Problem 1

Goat, head of cabbage and wolf

A peasant bought a goat, a head of cabbage and a wolf on the market. On the way home he had to cross the river. The peasant had a small boat, in which besides him only one of his purchases could fit.

Question: How can he transport all goods across the river, if he can't leave the goat alone with the cabbage and the wolf alone with the goat?

Solution:

$$wolf, goat, cabbage||$$

$$wolf, cabbage|\xRightarrow{goat}|$$

$$wolf, cabbage| \Leftarrow |goat$$

$$wolf|\xRightarrow{cabbage}|goat$$

$$wolf|\xLeftarrow{goat}|cabbage$$

$$goat|\xRightarrow{wolf}|cabbage$$

$$goat| \Leftarrow |wolf, cabbage$$

$$|\xRightarrow{goat}|wolf, cabbage$$

$$||wolf, goat, cabbage$$

Problem 2

Family

A father, a mother, and two children — a son and a daughter — must cross the river. A fisherman happened to pass nearby who could lend them his boat. However, only one adult or two children can fit in the boat.

Question: How can a family cross the river and return the boat to the fisherman?

Solution:

$$son, daughter, mother, father||$$

$$mother, father|\xRightarrow{son,daughter}|$$

$$mother, father|\xLeftarrow{son}|daughter$$

$$mother, son|\xRightarrow{father}|daughter$$

$$mother, son|\xLeftarrow{daughter}|father$$

$$mother|\xRightarrow{son,daughter}|father$$

$$mother|\xLeftarrow{son}|daughter, father$$

$$son|\xRightarrow{mother}|daughter, father$$

$$son|\xLeftarrow{daughter}|mother, father$$

$$|\xRightarrow{son,daughter}|mother, father$$

$$son, daughter, mother, father||$$

Problem 3

Missionaries and cannibals

Three missionaries and three cannibals must cross the river. They have one boat in which only two are accommodated. To avoid tragedy, you cannot leave more cannibals together than missionaries.

Question: How to cross a river?

Solution:

$$3\,missionaries, 3\,cannibals||$$

$$3\,missionaries, cannibal|\xRightarrow{2\,cannibals}|$$

$$3\,missionaries, cannibal|\xLeftarrow{cannibal}|cannibal$$

$$3\,missionaries|\xRightarrow{2\,cannibals}|cannibal$$

$$3\,missionaries|\xLeftarrow{cannibal}|2\,cannibals$$

$$missionary, cannibal|\xRightarrow{2\,missionaries}|2\,cannibals$$

$$missionary, cannibal|\xLeftarrow{missionary,cannibal}|missionary, cannibal$$

$$2\,cannibals|\xRightarrow{2\,missionaries}|missionary, cannibal$$

$$2\,cannibals|\xLeftarrow{cannibal}|3\,missionaries$$

$$cannibal|\xRightarrow{2\,cannibals}|3\,missionaries$$

$$cannibal|\xLeftarrow{cannibal}|3\,missionaries, cannibal$$

$$|\xRightarrow{2\,cannibals}|3\,missionaries, cannibal$$

$$||3\,missionaries, 3\,cannibals$$

Problem 4

Monkeys

Three people, one large and two small monkeys must cross the river. There is one boat that can fit no more than two of monkeys or people. Only humans and the big monkey can row. It is impossible to leave together more monkey than the people, otherwise the monkeys will devour people. Monkeys can jump ashore when the boat approaches the land.

Question: How do they cross the river?

Solution:

