Problem 1

100 prisoners

There are 100 prisoners in solitary cells. There's a central living room with one light bulb. No prisoner can see the light bulb from his or her own cell. Everyday, the warden picks a prisoner randomly, and that prisoner visits the living room. While there, the prisoner can toggle the bulb if he or she wishes. Also, the prisoner has the option of asserting that all 100 prisoners have been to the living room by now. If this assertion is false, all 100 prisoners are shot. However, if it is indeed true, all prisoners are set free. Thus, the assertion should only be made if the prisoner is 100% certain of its validity. The prisoners are allowed to get together one night in the courtyard, to discuss a plan. The initial state of the bulb is unknown to the prisoners.

Question: What plan should they agree on, so that eventually, someone will make a correct assertion?

Solution: The prisoners need to chose one of them, let's call him counter. He will be turning off light every time he goes to the living room and the light is on. He will be counting how many times he does it. If the light is off he does not touch it.

The other prisoners will be turning on light on their first two visits to the living room with the light off. They will do nothing on their next visits to the living room with the light off. If they visit the living room with the light on, they do nothing.

Once the counter turn the light off 200 times, each prisoner will visit the living room at least once (since the starting position of the light could be on).

Problem 2

Circle of rooms

There is a house in which the rooms are closed in a circle. All the rooms in this house are windowless and identical to each other but have a door to go from one room to another. You wake up in one of these rooms and you need to calculate how many such rooms are in the house to get out of this house. **You can only mark rooms by turning the light in the rooms on or off.** The initial position of the switches in each room is random, either on or off.

Question: How to figure out how many rooms are there in such a house?

Solution: You need to turn the light on in the room you woke up. Then you walk in one direction from room to room counting rooms until you see turned on light. You turn it off and walk back to the original room (you know how to find it, since you was counting rooms). If the light in the original room is off, you made the whole circle and you know the number of rooms in the house. If the light is on, you walk again in one direction from room to room counting rooms and turn off the light you found before. Then you continue walking again in one direction from room to room counting rooms until you find another room with light on. Repeat the procedure described above until you make the whole circle in the house.

Problem 3

Green eyed prisoners

Imagine an island on which a crazy dictator holds 100 people in custody, and they are all excellent mathematicians. It's impossible to run, but there is one strange rule. At night, any prisoner can ask the guard for release. If the prisoner has green eyes, he will be released. If not, he will be thrown into the throat of a volcano.

In fact, all 100 prisoners have green eyes. But prisoners have been living on the island since birth, and the dictator did everything so that no one could recognize the color of their eyes. There are no mirrors on the island, and all water containers are opaque. But, most importantly, prisoners are forbidden to talk to each other.

Nevertheless, every morning they see each other at roll call. Everyone knows that no one will even dare to ask for freedom without being absolutely sure of success. Unable to withstand the pressure of human rights organizations, the dictator reluctantly allows you to visit the island and talk with prisoners. But he sets the conditions: you make only one statement, and you do not give the prisoners new information.

After much thought, you say to the crowd of prisoners: "At least one of you has green eyes." The dictator is full of distrust, but calms himself: such a statement will not change anything. You are leaving, and life on the island seems to be taking its course. But one morning, several days after your visit, the island is empty - all the prisoners demanded their release last night.

Question: How did you manage to trick the dictator and how many days it took for prisoners to figure out the color of their eyes?

Solution: The prisoners did the following argument. If there is only one green eyed prisoner, he will ask the guard for release on the first night, since he won't see any other green eyed prisoners on roll call. Since nobody asked for release on the first night, there are at least two prisoners with green eyes. If there are exactly two prisoners with green eyes on the second night they both will ask for release, since they will see only one other green eyed prisoner. Continuing in the same fashion we will get to the point that on 99th day all prisoners will know there are at least 99 green eyed prisoners. If there are exactly 99 green eyed prisoners, they will all ask for release, since they will see only 98 other green eyed prisoners. Since nobody ask for release on 99th night, all prisoners are green eyed and they will ask for release on 100th day.

Problem 4

Four buttons

You are trapped in a very small room. In the middle of each side of the room there is a hole with a button inside. The button can either be on or off but you cannot see in the holes and cannot feel whether the button is on or off. You can stick your hands in any two of these four holes at a time and either push both buttons, push one button, or do nothing. The goal is to set all four buttons to the same state, whether they are all on or all off. If you are successful, the room will open up and you can escape. You will not know if you have succeeded until you remove both of your hands from the holes. However, if you remove both of your hands from the holes and you have not succeeded in setting all of the buttons to the same state, the room will spin around very quickly and disorient you so that you cannot tell which wall is which.

Question: How do you put all of the buttons in the same state and escape from the room?

