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MODEL AND CALIBRATION OF A DIESEL ENGINE AIR PATH WITH AN ASYMMETRIC TWIN SCROLL TURBINE

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ABSTRACT

A control-oriented model and its associated tuning methodology is presented for the air path of a six cylinder 13 L *diesel engine equipped with an asymmetric twin-scroll turbine, wastegate (WG), and exhaust gas recirculation (EGR). This model is validated against experimental engine data and shows good agreement. The small scroll of the asymmetric twin scroll turbine is fed by the exhaust of three cylinders via a split manifold that operates at higher pressure than the exhaust manifold feeding the larger turbine scroll. The asymmetric design with the high exhaust back pressure on three of the six cylinders gives the necessary EGR capability, with reduced pumping work, but leads to complex flow characteristics. The mean-value model describes the flows through the engine, the flow through the two turbine scrolls, the EGR flow, and the WG flow as they are defined, and defines the pressure of the manifolds they connect to. Using seven states that capture the dynamics of the pressure and composition in the manifolds and the speed of the turbo shaft, the model can be used for transient control, along with set point optimization for the EGR and WG flows for each speed and load condition. The relatively low order of the model makes it amenable to fast simulations, system analysis, and control design.*

NOMENCLATURE

VARIABLES AND QUANTITIES

A	Area (m^2)	AF	Air fuel ratio (-)
BSR	Blade-speed ratio (-)	F	Burned gas frac. (-)
J	Inertia (kgm^2)	l	Length(m)
M	Torque (N m)	N	Speed (rpm)
P	Pressure (Pa)	q_{hv}	Fuel heat val. (J/kg)
R	Gas const. (J/kgK)	r_c	Compr. ratio (-)
r	Asymmetry ratio (-)	c_p	Heat capacity (J/kgK)
T	Temperature (K)	V	Volume (m^3)
W	Mass flow (kg/s)	γ	Spec. heat ratio (-)
η	Efficiency (-)	Π	Pressure ratio (-)
ρ	Density (kg/m^3)	Ψ	Flow parameter (-)
ω	Angular vel. (rad/s)		

SUBSCRIPTS

a	Air	amb	Ambient
c	Compressor/Combustion	cyl	Cylinder
d	Displacement	EGR	Exhaust gas recirc.
e	Engine	ei	Engine/cylinder in
em	Exhaust manifold	ems	Small scroll ex. man.
eml	Large scroll ex. man.	eo	Engine/cylinder out
ex	Exhaust	f	Fuel
fr	Friction	im	Intake manifold
in	Into the volume	l	WG pneumatic line
out	Out of the volume	p	Pumping
s	Stoichiometric/Spring	t	Turbine
tc	Turbocharger	tl	Large turbine scroll
ts	Small turbine scroll	u	Control signal
vol	Volumetric	WG	Wastegate
,c	Corrected for oper. pt.		

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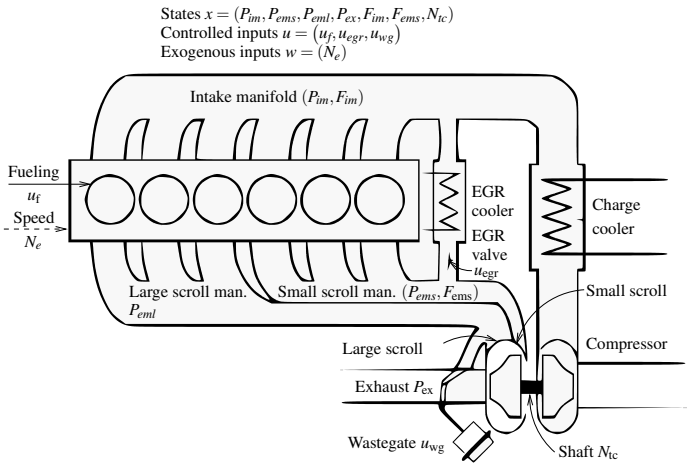

FIGURE 1. SCHEMATIC OF THE ENGINE AND OVERVIEW OF THE MODEL STATES, CONTROLLED INPUTS, AND EXOGENOUS INPUTS.

1 INTRODUCTION

A control-oriented model of the air path of a 13 L heavy duty diesel engine, represented by the schematic in Fig. 1, is presented. The engine is equipped with an electrically actuated exhaust gas recirculation (EGR) valve that diverts gas from the exhaust manifold into the intake manifold. This cooled EGR leads to lower peak in-cylinder temperatures and accordingly lower production of harmful nitrogen oxides [1,2]. To achieve the necessary EGR rates, sufficient back pressure is required. This increases the pumping work and conflicts with the air demand for smoke limitation for a single-entry turbine design [2]. To overcome these limitations, the considered engine is equipped with an asymmetric twin-scroll turbine [3,4]. The exhaust manifold is constructed so that the flow from three of the cylinders feeds into a small turbine scroll and the flow from the other three cylinders feeds into a larger scroll. The higher pressure in the small scroll manifold drives high-pressure EGR, through an EGR cooler, to the intake manifold. The pressure in the large scroll manifold, which is designed for the air demand, is lower and allows three of the cylinders to operate with reduced pumping work. The net effect is typically reduced overall pumping, and a corresponding improvement in fuel economy. The engine is also equipped with a pneumatically actuated wastegate valve, which bypasses the large scroll.

Control-oriented models have been formulated in [1, 5] for engines using single-entry turbines. The model presented here expands on the models described in [1, 2, 6] to account for the asymmetric turbine design, which creates complex flow characteristics due to the strong interaction between the flows [3, 4]. The model is built by dividing the engine into components and deriving sub-models, which are physics based when possible, and tuned individually to experimental steady-state data. Two parameters in the composite model are further tuned to give good agreement between data and the final model in the closed-loop simulation.

