An Initial Assessment of Fuzzy Logic Vessel Path Control

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Abstract - A fuzzy logic controller for ship path control in restricted waters is developed and evaluated. The controller uses inputs of heading, yaw rate, and lateral offset from the nominal track. A Kalman filter is used to produce the input state variables from noisy measurements. The controller produces a commanded rudder Input variable fuzzification, fuzzy angle. associative memory rules, and output set defuzzification are described. The controller is benchmarked against a conventional Linear Quadratic Gaussian (LQG) optimal controller and Kalman filter control system. An initial startup transient and regulator control performance with an external hydrodynamic disturbance are The fuzzy controller yields evaluated. competitive performance.

I. INTRODUCTION

The maneuvering of a ship along a prescribed path in restricted waters is important from operational, safety, and environmental viewpoints. A ship is subjected to short-term, essentially zero-mean disturbances due to passing ships, current and wind variations, and bank and bottom changes. It is also subjected to more long-term, non-zero-mean disturbances due to steady current and wind, second-order wave forces, and banks. The dynamic characteristics of the ship change significantly with depth under keel, draft, trim, speed, and nearness to banks. The effects of the bank and bottom

boundaries can be viewed either as changes in the dynamic characteristics of the ship or as external force and moment disturbances. Uncertainty exists about the nonlinear mathematical model of the ship in these varying conditions.

Earlier we studied the use of conventional Linear Quadratic Gaussian (LQG) optimal controllers, Minimum Variance adaptive controllers, and multivariable integral controllers for ship path control [1,2,3]. More recently, Papoulias and Healey [4,5] have investigated the use of multivariable sliding mode control for the path control of surface ships. This approach shows promise for robust control of nonlinear systems with modeling uncertainty.

Recent Sea Grant long-range planning emphasized the investigation of developments in Artificial Intelligence (AI) for possible application to the marine field. Of these, fuzzy logic control offers an effective alternative for the control of nonlinear systems where the model is not well known [6,7,8]. This paper describes the initial efforts to investigate the effectiveness of fuzzy logic for ship path control. In the future, we will investigate the use of neural networks in combination with fuzzy logic control to provide an element of learning and adaptivity in the control.

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II. SYSTEM MODEL

The system model is the typical ship maneuvering coordinate system fixed at the center of gravity of the vessel as shown in Fig. 1. The five states are heading angle ψ , yaw rate $r = d\psi/dt$, side slip angle β , lateral offset of the vessel from the nominal path η , and rudder angle δ . A first-order model with a commanded rudder angle δ _C models the steering gear. The vessel can be subjected to an external yaw moment N and external lateral force Y due to a passing ship or other hydrodynamic influence. The non-dimensionalized linear state equations are as follows [1]:

where
$$\mathbf{x} = [\mathbf{y}', \mathbf{r}', \boldsymbol{\beta}', \boldsymbol{\eta}', \boldsymbol{\delta}']^{\mathrm{T}}$$

$$\mathbf{y} = \delta_{\mathbf{c}}$$

$$\mathbf{w} = [\mathbf{N}', \mathbf{Y}']^{\mathrm{T}}$$

$$0 \quad 1 \quad 0 \quad 0 \quad 0$$

$$0 \quad \text{for for for } 0 \quad \text{for } 0$$

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & f_{22} & f_{33} & 0 & f_{25} \\ 0 & f_{32} & f_{33} & 0 & f_{35} \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1/T_r \end{bmatrix}$$

$$G = [0 \ 0 \ 0 \ 0 \ 1/T_{\mathbf{r}}]^T$$

$$\Gamma^{T} = \begin{bmatrix} 0 & \gamma_{21} \gamma_{31} & 0 & 0 \\ 0 & \gamma_{22} \gamma_{32} & 0 & 0 \end{bmatrix}$$

The specific vessel used here is the 150,000 deadweight ton, 290m x 47.5m x 16.0m (LxBxT) tanker *Tokyo Maru* which was studied extensively by Fugino [9]. We used this vessel in our earlier work. We have shown that, for best overall performance, conventional LQG controllers should be designed for the least course stable water depth to ship draft ratio [1] which for this vessel is H/T = 1.89. The coefficients of the equations of motion at this water depth and a Froude Number of 0. 116 or 12 knots are as follows:

$$\begin{array}{lll} f_{22} = -1.7657 & \gamma_{21} = 477.68 \\ f_{23} = 5.7359 & \gamma_{22} = -5.0043 \\ f_{25} = -0.88074 & \gamma_{31} = 21.141 \\ f_{33} = -0.52766 & \gamma_{32} = -28.233 \\ f_{35} = -0.15607 \\ 1/T_r = 4.6980 & \end{array}$$