Solution: You can follow the following algorithm

- Don't do anything. This move will help you if the buttons are already in the same state.
- Push 2 opposite buttons. This move will help you if opposite buttons are in the same state.
- Push 2 adjacent buttons. After this step you have either 2 opposite buttons in the same state or 3 buttons in the same state.
- Push 2 opposite buttons. After this step you can only have 3 buttons in the same state.
- Push one button. After this step you can only have 2 buttons in the same state.
- Push 2 opposite buttons.
- Push 2 adjacent buttons.
- Push 2 opposite buttons.

After that you necessary have 2 buttons in the same state.

Problem 5

100 dwarfs

The giant caught 100 dwarfs and was going to eat them, but decided to give each of them a chance to survive, offering them to play a game with the following rules: the dwarfs will stand in one column, and then the giant will put on each of them a hat of white, black or red color randomly. Each dwarf in the column can see the hats in front of him, but cannot see his hat and the hats behind him. The giant offer each dwarf to make a guess (starting from the last in the column that sees all the dwarfs) what color the hat is on his head (it is allowed to say one word loudly: "white", "black" or "red"). If the dwarf guesses correctly, then he remains alive, if not, then he immediately is eaten in front of other dwarfs.

Question: How many dwarfs will be eaten in the worst case scenario for them if they act together and reasonably?

Solution: Only one dwarf will be eaten.

Let's assign numbers to different colors. White=0. Black=1, Red=2. The last dwarf counts the sum of the colors he sees in from of him and compute the remainder of division of this sum by 3. He converts it to the color and says out loud.

The second dwarf computes the sum of the colors in from of him and the remainder of its division by 3. Comparing it with the answer of the second dwarf, he understands which number he need to add to his sum to get the sum of first dwarf. It number corresponds to the color of his hat, so he tells the giant the color of his hat.

The other dwarfs follow the same procedure.

Problem 6

Prisoners and hats

The evil jailer gets new batch of 100 prisoners. In order not to torment them with too long waiting time, the jailer offered them a game:

"You will all be arranged in a circle. Then I personally will put a hat on each of you. The hats have numbers from 1 to 100 written on them. The numbers on the hats can be repeated. Everyone will see the numbers on all the other prisoners, but will not be able to see the number on their hat. After that, I will give everyone the opportunity to say (not out loud, but only in my ear) exactly one number. If at least one prisoner guesses the number on his hat, I will immediately let everyone go, and if not, I will send you all to uranium mines. But if I notice clues and winks, I order to execute everyone immediately. "

After developing an action strategy, the prisoners agreed to play the game.

Question: How should they act in order to be liberated?

Solution: Let's compute the sum of all numbers on the hats and the remainder of its division by 100. The prisoners don't know this number, but since there are only 100 possible remainders if they all guess different remainders, one of them will be right.

Once you know that remainder, you can compute the sum of all numbers you see and compute its remainder of its division by 100. Knowing this 2 remainders, you will know the number on your hat.

Problem 7

Chessboard

You, and your friend, are incarcerated. Your jailer offers a challenge. If you complete the challenge you are both free to go. Here are the rules:

The jailer will take you into a private cell. In the cell there is a chessboard and a jar containing 64 coins.

The jailer will take the coins, one-by-one, and place a coin on each square on the board. He will place the coins randomly on the board. Some coins will be heads, and some tails (or maybe they will be all heads, or all tails; you have no idea. It's all at the jailers whim. He may choose to make a pattern himself, he may toss them placing them the way they land, he might look at them as he places them, he might not. If you attempt to interfere with the placing of the coins, it is instant death for you. If you attempt to coerce, suggest, or persuade the jailer in any way, instant death. All you can do is watch.

Once all the coins have been laid out, the jailer will point to one of the squares on the board and say: "This one!" He is indicating the magic square. This square is the key to your freedom.

The jailer will then allow you to turn over one coin on the board. Just one. A single coin, but it can be any coin, you have full choice. If the coin you select is a head, it will flip to a tail. If it is a tail it will flip to a head. This is the only change you are allowed to make to the jailer's initial layout.

You will then be lead out of the room. If you attempt to leave other messages behind, or clues for your friend ... yes, you guessed it, instant death!

The jailer will then bring your friend into the room.

Your friend will look at the board (no touching allowed), then examine the board of coins and decide which location he thinks is the magic square.

He gets one chance only (no feedback). Based on the configuration of the coins he will point to one square and say: "This one!"

If he guesses correctly, you are both pardoned, and instantly set free. If he guesses incorrectly, you are both executed.

The jailer explains all these rules, to both you and your friend, beforehand and then gives you time to confer with each other to devise a strategy for which coin to flip.

Question: How do you escape?