2 MODEL STRUCTURE

The model presented here is a control-oriented, mean-value, cycle-averaged, and lumped parameter model, which accounts for the air path behavior by parameterizing models for each of the components individually. The model components are the intake manifold, both exhaust manifolds, post-turbine exhaust manifold, cylinders, EGR path, and turbocharger. The cylinder component includes models for the inflow, produced torque, and exhaust temperature. The turbocharger model includes the compressor, shaft intertia, twin-scroll turbine, and wastegate. After parameterizing the components, they are assembled into a composite model, which then undergoes some additional tuning.

The composite model is a nonlinear continuous time model,

$$\frac{dx}{dt} = f(x, u, w) \tag{1}$$

where the vectors x are the states, u the controlled inputs, and w the exogenous input that cannot be controlled by the engine control unit (ECU). The states are

$$x = (P_{im}, P_{ems}, P_{eml}, P_{ex}, F_{im}, F_{ems}, N_{tc}) \tag{2}$$

where the symbols denote, in order, the intake manifold pressure, the small scroll exhaust manifold pressure, the large scroll exhaust manifold pressure, the exhaust system pressure, the burned gas fraction in the intake manifold, the burned gas fraction in the small scroll exhaust manifold, and the turbocharger shaft speed. The states are also shown in Fig. 1. The control inputs are

$$u = \left(u_f, u_{egr}, u_{wg}\right) \tag{3}$$

where the fueling control input is denoted by u_f, and the EGR and WG valves are denoted by u_{egr} and u_{wg}, respectively. The EGR and WG valves can take values in the range $u \in [0, 1]$ with 0 indicating that the valve is fully closed and 1 indicating that it is fully open. The exogenous input w is the engine speed, N_e, which is assumed to be slowly varying and available as a measurement.

2.1 Tuning

Tuning of the parameters in each sub model is performed by minimizing the sum of squared prediction errors. For the measurement vector y and the vector of values predicted by the model $\hat{y}$ the model fit is evaluated based on the relative error R, which for an operating point k is

$$R(k) = \frac{y(k) - \hat{y}(k)}{\bar{y}} \tag{4}$$

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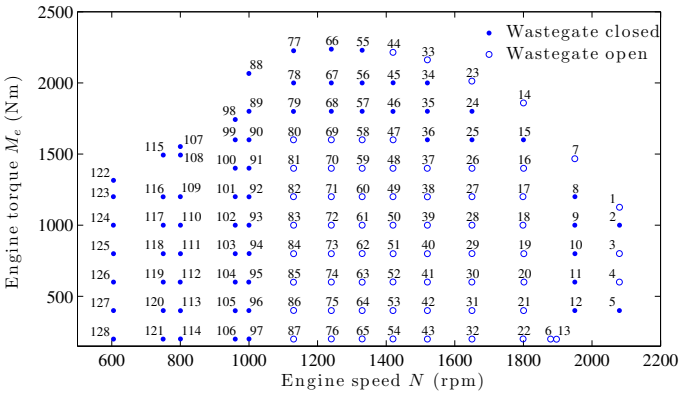

FIGURE 2. THE ENGINE SPEED AND TORQUE FOR THE OPERATING POINTS IN THE STEADY-STATE MEASUREMENTS, DATA SET A. THE DOTS ARE WITH THE WASTE GATE CLOSED (DATA SET A') AND THE CIRCLES ARE WITH THE WASTEGATE OPEN.

where $\bar{y}$ is the average of all steady state measurements. This reduces the impact of errors for points with low-valued measurements. The average and maximum of the absolute value of R over all operating points k is finally used to judge the model fit.

Due to the strong interactions and the inherent feedback via interacting flows, small errors are compounded when putting the components together. This is handled by systematically tuning parameters for the complete model, as described in Sec. 7.

2.2 Experimental Data sets

The first data set, set A, is an engine map with 128 experimentally determined steady state operating points, provided by Detroit Diesel. An overview of this data is given in Fig. 2. A subset of data set A, denoted by A', where the wastegate is closed, is used for tuning the turbine model. A different subset of data set A, denoted by A", where the wastegate is open is used for fitting the wastegate flow model. The subsets A' and A" are disjoint and fully span A. The compressor model is tuned using flow bench data from the turbocharger manufacturer, which is set B. The model for the wastegate opening uses a data set with two sweeps of the wastegate actuator while keeping all other actuators fixed.

3 MANIFOLD VOLUMES

Six of the model states represent pressure and composition dynamics in the various engine system volumes. These volumes are the intake and exhaust manifolds and the post-turbine exhaust manifold.

The dynamic models for these systems are based on the standard "filling and emptying" models [7, Ch. 14.2–14.3] and are assumed to be isothermal. Assuming the ideal gas law and mass conservation hold, the state equations are