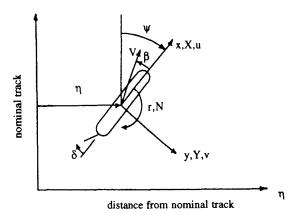


Fig. 1 Ship Maneuvering Coordinate System

III. CONVENTIONAL LQG CONTROLLER

The performance of the fuzzy logic controller will be benchmarked against the performance of a Linear Quadratic Gaussian (LQG) optimal controller and Kalman filter. The optimal controller is defined by the quadratic cost function,

$$J = [\underline{x} A \underline{x}^{T} + \underline{u} B \underline{u}^{T}]/2$$
where
$$a_{44} = 772.463$$

$$a_{55} = 131.332$$

$$b_{11} = 131.332$$
(2)

Using the design logic of Bryson and Ho [10] these weights assume 5 degrees of rudder would be applied when the lateral offset is 10.43 m or about 22% of the 47.5 m ship beam. The resulting optimal control gain matrix becomes:

$$C_x = [5.5421, 2.6601, 6.3894, 2.4252, -0.8498]^T$$

The system measurements are the heading, yaw rate, and lateral offset from the nominal path. These are given by,

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{y} \tag{3}$$
where
$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The measurement white noise components have power spectral density matrix R with non-zero diagonal components $r_{11} = 1.298 \times 10^{-8}$, $r_{22} = 2.860 \times 10^{-7}$, and $r_{44} = 4.559 \times 10^{-7}$. The side slip angle β is not measured. We have shown using relative observability that the system is fully observable without β . This precludes the need for Doppler sonar [1].

The design external hydrodynamic disturbance is for a passing ship as shown in Fig. 2. The RMS values of these disturbances between non-dimensional time (ship lengths) -2.0 and 1.4 are 8.798x10⁻⁵ and 21.178x10⁻⁵ for N and Y, respectively. Assuming a correlation time of 1 for each disturbance, the design external disturbance power spectral density matrix Q has diagonal components $q_{11} = 1.548 \times 10^{-8}$ and $q_{22} = 8.970 \times 10^{-8}$. The resulting Kalman filter gain matrix is as follows:

$$\mathbf{K} = \begin{bmatrix} 4.6883 & 0.9507 & 0.0035 \\ 20.9479 & 109.7887 & -0.4755 \\ 2.7730 & 9.0086 & -8.6949 \\ 1.2389 & -0.7579 & 4.1275 \\ 0.0000 & 0.0000 & 0.0000 \end{bmatrix}$$

This LQG controller is similar to those studied in our earlier work except that here we have not used shaping filters to model the external disturbances. This approach will allow the same Kalman filter to be used with both controllers and will provide a valid comparison between the two controllers.

IV. FUZZY LOGIC CONTROLLER

Conventional control design requires a mathematical model of the physical system. Generating the model may often be difficult as in the case of a model for a ship path controller where robust control of a nonlinear system is required. In contrast, fuzzy logic controllers are modelless. In fuzzy logic control design, the designer need only establish linguistically (perhaps from expert knowledge) how the control output should vary with the input. Further, traditional nonlinear features such as deadbands and limiters can be included.

In many cases, the simplicity of the fuzzy logic controller results in a faster and more accurate response than a traditional controller. The simplicity also has the advantage of translating to a lower design cost. The best candidates for fuzzy control are systems where the dynamic behavior is complicated yet the dynamics are of low order.

The fuzzy logic ship path controller receives three crisp numerical inputs: heading angle w, yaw rate r, and lateral offset n from the Kalman filter. It categorizes these inputs qualitatively (fuzzification); determines each input's quantitative value (by assigning degrees of membership); applies antecedent-consequent logic rules corresponding to the input set combinations (firing rules); determines a qualitative output for each active rule and determines the degree to which the rule is fired (correlation-minimum inference procedure); and lastly determines one crisp output for commanded rudder angle $\delta_{\rm C}$ (defuzzification).