Solution:

Let's assign the 6 digit binary number to the chessboard pattern created by jailer. It will encode the parity of different regions of the board. The regions are obtained by subsequent splitting of board in half. We will call it shortly parity of the board.

Let's enumerate columns from left to right and rows from top to bottom.

- If there is an odd number of heads in the first 4 columns, we pick the first digit to be 1. If there is an even number of heads in the first 4 columns, we pick the first digit to be 0.

- If there is an odd number of heads in the first 4 rows, we pick the second digit to be 1. If there is an even number of heads in the first 4 rows, we pick the second digit to be 0.

- If there is an odd number of heads in the columns 1, 2, 5 and 6, we pick the third digit to be 1. If there is an even number of heads in the columns 1, 2, 5 and 6, we pick the third digit to be 0.

- If there is an odd number of heads in the rows 1, 2, 5 and 6, we pick the fourth digit to be 1. If there is an even number of heads in the rows 1, 2, 5 and 6, we pick the fourth digit to be 0.

- If there is an odd number of heads in the columns 1, 3, 5 and 7, we pick the fifth digit to be 1. If there is an even number of heads in the columns 1, 3, 5 and 7, we pick the fifth digit to be 0.

- If there is an odd number of heads in the rows 1, 3, 5 and 7, we pick the sixth digit to be 1. If there is an even number of heads in the rows 1, 3, 5 and 7, we pick the sixth digit to be 0.

Flipping one coin we can change the parity of the board to any 6 digit number. Let's see how.

- If we want to change the first digit, we need to flip the coin in the first 4 columns. If we don't want to change the first digit, we need to flip the coin in the last 4 columns.

- If we want to change the second digit, we need to flip the coin in the first 4 rows. If we don't want to change the second digit, we need to flip the coin in the last 4 rows.

- If we want to change the third digit, we need to flip the coin in the columns 1, 2, 5 or 6. If we don't want to change the third digit, we need to flip the coin in the columns 3, 4, 7 or 8.

- If we want to change the fourth digit, we need to flip the coin in the rows 1, 2, 5 or 6. If we don't want to change the fourth digit, we need to flip the coin in the rows 3, 4, 7 or 8.

- If we want to change the fifth digit, we need to flip the coin in the columns 1, 3, 5 or 7. If we don't want to change the fifth digit, we need to flip the coin in the columns 2, 4, 6 or 8.
- If we want to change the sixth digit, we need to flip the coin in the rows 1, 3, 5 or 7. If we don't want to change the sixth digit, we need to flip the coin in the rows 2, 4, 6 or 8.

Finally, we need to mention that 6 digit binary number corresponds to numbers from 0 to 63 and can describe any square on the chessboard.

So let's sum up our strategy. First prisoner compute the parity of the board created by jailer and the 6 digit number corresponding to the coin pointed out by jailer. Flipping one coin the first prisoner makes these two numbers equal to each other. The second prisoner reads of the board its parity and finds based on this number the square pointed out by jailer.

Problem 8

Prisoners and boxes

In this puzzle there are 100 prisoners, each given a distinct number 1-100. The jailer has decided to give all the prisoners a chance to escape. He prepares a challenge, and if every single one of the prisoners passes, they are all free to go. If even one of them fails, they all die.

The Challenge: The jailer goes into a secret room and prepares 100 boxes with lids. He labels these boxes 1-100. Then he prepares 100 tickets, one for each prisoner, and labels these tickets 1-100. Finally, he shuffles the tickets thoroughly, and puts one ticket in each box, closing the lid behind it. The prisoners cannot see any of these preparations.

The challenge is now on. He fetches each prisoner, one-by-one, into the box room and tells the prisoner that they must find the box that contains their ticket. They attempt to do this by opening boxes. They are allowed to open up to half the boxes in their search. If they succeed in finding their own number, they win. If they have not found their ticket after examining 50 boxes, they fail.

In order for the prisoners to escape, all prisoners have to win. After opening a box and examining its contents, the lid is closed again. The position of tickets cannot be changed. No messages can be left behind for prisoners yet to come.

The prisoners are allowed to confer before the challenge begins.

A comrade (not party to the challenge) is able to sneak into the room beforehand, examine the locations of all the tickets and (optionally) and swap just two of the tickets over (but not able to communicate what change he made).

Question: What strategy should prisoners and their comrade use to escape?

Solution: The prisoners should open the boxes according to the following rule. Each prisoner starts with the box labeled with their number. They take ticket out of the box and open next box labeled with this ticket. In such process the boxes gets ordered and all the prisoners follow the same order in opening boxes. If the prisoners were allowed to open all boxes, they would split all boxes in several loops. If all the loops lengths are less than 50, all prisoners wins. So if their comrade find the largest loop and split it in half, the prisoners will necessary win.