$$\frac{dP}{dt} = \frac{RT}{V}\left(\sum W_{in} - \sum W_{out}\right) \tag{5}$$

$$\frac{dF}{dt} = \frac{RT}{PV}\left(\sum W_{in}(F_{in} - F)\right) \tag{6}$$

for burned gas fraction F, pressure P, ideal gas constant R, and inlet temperature T of each volume. The burned gas fraction is defined as the mass fraction of combustion products, excluding excess oxygen, in the mixture. R for the intake manifold is assumed to be for air; R for the exhaust manifolds is based on a typical composition for the exhaust products. The intake manifold has inlet temperature T_{im} which, as in [8], is assumed to be constant. The exhaust manifold temperature is calculated in Sec. 4.3. The summations are over the flows, denoted by W. The intake manifold has inflows from the EGR cooler and from the charge cooler. Both of these coolers are assumed to be ideal and, as in [5], to not impact the mass flow through them, except in temperature. These flows can thus be represented by the flow through the EGR (W_{egr}, calculated in Sec. 5) and compressor (W_c) . The intake manifold has outflow to the engine inlet (W_{ei}), which is determined via volumetric efficiency in Sec. 4.1. The compressor flow consists of air ($F_c = 0$), so the only non-zero F_{in} term comes from the EGR flow. The small scroll exhaust manifold has inflow from the connected engine cylinder bank (W_{ems}) and outflows through the EGR (W_{egr}) and small turbine scroll (W_{ts}). The input composition, F_{in}, to the small scroll exhaust manifold composition state, F_{ems}, is the engine out burned gas fraction

$$F_{eo} = \frac{(1+\mathrm{AF}_s)W_f + F_{im}W_{ei}}{W_f + W_{ei}} \tag{7}$$

where AF_s is the stoichiometric fuel ratio, assuming a lean operation. The large scroll exhaust manifold has inflow from the connected engine cylinder bank (W_{eml}) and outflows through the WG (W_{wg}) and large turbine scroll (W_{tl}). The flows through the wastegate and turbine are described later in Sec. 6. Finally, the exhaust system has inflow from the turbine ($W_t = W_{ts} + W_{tl}$) and WG (W_{wg}). Given the low pressure drop and flow velocity, the exhaust outflow is obtained using the standard incompressible flow model, as is done in [6].

4 CYLINDER MODEL

The engine inflow, outflow, torque, and temperature are calculated for the entire engine, which corresponds to the cycle-mean-value, treating the engine as a pump, rather than cylinder-by-cylinder.

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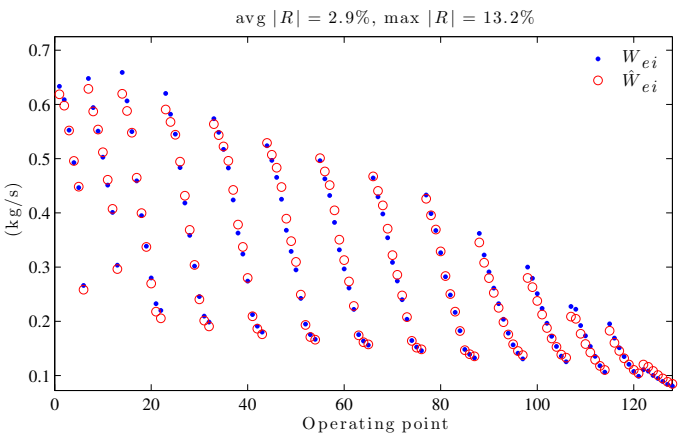

FIGURE 3. CYLINDER INFLOW MODEL ($\hat{W}_{ei}$) VS. DATA (W_{ei})

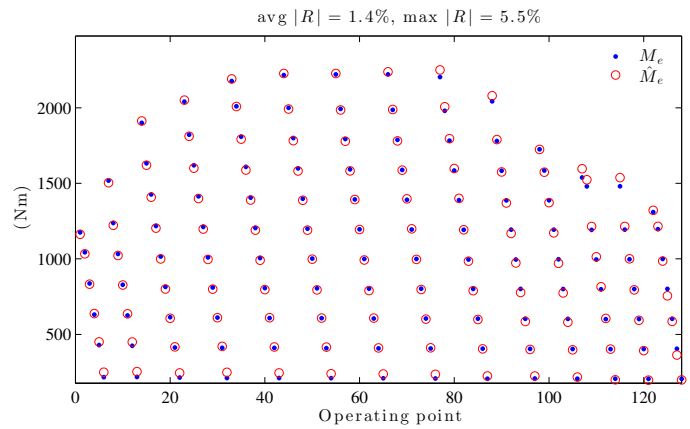

FIGURE 4. ENGINE TORQUE MODEL ($\hat{M}_e$) VS. DATA (M_e)

4.1 Engine Flow Rate

It is assumed that the flow is symmetric with respect to each set of three cylinders connected to the small scroll and large scroll exhaust manifolds. This assumption is made based on the fact that the residual gas fraction trapped in this high compression ($r_c = 17$) engine is very small. Hence the difference of the gas fraction, and consequently the inflow, between the two sets of cylinders facing different back pressure is estimated to be less than 1.5%.

The flow into the cylinders is calculated through the volumetric efficiency [7, Ch. 6.2],

$$W_{ei} = \eta_{vol} \frac{P_{im} N_e V_d}{120 R T_{im}} \tag{8}$$

with N_e in rpm and V_d is the displacement volume. The volumetric efficiency is given by an empirical regression in engine speed and intake manifold pressure

$$\eta_{vol} = c_0 + c_1\sqrt{N_e} + c_2\sqrt{P_{im}} + c_3 N_e + c_4 P_{im} \tag{9}$$

with c_i being tuning parameters. This parameterization is motivated by [6] which indicates N_e, P_{im} as the leading factors for η_{vol}. Note that (9) does not have a modeled dependence on the exhaust manifold pressure. The outflow from the engine is $W_{eo} = W_{ei} + W_f$. Since the flow into the cylinders is assumed to be equal, the outflow from each cylinder will also be equal, and the small scroll and large scroll exhaust manifold inflows are thus $W_{ems} = W_{eml} = \frac{1}{2} W_{eo}$.

The parameters for volumetric efficiency in (9) are determined by linear least-squares using data set A. The resulting fit for the flow rate in (8) shows good agreement between the modeled $\hat{W}_{ei}$ and the measured W_{ei}, with an average relative error of only 2.9%, as is shown in Fig. 3.