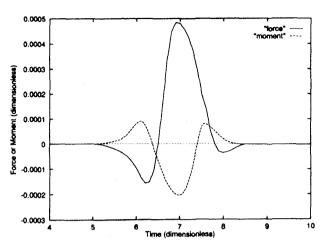


Fig. 2 Force and Moment from Design Passing Ship Disturbance

A. Fuzzification

The first response of the fuzzy controller is to assign each of the three numerical inputs (heading angle w, yaw rate r, and lateral offset n) to a linguistically described input variable fuzzy set. These fuzzy sets are defined as follows:

 heading angle ψ (degrees) 	
LN = -90.00 to -28.00	SP = +0.00 to +17.00
MN = -40.00 to -10.00	MP = +10.00 to +40.00
SN = -17.00 to 0.00	LP = +28.00 to +90.00

SN = -17.00 to 0.00

ZE = -5.00 to + 5.00

where LN = large negative, MN = medium negative, SN = small negative, ZE = zero, SP = small positive, MP = medium positive, and LP = large positive

yaw rate r (degrees/sec)

SP = 0.00 to +0.25LN = -0.20 or less LP = +0.20 and larger SN = -0.25 to 0.00

ZE = -0.10 to +0.10

lateral offset η (dimensionless on ship length)

LN = -0.2 or less SP = 0.0 to +0.4LP = +0.2 and larger SN = -0.4 to 0.0

ZE = -0.1 to +0.1

Similarly, the output variable commanded rudder angle $\delta_{\rm C}$ (degrees) linguistically defined fuzzy sets are as follows:

LN = -35.0 to -15.0SP = 0.0 to +20.0SN = -20.0 to 0.0LP = +15.0 to +35.0ZE = -5.0 to + 5.0

The number of fuzzy sets for each input or output variable is somewhat arbitrary, reflecting the ease of design. The literature [6,7] advises that the increase in accuracy that might be obtained from having many fuzzy sets may be negligible and unwarrented computationally. Three sets for any of the input variables seemed too little in this case; nine seemed too many. Consideration was given to the numerical range of possible values. Thus the heading angle ψ has seven sets, while the other variables have five.

B. Assigning Degrees of Membership

The fuzzy controller assigns to each of the three inputs a degree of membership within each linguistic fuzzy set. Note that often the degree of membership is zero, since the most fuzzy sets to which an input can belong and have a non-zero degree of membership is two. The degree of membership will depend on the fuzzy set's membership function. The input fuzzy set membership functions are defined in Figs. 3, 4, and 5 for ψ , r, and η , respectively. They are symmetrical about the input variable value 0.

The following guidelines were used to prepare the membership functions: the membership function for each ZE fuzzy set has zero as its center; membership functions for fuzzy sets nearer equilibrium (0,0,0) conditions are narrower than those farther from equilibrium (this results in finer control near equilibrium); the degrees of membership in complementary fuzzy sets add to unity.

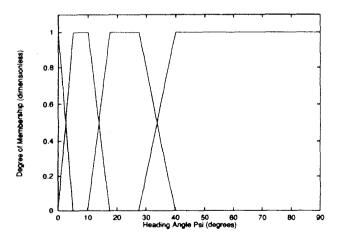


Fig. 3 Membership functions for heading angle w

C. Fuzzy Associative Memory Rules

The fuzzy ship path controller correlates each group of fuzzy input sets to an output fuzzy set through a Fuzzy

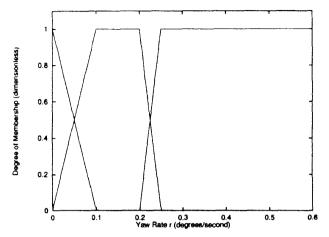


Fig. 4 Membership functions for yaw rate r

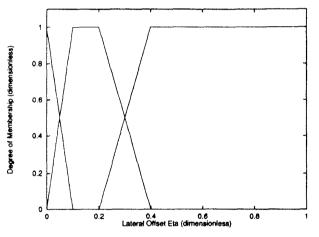


Fig. 5 Membership functions for lateral offset n

Associate Memory (FAM) Rule. A FAM rule is a logical if-then statement such as if these three antecedent components (group of fuzzy input sets) occur, then this consequence (fuzzy output set) should be used. The fuzzy ship path controller uses 175 rules, corresponding to 7x5x5 different combinations of the three input fuzzy sets. Matrices of rules are given in Fig 6. Note that symmetry requires the designer to prepare only 88 rules, with symmetry determining the remaining 87 rules. The 175 rules were written using general maneuvering algorithms which contained the expertise that might otherwise be provided by an expert ship operator.