$$3\,humans, 2\,small\,monkeys, big\,monkey||$$

$$3\,humans, small\,monkey|\xrightarrow{big\,monkey, small\,monkey}|$$

$$3\,humans, small\,monkey|\xleftarrow{big\,monkey}|small\,monkey$$

$$3\,humans|\xrightarrow{big\,monkey,\,small\,monkey}|small\,monkey$$

$$3\,humans|\xleftarrow{big\,monkey}|2\,small\,monkeys$$

$$human, big\,monkey|\xrightarrow{2\,humans}|2\,small\,monkeys$$

$$human, big\,monkey|\xleftarrow{human, small\,monkey}|human, small\,monkey$$

$$human, small\,monkey|\xrightarrow{human, big\,monkey}|human, small\,monkey$$

$$human, small\,monkey|\xleftarrow{human, small\,monkey}|human, big\,monkey$$

$$2\,small\,monkeys|\xrightarrow{2\,humans}|human, big\,monkey$$

$$2\,small\,monkeys|\xleftarrow{big\,monkey}|3\,humans$$

$$small\,monkey|\xrightarrow{big\,monkey,\,small\,monkey}|3\,humans$$

$$small\,monkey|\xleftarrow{big\,monkey}|3\,humans,\ small\,monkey$$

$$|\xrightarrow{big\,monkey,\,small\,monkey}|3\,humans, small\,monkey$$

$$||3\,humans, 2\,small\,monkeys, big\,monkey$$

Problem 5

Tourists and the lantern

Four tourists need to cross the bridge at night. They have only one lantern, without which it is impossible to cross the bridge, in addition, the bridge is so old that it can hold no more than two people. It is known that the first of the tourists crosses the bridge in 1 minute, the second in 2, the third in 5 and the fourth in 10 minutes. Moreover, if two of them go at the same time, then they move at the speed of the slower of them.

Question: In what minimum time can all four tourists cross the bridge?

Solution:

$$first\,tourist,\ second\,tourist,\ third\,tourist,\ fourth\,tourist||$$

$$third\,tourist,\ fourth\,tourist|\xRightarrow[2\,min]{first\,tourist,\,second\,tourist}|$$

$$third\,tourist,\ fourth\,tourist|\xLeftarrow[1\,min]{first\,tourist}|second\,tourist$$

$$first\,tourist|\xRightarrow[10\,min]{third\,tourist,\,fourth\,tourist}|second\,tourist$$

$$first\,tourist|\xLeftarrow[2\,min]{second\,tourist}|third\,tourist,\ fourth\,tourist$$

$$|\xRightarrow[2\,min]{first\,tourist,\,second\,tourist}|third\,tourist,\ fourth\,tourist$$

$$||first\,tourist,\ second\,tourist,\ third\,tourist,\ fourth\,tourist$$

Total time spent is 17 min.

Problem 6

Climber

The climber needs to descend from a sheer cliff 100 meters high. He has one rope 75 meters long. In addition, on the edge of the cliff and on the cliff itself, at a height of 50 m, a tree is growing to which the rope can be tied.The climber can cut the rope and tie it together.

Question: How should he act to descend from a cliff without damage?

Solutions: The climber should perform the following actions:

- cut the rope on 2 pieces, 25 meters and 50 meters long.
- tie the loop at the end of 25 meters long piece.
- put 50 meters piece through that loop.
- tie 50 meters piece in the loop.
- tie the loose end of 25 meters long piece to the edge of the cliff.

As the result the climber will get 50 meters long piece of rope attached to the edge of the cliff. He can climb down the tree, then cut 50 meters long loop and pull it down. It will let him to climb down the rest of the cliff.

Problem 7

Camel in the desert. Part 1.

A camel must pass through the desert to the nearest city, which is located at a distance of 1000 km. At the beginning of the journey, he has 2000 liters of water and he can carry no more than 1000 liters of water. For every kilometer traveled, he drinks one liter of water. A camel can leave any amount of water in the desert and then take it.

Question: How much water can he transfer to the city?

Solution: The camel can make 2 trips, since it can not carry more than 1000 liters of water. During the first trip it can leave some water in the desert and during the second trip it can pick it up. Denote x the distance to the drop off and y the amount dropped. Schematically we have. First trip:

$$\begin{array}{ccc} 1000 & \stackrel{x}{\Longrightarrow} & 1000-x \\ & & \Downarrow y \\ 1000-2x-y & \stackrel{x}{\Longleftarrow} & 1000-x-y \end{array}$$

Second trip:

$$1000 \quad \stackrel{x}{\Longrightarrow} \quad 1000-x \quad \Uparrow x \quad 1000 \quad \xrightarrow{1000-x} \quad x$$

We see that the camel picked up all the water on the second trip, so $y = x$. We want the camel to return from the first trip empty so we have $x = 333\frac{1}{3}$ liters. As the result, camel transferred $333\frac{1}{3}$ liters.

Problem 8

Camel in the desert. Part 2.

A camel must pass through the desert to the nearest city, which is located at a distance of 1000 km. At the beginning of the journey, he has 3,000 liters of water and he can carry no more than 1,000 liters of water. For every kilometer traveled, he drinks one liter of water. A camel can leave any amount of water in the desert and then take it.

Question: How much water can he transfer to the city?