4.2 Engine Torque

The engine torque is modeled by

$$M_e = M_c - M_p - M_{fr} \tag{10}$$

where M_c is the indicated gross torque from combustion, M_p is the pumping torque, and M_{fr} is the friction torque. These are presented in [8] and given by

$$M_c = \frac{3}{2\pi}\left(u_f q_{hv} \eta_{cyl}\left(1 - \frac{1}{r_c^{\gamma_{cyl}-1}}\right)\right) \tag{11}$$

$$M_p = \frac{V_d}{4\pi}\left(\frac{P_{ems} + P_{eml}}{2} - P_{im}\right) \tag{12}$$

$$M_{fr} = \frac{V_d}{4\pi}\left(c_0 + c_1 N_e + c_2 N_e^2\right) \tag{13}$$

where u_f is kg/cycle·cylinder and $q_{hv} = 42.7\,\mathrm{MJ/kg}$. The parameters are the lumped efficiency η_{cyl}, friction parameters (c_0, c_1, c_2), and the specific heat ratio γ_{cyl}.

Data set A is used and the parameters $(\eta_{cyl}, c_0, c_1, c_2)$ are determined from the linear regression obtained when rearranging (10) as

$$M_e + M_p = M_c - M_{fr} \tag{14}$$

where the left hand side is known from measurements and the right hand side contains all the parameters in a linear way. The specific heat ratio γ_{cyl} is chosen based on typical composition and temperature. The model fit is very good, with an average relative error of 1.4% and a maximum error of 5.5%, as shown in Fig. 4.

4.3 Exhaust Temperature

The measured temperatures of the exhaust flowing into each manifold differ in data set A by an average of 9 °C and are therefore assumed to be equal. W_f, AF_{cyl} and the pressure ratio are

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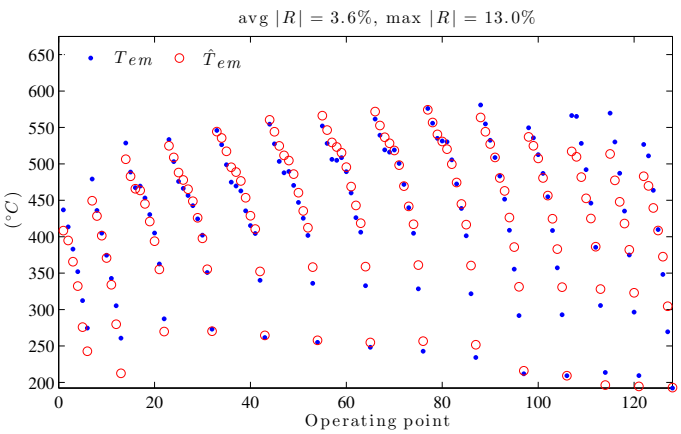

FIGURE 5. EXHAUST TEMP. MODEL ($\hat{T}_{em}$) VS. DATA (T_{em})

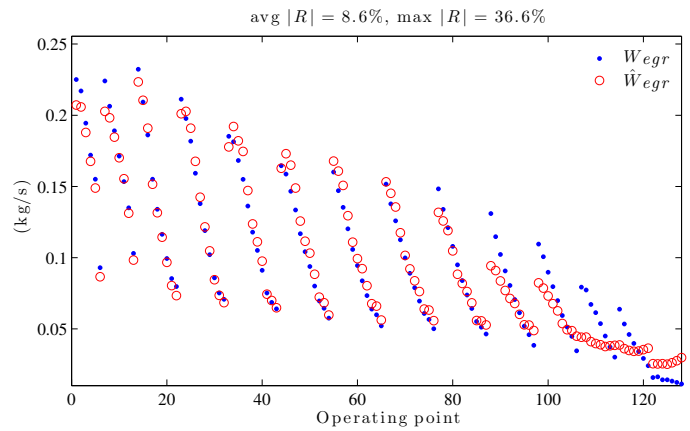

FIGURE 6. EGR FLOW MODEL ($\hat{W}_{egr}$) VS. DATA (W_{egr})

observed to be principal factors in the exhaust temperature. It is therefore modeled by the empirical regression

$$T_{em} = T_{ems} = T_{eml} = c_0 + c_1 \frac{P_{ems} + P_{eml}}{2P_{im}} + c_2 W_f + c_3 \mathrm{AF}_{cyl} \quad (15)$$

where

$$\mathrm{AF}_{cyl} = (1 - F_{im}) \frac{W_{ei}}{W_f} \quad (16)$$

where $W_f = u_f N_e / 20$.

The model in (15) was fit using linear least squares with data set A. The fit had an average error of 3.6% and is shown in Fig. 5

5 EGR FLOW

The EGR flow is modeled using the standard orifice equation for compressible flow presented in [1]

$$W_{egr} = A(u_{egr}) \frac{P_{ems}}{\sqrt{R_{ex} T_{ems}}} \Psi(\Pi) \quad (17)$$

where $\Pi = P_{im}/P_{ems}$. The model is tuned following the approach in [8] but here Ψ is given by (18). Due to pulsation effects, pressure ratios near or above unity are observed. The standard model for Ψ is thus not amenable, as it would predict zero flow or backflow, and a more flexible quadratic model has been used instead

$$\Psi = p_0 + p_1 \Pi^* + p_2 \Pi^{*2} \quad (18)$$

$$\text{where } \Pi^* = \max \left[\left(\frac{2}{\gamma_{ex} + 1} \right)^{\frac{\gamma_{ex}}{\gamma_{ex} - 1}}, \frac{P_{im}}{P_{ems}} \right].$$