The controller applies ('fires') the antecedent-consequent rules to every group of three input linguistic fuzzy sets and outputs a commanded rudder angle δ_C linguistic fuzzy set for each group. For example, for the crisp input vector (ψ, r, η) of (0, 0.15, 0.05), the controller yields fuzzy set degree of membership values of 1.0 in ZE for ψ , 1.0 in SP for r, and

Lateral Offset II (Eta) . LP

	Heating Angle y (Psi)							
		مِا	МР	SP	ZE	SN	MN	LN
Vaw Rate r	LP	LP	Ľ.	LP	J.	J.	LP	LN
	SP	LP	LP	LP	LP	SP	ZE	LN
	ZE	LP	LP	LÞ	SP	SP	SN	LN
	SN	LP	LP	u	SP	ZE	LN	LN
	LN	LP	u	LP	ΖE	LN	LN	ĽN

Lateral Office n (Eta) = SP

		Heading Angle w (Pti)						
		LP	MP	SP	ZE	SN	MIN	ĽX
Yaw Rate r	LP	5	LP	LP	LP	Ľ	SP	ĽN
	SP	LP.	LP	LP	LP	ZE	SN	LN
	ZE	UP	LP	SP	SP	SN	SN	LN
	SN	LP	UP.	SP	ZE	LN	LN	LN
	LN	LP	LP	7 <u>E</u>	LN	LN	LN	LN

Lateral Offset it (Eta) = ZE

	Heading Angle w (Psi)								
		L.P	МР	SP	ZE	SN	MN	LN	
Yaw Rate 1	LP	LP.	Ŀ	LP	LP	SP	LN	LN	
	SP	LP	LP	LP	SP	SN	LN	LN	
^	ZE	LP	LP	SP	ΖE				

Remainder of Rules are Symmetrical to Previous Matrices

Fig. 6 Control rules for n input fuzzy sets LP, SP, and ZE

0.5 in ZE and 0.5 in SP for η . This results in two rules firing: {ZE, SP, ZE} yields δ_C fuzzy set SP and {ZE, SP, SP} yields δ_C fuzzy set LP.

D. Correlation-Minimum Inference Procedure

The correlation-minimum inference procedure [6] is used to determine the degree to which a rule is fired. Each group of three input variable fuzzy sets has itself a corresponding degree of membership in an antecedent group. For degrees of membership of 1.0 in the LP heading angle ψ fuzzy set, 1.0 in the ZE yaw rate r fuzzy set, and 1.0 in the SN lateral offset η fuzzy set, the degree of membership of the input variables taken collectively in the antecedent group {LP,ZE,SN} would be 1.0. The degree to which the rule {LP,ZE,SN;LP} is fired would be 1.0.

The correlation-minimum inference procedure furthermore observes that, for conjunctive (i.e., AND) related component antecedents, the greatest degree of membership that ψ , r, and η have collectively in an antecedent group shall be the smallest of the individual degrees of membership of the antecedent group's components. Thus if ψ belongs to fuzzy set ZE with degree 1.0, AND r belongs to fuzzy set SN with degree 0.7, AND η belongs to fuzzy set LP with degree 0.8, the degree to which ψ , r and η belong collectively to the antecedent group {ZE,SN,LP} would be 0.7. The degree to which the rule {ZE,SN,LP;SP} is fired would be 0.7.

The firing of a rule yields a commanded rudder angle $\delta_{\rm C}$ fuzzy set degree of membership. The membership functions for the δ_{c} output fuzzy sets are defined in Fig. 7. The controller may fire as many as eight (2x2x2) rules for any input group (w.r.n). Thus it may produce as many as eight output commanded rudder angle δ_c fuzzy set degrees of membership. The degree to which each rule fires determines the contribution of its output fuzzy set. The controller clips the output commanded rudder angle δ_C fuzzy set's triangle or trapezoid membership function at the value of its associated degree of membership. Thus in the above example, the controller would clip the SP commanded rudder angle δ_c fuzzy set membership function at a height of 0.7. As many as eight clipped or unclipped output fuzzy set membership functions provide the basis for the final commanded rudder angle $\delta_{\rm c}$.

