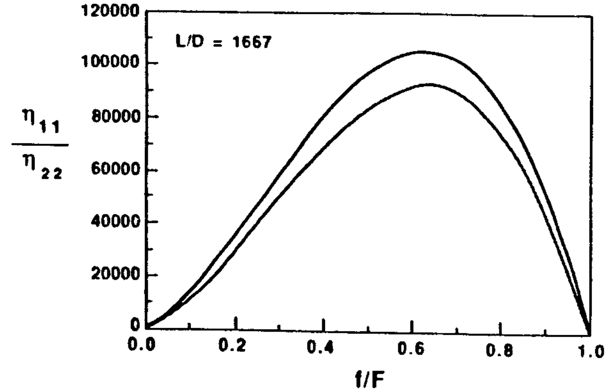

Figure 10. Anisotropy ratio, η_{11}/η_{22}.

gational viscosity, η_{11} in the fiber direction upon the square of the fiber aspect ratio, while all other viscosities show no such dependence, indicates that the anisotropy ratio of the material, η_{11}/η_{22} is in the range of 10^5 for $L/D = 1667$ and $f/F = 0.6$ as shown in Figure 10. Finally, the ratio of the axial shear and transverse elongational viscosities ranges between 3/10 and 1/10 for the normalized fiber volume fractions of 0.25–0.83 as shown in Figure 11.

For large values of L/D, certainly above 100, $\eta_{11} \gg \eta_{22}$. This means that the transverse shear viscosity, η_{23}, will be equal to one quarter of the transverse elongational viscosity, η_{22}. In elongational flow, the transverse contraction needed to maintain constant volume will be equally divided between the two transverse directions, with the strain rate ratio equal to 0.5. But, in transverse elongational flow, there will be zero axial elongation, and the incompressibility is maintained by the strain rate ratio λ_{23} being unity.

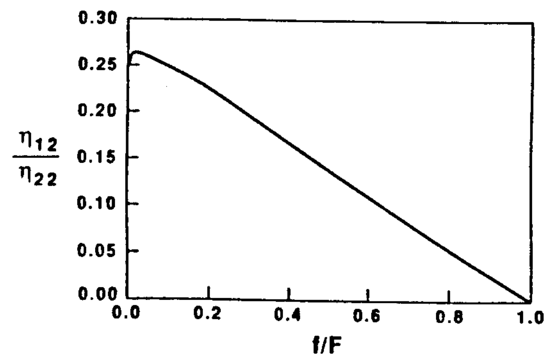

Figure 11. Anisotropy ratio, η_{12}/η_{22}.

CONCLUSIONS

The constitutive relationship for an ordered assembly of discontinuous fibers in a viscous fluid has been developed in terms of a viscous compliance matrix with three independent parameters. The effective viscosities were shown to be functions of the fiber aspect ratio, the fiber volume fraction and the matrix fluid shear viscosity. The relationships developed were based upon assumed homogeneous velocity fields, collimated fiber assemblies with cross-sectional geometries of both square and hexagonal arrays. The elongational viscosity was found to vary as the square of the fiber aspect ratio, while the other terms of the viscosity matrix were not related to fiber geometry. All terms of the viscosity matrix showed a complex relationship to fiber volume fraction. Finally, the ratio between terms of the anisotropic viscosities for typical contemporary fiber systems volume fraction 0.3–0.6 and fiber aspect ratios of 10^3 was shown to range from 10^{-1} to 10^6.

Of course the developed relationships do not provide true quantitative predictions of the anisotropic viscosities for the collimated fiber assembly in a polymeric fluid. This is true for three primary reasons. First, the simplifying assumptions regarding the uniform velocity fields and the arrangement of fibers in the assembly may not be satisfied. Second, the simple viscous fluid model does not allow for the anticipated viscoelastic and/or nonlinear behavior of these material systems. Third, there were several simplifying mathematical approximations involved in the development dealing primarily with the effective shearing strain rate of the fluid. However, the models developed may well serve as a guide in establishing the influence of the fiber assembly geometric parameters, as well as the fluid properties upon anisotropic viscosities of the assembly. Finally, the models allow for determination of the relative magnitudes of the anisotropic viscosity terms.

NOMENCLATURE

Symbol	Term	Units (FLT)
B	Constant	—
D	Fiber diameter	L
F	Maximum fiber volume fraction	—
f	Fiber volume fraction	—
L	Fiber length	L
S	Fiber spacing	L
β_{ij}	Viscous compliance matrix	L^2/FT
η_{ij}	Viscosity terms	FT/L^2
η	Fluid shear viscosity	FT/L^2
σ_i	Normal stress component	F/L^2
$\dot{\epsilon}_i$	Normal strain rate component	1/T
$\dot{\gamma}$	Fluid shear strain rate	1/T
$\dot{\gamma}_{12}$	Fiber assembly shear strain rate	1/T
τ	Fluid shear stress	F/L^2
λ_{ij}	Strain rate ratios	—

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