The effective area is parameterized using the model in [8]:

$$A = a_0 + a_1 v + a_2 v^2 \quad (19)$$

$$\text{where } v = \min(-\frac{a_1}{2a_2}, u_{egr}) \quad (20)$$

with $(p_0, p_1, p_2, a_0, a_1, a_2)$ being tuning parameters.The effective EGR area parameters are found to satisfy $a_1 > 0$ and $a_2 < 0$. The EGR flow rate model is parametrized by using data set A. In the data set, the EGR rates are determined from the pressure differential over a venturi flow meter placed after the EGR cooler. The flow through an ideal venturi is calculated by

$$W_{egr} = A_2 \sqrt{\frac{2\rho \Delta P}{1 - (\frac{A_2}{A_1})^2}} \quad (21)$$

where (A_1, A_2) are the cross section areas of the inlet and throat of the venturi, ΔP is the measured differential pressure between inlet and throat, and ρ is the gas density. Typically the flow is scaled with a coefficient to account for the real non-ideal flow; however, no flow-bench data were available to enable such a scaling. The geometry of the venturi is known, which gives the areas, and the upstream conditions are used to calculate the density (ρ). At operating points where the engine speed is near or below 1000 RPM, the venturi model does not account well for all the necessary effects, such as flow pulsation. Despite the trend mismatch for low engine speeds, the developed model is shown to predict the flow reasonably well under the range of points tested.

With the EGR flow given by (17), the model parameters are tuned by non-linear least squares minimization. The fit has an average error of 8.6% and is shown in Fig. 6.

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6 TURBO CHARGER

The turbocharger model is comprised of mass flow and efficiency models for the compressor and turbine, a model of the rotational dynamics, and a model of the WG flow. For the wastegate, a model for the opening threshold is formulated that is used to determine data set A' that corresponds to zero WG flow. The modeling for the compressor flow efficiency was done according to the Jensen & Kristensen model [9] without alteration. The average model error for compressor flow is 2.5% and for efficiency is 1.4%. Turbine modeling is ordinarily done similar to the compressor, using the flow and efficiency maps provided by the manufacturer. The maps provided for the turbine only covered conditions where one of the scrolls was blocked, or where both were equally pressurized. This data failed to capture the complex interaction between the flows and was thus not used for modeling of the turbine.

6.1 Turbine flow

As described previously, the turbine consists of two scrolls through which exhaust can flow. The scrolls have a high degree of asymmetry and are thus modeled separately. The novel model presented here is motivated by the work in [4], where flow bench measurements of an asymmetric twin-scroll turbine, similar to the one used here, are analyzed. One conclusion in [4] is that the flow through each scroll behaves like a single-entry turbine when the ratio of the scroll flows is constant.

The flow through the large scroll is obtained by letting the parameters of the single-entry turbine model in [6] vary with an additional asymmetry ratio, r. This ratio is approximated as the ratio of the corrected flow through each of the scrolls

$$r = \frac{W_{ts,c}}{W_{tl,c}}. \tag{22}$$

Given a flow ratio r, the scroll flows are modeled by

$$W_{tl,c} = (d_0 + d_1 r)\sqrt{1 - \Pi_{tl}^{d_2 + d_3 r}} \tag{23}$$

$$W_{ts,c} = r W_{tl,c} \tag{24}$$

where the pressure ratios are given by

$$\Pi_{ts} = \frac{P_{ex}}{P_{ems}}, \quad \Pi_{tl} = \frac{P_{ex}}{P_{eml}}. \tag{25}$$

Since r depends on the flows through each scroll, it is parametrized with an empirical, tri-linear model in terms of the scroll pressure ratios and the shaft speed $(\Pi_{ts}, \Pi_{tl}, N_{tc,c})$ in order to arrive at a causal model.

The model uses the following corrected quantities for mass flows and shaft speed, which are normalized for the temperature

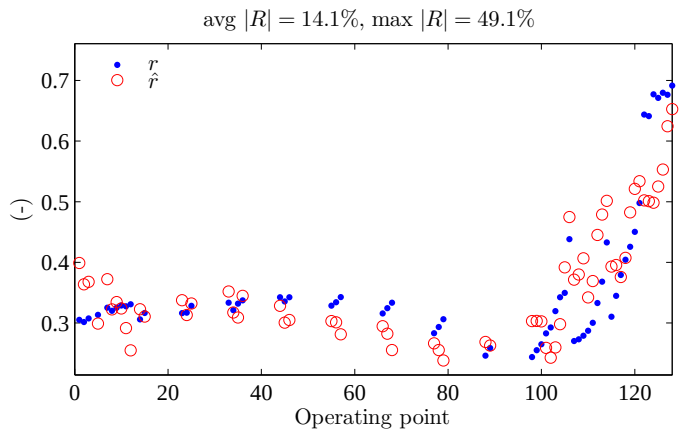

FIGURE 7. ASYMMETRY RATIO MODEL ($\hat{r}$) VS. DATA (r)

and pressure at the operating point

$$W_{ts,c} = W_{ts}\frac{\sqrt{T_{ems}}}{P_{ems}} \quad W_{tl,c} = W_{tl}\frac{\sqrt{T_{eml}}}{P_{eml}} \quad N_{tc,c} = N_{tc}\sqrt{\frac{T_{ref}}{T_{em}}} \tag{26}$$

where T_{em} is the average value of T_{ems} and T_{eml}. The true flows and shaft speed can be found by inverting these relations.