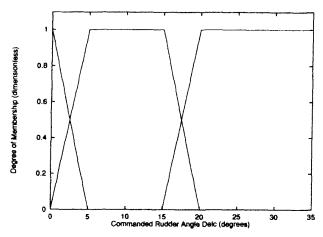


Fig. 7 Membership functions for commanded rudder angle δ_c

E. Defuzzification

To get the final crisp commanded rudder angle δ_c from the clipped output fuzzy set membership functions, the controller performs the process called defuzzification [6].

Here, the controller computes a weighted average of the (up to eight) active output fuzzy set membership functions. Specifically, for each output fuzzy set membership function (clipped or unclipped), the controller determines the area and centroid and then calculates the centroid of the sum of the areas as illustrated in Fig 8. This centroid is the crisp output commanded rudder angle $\delta_{\rm C}$.

F. Summary of Control Algorithm

In summary, the controller proceeds through the following steps:

- 1. Determines all possible non-zero fuzzy set memberships of ψ , r, and η .
- 2. Determines all possible active antecedent groups.
- Determines the minimum degree of membership associated with each antecedent group.
- 4. Fires the rule corresponding to each antecedent group and identifies the output commanded rudder angle δ_c fuzzy set and degree of membership.
- Clips each output commanded rudder angle δ_C fuzzy set membership function at the minimum degree of membership of the three antecedent group components.
- Calculates the centroid of the sum of the output commanded rudder angle δ_C clipped triangle or trapezoid membership functions. This centroid is the crisp output commanded rudder angle δ_C.

The Fuzzy Associative Memory Rules establish discrete levels of control for particular combinations of input variables (ψ,r,η) . Expert knowledge or physics can aid in defining these rules. The combined process of fuzzification, rule

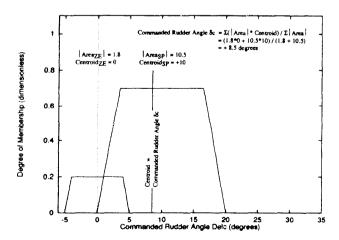


Fig. 8 Output membership function clipping and centroid process

firing, correlation-minimum inferencing, and defuzzification then have the effect of interpolating these levels of control to give a smoothly varying nonlinear control law for the input space. In this case, this is in three dimensions. Here the resulting control law can be viewed as control surfaces when one of the control inputs is set to a constant. Fig. 9 shows the global control surface for yaw rate r = 0. Fig. 10 shows a magnification of the local region of this control surface for yaw rate r = 0 near the desired equilibrium point $(\psi, r, \eta) = (0,0,0)$. Fig. 10 shows the global control surface for yaw rate r = -0.3.

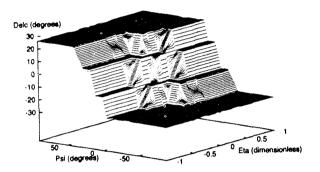


Fig. 9 Global control surface for yaw rate r = 0

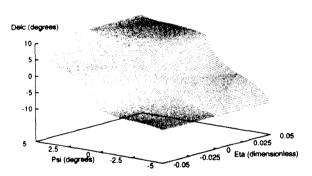


Fig. 10 Local regulator control surface for yaw rate r = 0

V. SIMULATION RESULTS

To provide an initial evaluation of the effectiveness of the fuzzy ship path controller, we have simulated the $Tokyo\ Maru$ in two cases: first, an initial startup transient where the vessel begins offset one beam from the desired track; and second, a passing ship situation where the ships pass beam-to-beam at nondimensional time t'=7. We use the conventional LQG controller defined in Section II as a benchmark.

To evaluate the fuzzy ship path controller, we replace the optimal control law $\delta_c = C_x \mathbf{x}$ with the fuzzy control law

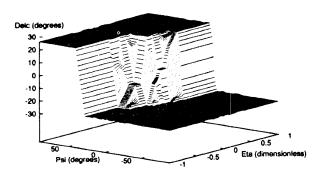


Fig. 11 Global control surface for yaw rate r= -0.3

 $\delta_{\rm C} = {\rm f_Z}(\underline{x})$. Both cases are subject to measurement noise and the Kalman filter is used to process the noisy measurements to produce the state input to the controller. The real value of the fuzzy controller is its ability to operate in large maneuvers where the system behavior is nonlinear. In these initial comparisons, however, the maneuvers are small (the regulator problem) and the classical LQG controller should be in its intended operating range. Further, a simulation using the linearized equations of motion will be valid.