Solution: Now the camel can make 3 trips. During the first and the second trip it can leave some water in the desert and during the second and the third trip it can pick it up. The amount of water left on the first trip will be used to fill up the camel on the second trip on the way there and back and also on the third trip. Denote it y. The amount of water dropped on the second trip will be used only on the third trip. Denote it s. Denote also x the distance to the first drop off and $x+z$ the distance to the second drop off. Schematically we have. First trip:

$$\begin{array}{ccc} 1000 & \overset{x}{\Longrightarrow} & 1000-x \\ & & \Downarrow y \\ 1000-2x-y & \overset{x}{\Longleftarrow} & 1000-x-y \end{array}$$

Second trip:

$$\begin{array}{ccccccc} 1000 & \overset{x}{\Longrightarrow} & 1000-x & \Uparrow x & 1000 & \overset{z}{\Longrightarrow} & 1000-z \\ & & & & & & \Downarrow s \\ 1000-2z-s & \overset{x}{\Longleftarrow} & 1000-2z-s+x & \Uparrow x & 1000-2z-s & \overset{z}{\Longleftarrow} & 1000-z-s \end{array}$$

Third trip:

$$1000 \overset{x}{\Longrightarrow} 1000-x \;\Uparrow x\; 1000 \overset{z}{\Longrightarrow} 1000-z \;\Uparrow z\; 1000 \xrightarrow{1000-x-z} x+z$$

We see that from the first drop off camel needed to pick up 3 times x liters, so $y=3x$. From the second drop off the camel needed z liters, so $s=z$. We want the camel to return from the first and the second trip empty so we have $x=200$ liters and $z=333\frac{1}{3}$ liters. As the result, camel transferred $533\frac{1}{3}$ liters.

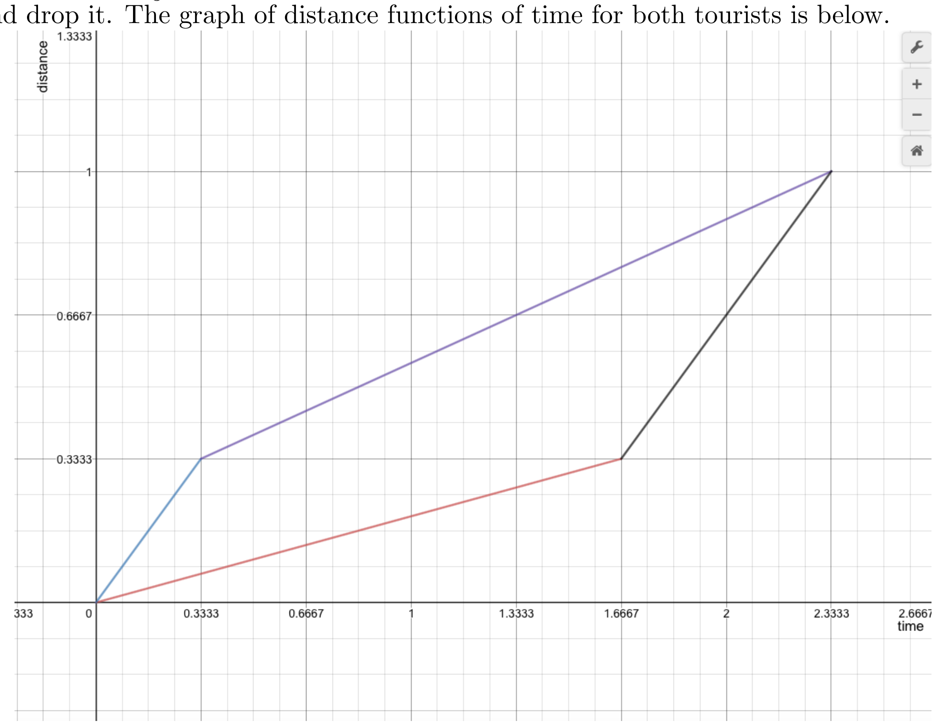
Problem 9

Tourists and the bike. Part 1.

Two tourists need to cross the bridge. It is known that first of the tourists crosses the bridge in 3, the second in 5. In addition, they have a bike that allows any of the tourists to cross the bridge in 1 minute.

Question: In what minimum time can both tourists cross the bridge?

Answer: $2\frac{1}{3}$min. The first tourist need to ride the bike part of the bridge and drop it. The graph of distance functions of time for both tourists is below.



Problem 10

Tourists and the bike. Part 2.

Three tourists need to cross the bridge. It is known that first of the tourists crosses the bridge in 3, the second in 5 and the third in 10 minutes. In addition, they have a bike that allows any of the tourists to cross the bridge in 1 minute.

Question: In what minimum time can all three tourists cross the bridge?

Answer: $3\frac{5}{11}$ min. The third tourist need to ride the bike part of the bridge and drop it. The first tourist need to pick up the bike and ride it back to help the second tourist. The graph of distance functions of time for all tourists is below.