6.1.1 Tuning

The turbine flow model in (23) is parameterized by calculating the turbine flow from data using mass conservation. The following equations hold under such steady state conditions.

$$W_{eo} - W_f = W_{ei} = W_c + W_{egr} \tag{27}$$

$$W_{ems} = W_{ts} + W_{egr} \tag{28}$$

$$W_{eml} = W_{tl} + W_{wg} \tag{29}$$

Since the flows through each bank are assumed to be equal

$$W_{ts} + W_{egr} = \frac{1}{2}(W_c + W_{egr} + W_f) \tag{30}$$

$$W_{tl} + W_{wg} = \frac{1}{2}(W_c + W_{egr} + W_f) \tag{31}$$

where (W_c, W_{egr}, W_f) are known and (W_{ts}, W_{tl}, W_{wg}) are unknown. To fit the turbine flow models, the operating points with closed wastegate are selected (data set A', see Sec. 2.2), which yields

$$W_{ts} = \frac{1}{2}(W_c - W_{egr} + W_f) \tag{32}$$

$$W_{tl} = \frac{1}{2}(W_c + W_{egr} + W_f). \tag{33}$$

Here all the terms on the right hand side are available as measurements or calculations from known values. The asymmetry parameter r is calculated from flow data as the flow ratio given by (22). The 4 parameters in the mass flow model (23) can thus

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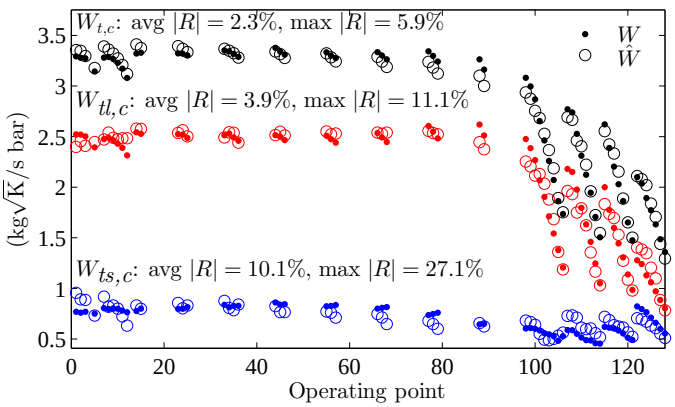

FIGURE 8. TURBINE FLOW MODEL ($\hat{W}$) VS. DATA (W)

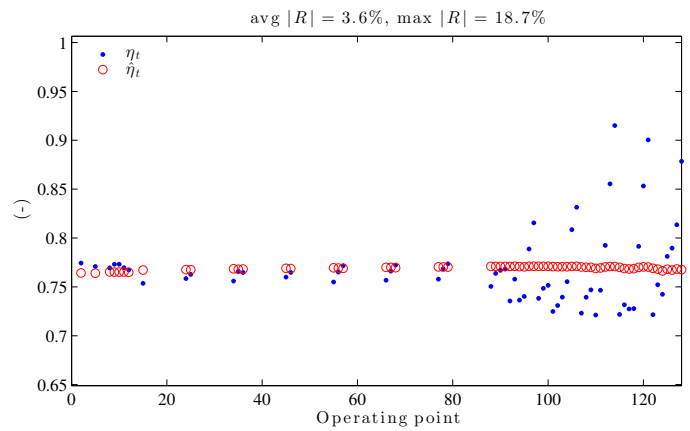

FIGURE 9. TURBINE EFFICIENCY MODEL ($\hat{\eta}_t$) VS DATA. (η_t)

be determined through nonlinear least-squares.

After that, the 8 parameters in the tri-linear model for r are determined by linear least squares. The model agreement for r is shown in Fig. 7. While the relative error is large, the impact of this error on the rest of the system is small. This is shown by the flow model agreement in Fig. 8. Note that the missing operating points in these figures are where the wastegate is open. Moreover, note that although the fit for the small scroll has fairly large relative errors, the absolute values and trends in the total flow are good because the majority of the total flow is defined by the other scroll, and that is well captured.

6.2 Turbine Efficiency

The turbine efficiency η_t is calculated from the model presented in [10, 11], as the second order polynomial in blade speed ratio (BSR), where

$$BSR = \frac{\omega_{tc} r_t}{\sqrt{2 c_p T_{em} \left(1 - \Pi_t^{\frac{\gamma_{em}-1}{\gamma_{em}}}\right)}}. \tag{34}$$

Here ω_{tc} is the angular speed of the turbocharger shaft, r_t is the turbine effective radius, c_p, γ_{em} are thermodynamic constants representative of the gas in the exhaust manifolds. The following mean values are used for upstream temperature and pressure ratio:

$$T_{em} = \frac{T_{ems} + T_{eml}}{2}, \quad \Pi_t = \frac{\Pi_{ts} + \Pi_{tl}}{2}. \tag{35}$$

6.2.1 Tuning Data set A' is also used for tuning the efficiency. The three parameters of the quadratic function in BSR are fit to calculated efficiency using linear least squares. The efficiency is calculated from the data by the ratio of the compressor power and the power from the turbine with ideal efficiency,

$$\eta_t = \frac{W_c c_{p,a}(T_{c,in} - T_{c,out})}{W_t c_{p,em} T_{em} (1 - \Pi_t^{1-1/\gamma_{em}})} \tag{36}$$

where $(T_{c,in}, T_{c,out})$ are the measured compressor inlet and outlet temperatures, and $W_t = W_{ts} + W_{tl}$ is the total turbine flow. This calculation of turbine efficiency includes a mechanical efficiency for the turbocharger, which is assumed close to one. The mean values of (35) are used for T_{em} and Π_t. Constant values for typical composition and temperature are used for the properties $(c_{p,a}, c_{p,em}, \gamma_{em})$. The three parameters of the regression are given by linear least squares. The BSR varies less than 0.15 in the data set, and there is some scatter for lower BSR. Thus, the model can only capture the mean efficiency as seen in Fig. 9.