A. One Beam Offset Initial Startup Simulation

The Tokyo Maru was simulated to begin with the initial state $\mathbf{x}^{\mathrm{T}} = (0,0,0,0.16,0)$ which placed the ship about one beam to the right of the nominal path. The resulting lateral offset η histories are shown in Fig. 12. The solid line is the ship with the fuzzy logic controller; the dashed line is with the optimal controller. The ship reaches the nominal path at five ship lengths and quickly settles to the path with either controller. The fuzzy logic controller performance is comparable to that of the optimal controller. The small initial oscillation with the optimal controller results because the rudder first produces an excessive yaw rate which yields a counter feedback. As the system dynamics integrate further. this initial overshoot in yaw rate is eliminated and the vessel proceeds with its turn. It should be noted here that LQG controllers using shaping filters so that the Kalman filter also estimates the external lateral force and moment do not exhibit this initial oscillation [1].

The associated actual rudder angle δ histories are shown in Fig. 13. The fuzzy logic controller uses less and more logically consistent rudder. It responds somewhat more to the noisy inputs after the initial startup transient has died out. The initial use of correct and then counter rudder is evident in the optimal controller response.

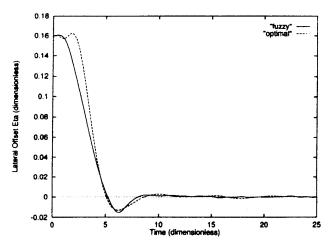


Fig. 12 Lateral offset η with $\eta = 0.16$ startup

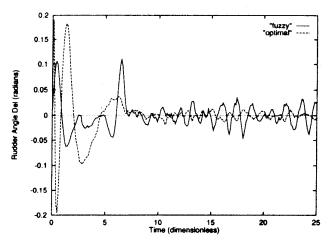


Fig. 13 Rudder angle δ with $\eta = 0.16$ startup

B. Passing Ship Simulation

The Tokyo Maru was simulated to begin with the initial equilibrium state $\mathbf{x}^T = (0,0,0,0,0)$ with another ship passing beam-to-beam at nondimensional time t' = 7. The resulting external hydrodynamic force and yaw moment are shown in Fig. 2. These represent fairly small disturbances for this particular ship. The resulting lateral offset η histories are shown in Fig. 14. Again, the solid line is the ship with the fuzzy logic controller; the dashed line is with the optimal controller. The fuzzy logic controller responds much better to this external disturbance. The resulting excursion from the nominal path is less than 3m which is about half that experienced with the optimal controller.

The associated actual rudder angle δ histories are shown in Fig. 15. As expected, the fuzzy logic controller utilizes more rudder to provide the improved lateral offset response. A maximum of 8 degrees of starboard rudder is used. The optimal controller uses a maximum rudder of only about 6 degrees.

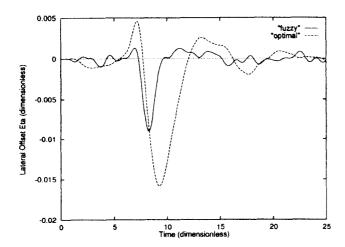


Fig. 14 Lateral offset η with design passing ship disturbance

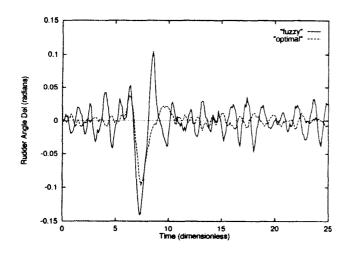


Fig. 15 Rudder angle 8 with design passing ship disturbance

VI. CONCLUSIONS

A fuzzy logic surface ship path controller has been developed. Its performance has been compared to that of a conventional LQG controller. In both cases, a Kalman filter was used to process the noisy measurements. In an initial offset startup transient the performance was almost identical. When the ship is subjected to a passing ship external hydrodynamic force and moment disturbance, the fuzzy logic controller exhibited superior performance. The nonlinear fuzzy logic controller can also handle a much large range of maneuvers effectively. The study of its performance in larger maneuvers requiring a nonlinear simulation model is continuing.

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