The efficiency model is used for calculating the temperature drop $T_{em} - T_{ex}$, where T_{ex} is the temperature in the exhaust system after the turbine. To further validate the turbine efficiency model, the measured T_{ex} is therefore compared with the temperature computed from the definition of the turbine efficiency

$$T_{ex} = T_{em} \left(1 + \eta_t \left(\Pi_t^{1-1/\gamma_{ex}} - 1\right)\right) \tag{37}$$

using measurements of (T_{em}, Π_t) and the efficiency model. The results, shown in Fig. 10, show a fairly good fit. The predictions are consistently higher than the measurements and the difference increases with temperature, which is consistent with the gas cooling before it reaches the sensor downstream of the turbine.

6.3 Rotational Dynamics

The turbocharger includes a rotating shaft which connects the turbine to the compressor. Without friction, the shaft acceleration is given by the difference in torque provided by the turbine and the torque consumed by the compressor, and follows from

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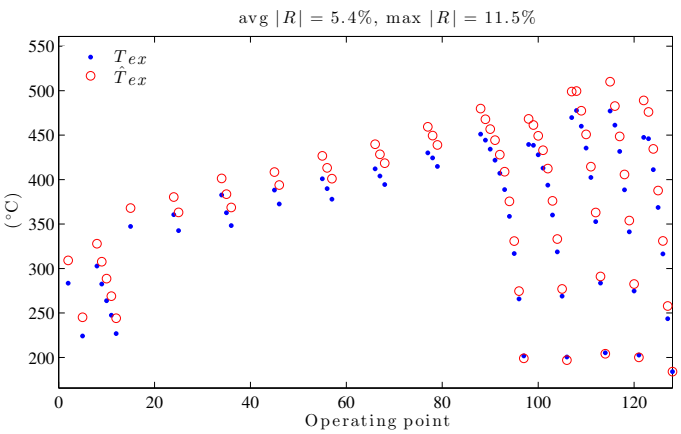

FIGURE 10. POST-TURBINE TEMPERATURE MODEL ($\hat{T}_{ex}$) VS. DATA (T_{ex})

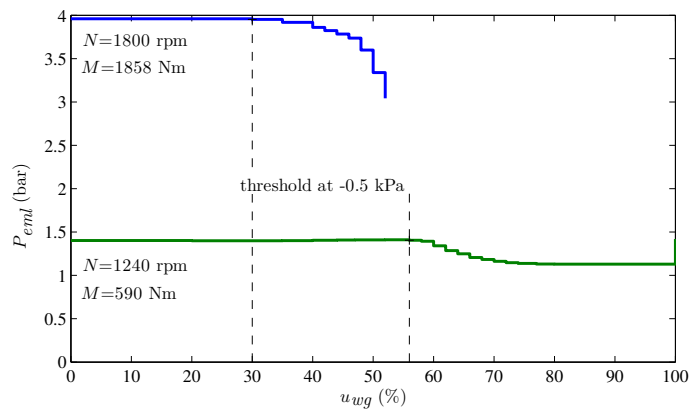

FIGURE 11. SWEEPS OF THE WASTEGATE u_{wg} FOR DETERMINING THE OPENING THRESHOLD OF u_{wg}

Newton's second law

$$J_{tc}\frac{dN_{tc}}{dt} = M_t - M_c \tag{38}$$

where N_{tc} (rad/s) is the shaft speed. The turbine and compressor torques are given by

$$M_t = W_t c_{p,e} T_{em} \eta_t \left(1 - \Pi_t^{1-1/\gamma_{ex}}\right)/N_{tc} \tag{39}$$

$$M_c = W_c c_{p,a} T_{amb} \left(\Pi_c^{1-1/\gamma_a} - 1\right)/(N_{tc}\eta_c) \tag{40}$$

where $W_t = W_{ts} + W_{tl}$ is the flow through both scrolls and (T_{em}, Π_t) are the temperature and pressure ratios averaged for the two exhaust manifolds given in (35).

6.4 Wastegate opening

The wastegate allows exhaust gas to bypass the turbine, thus decreasing the pressure in the exhaust manifold without boosting the intake manifold. Since there are no sensors to measure the flow through the wastegate or the turbine, if the wastegate is open, it is not possible to determine the flow through the large turbine scroll or the wastegate independently. A model of the wastegate opening condition is thus needed to parameterize the model developed in Sec. 6.1. The wastegate valve is governed by a pneumatic actuator which acts to open the valve and a spring which acts to close it. The opening condition is therefore given by the torque balance

$$T_s = A_1 l_1 (P_l u_{wg} - P_{amb}) + A_2 l_2 (P_{eml} - P_{ex}) \tag{41}$$

where T_s is the torque from the spring, (A_1, l_1) are the corresponding area and length for the wastegate actuator, P_l is the constant line pressure, (A_2, l_2) are the valve area and valve moment arm, and P_{ex} is the pressure in the exhaust system after the turbine. With $k = (A_1 l_1)(A_2 l_2)^{-1}$, (41) is written as

$$P_s = k(P_l u_{wg} - P_{amb}) + P_{ems} - P_{ex} \tag{42}$$

where $P_s = {}^{T_s}/_{A_2 l_2}$ is the equivalent pressure due to the spring and the constant (P_s, k) are determined from two sweeps of the wastegate, shown in Fig. 11. In the figure, the wastegate is said to open when the P_{eml} has reduced by 0.5 kPa. With this model, if u_{wg} is below or equal to the threshold

$$u_{wg}^* = \frac{P_s - (P_{ems} - P_{ex})}{kP_l} + \frac{P_{amb}}{P_l} \tag{43}$$

the wastegate is closed and no flow is passing through. The state of the wastegate in data set A is shown in Fig. 2.

6.5 Wastegate Flow

The wastegate flow is obtained from the standard orifice compressible flow model in [6]

$$W_{wg} = A(u_{wg}) \frac{P_{eml}}{\sqrt{R_{ex} T_{eml}}} \Psi(\Pi) \tag{44}$$

with the standard parameterization of Ψ. Motivated by trends in experimental data, and by the geometry of the valve, the area is modeled as a sigmoid function of the WG actuator signal

$$A_{wg} = c_0 + c_1 \tanh\left(c_2(\bar{u}_{wg} - c_3)\right) \tag{45}$$

where $\bar{u}_{wg} = u_{wg} - u_{wg}^*$. The effective area in (45) is modified slightly by adding a linear region below a threshold to ensure that the curve passes through the origin.

6.5.1 Tuning

This model was parameterized with a steady state mass-flow balance. In steady state, the flow through

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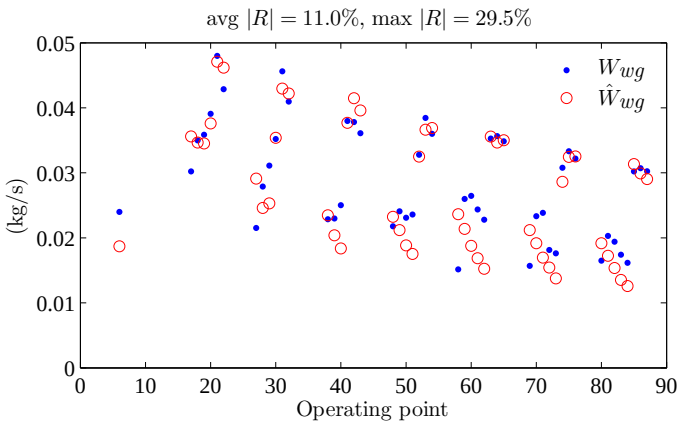

FIGURE 12. WASTEGATE FLOW MODEL ($\hat{W}_{wg}$) VS.DATA (W_{wg})

the wastegate can be found by

$$W_{wg} = W_c + W_f - \hat{W}_t \tag{46}$$

where $\hat{W}_t$ is the total predicted turbine flow. Equation (44) can then be inverted to solve for the calculated effective area. Because of the uncertainty in the turbine model, points with calculated $W_{wg} < 0.015\text{kg/s}$ were omitted from the fitting; the remaining points were used to parameterize the model using a nonlinear least squares method. Figure 12 shows the agreement between the calculated flow value and the predicted value from the model, for the points used to fit the model. This model captures the trends in the data well, though it still has moderately large average relative error.

7 COMPLETE MODEL TUNING

Table 1 shows the data set used, avg $|R|$, and max $|R|$ for each component in the model, where R is the relative error defined in (4). The average error is 4% or lower for all components except for the EGR, WG, and small turbine scroll flows, which have an average $|R|$ of 8.6%, 11.0% and 10.1% respectively. These components also show the largest maximum $|R|$ of 36.6%, 29.5%, and 27.1%, respectively. The turbine efficiency model also stands out compared to the other components with a maximum $|R|$ of 18.7%. The rest of the components have maximum $|R|$ in between 5% and 13.2%.

Each of the components in the engine model interacts with the others in a complicated way that will compound errors in each of the components. As such it is important to check the closed loop behavior of the whole model, and to retune as appropriate to minimize the resulting errors.

Connecting the model without any returning resulted in large relative error in most of the states. As such, the full model was tuned to minimize the sum of squared errors of the 5 measured states. This was done in two steps. The first step was to scale the

Component	Source	Avg.	Max.
Cylinder inflow	A	2.9	13.2
EGR flow	A	8.6	36.6
Engine torque	A	1.4	5.5
Exhaust temperature	A	3.6	13.0
Compressor flow	B	2.5	13.1
Compressor efficiency	B	1.4	7.9
Small turbine scroll flow	A'	10.1	27.1
Large turbine scroll flow	A'	3.9	11.1
Turbine efficiency	A'	3.6	18.7
Wastegate flow	A"	11.0	29.5

TABLE 1. AVERAGE AND MAXIMUM $|R|$ (%) FOR EACH SUB MODEL.

Operating Points	Source	P_{im}	P_{ems}	P_{eml}	P_{ex}	N_{tc}
Initial Model	A'	12.5	18.1	5.5	0.6	8.3
Tuned Model	A'	3.9	5.8	4.5	0.6	4.8
Initial Model	A"	3.4	4.2	4.9	0.9	4.3
Tuned Model	A"	3.2	4.9	4.3	0.8	3.5
Final Model	A	3.6	5.4	4.4	0.7	4.2

TABLE 2. SUMMARY OF FULL MODEL TUNING COMPONENT AVERAGE $|R|$

turbine efficiency (η_t) for points with closed wastegate. The best fit was found by scaling η_t by 0.908. The wastegate flow (W_{wg}) was then similarly scaled for open wastegate points. The best fit was found at a scaling of 0.883. These parameters were chosen for tuning because they were part of the turbocharger component, which was the least certain. They also had moderate individual uncertainties and strong authority on the model. The retuning process is summarized in Table 2 which compares the errors in the 5 measured states at each stage of the tuning process. Figure 13 shows the agreement between the measured states and the predicted states from the model. The overall fit is good and includes operating points for which the wastegate is open and for which it is closed.

8 CONCLUSIONS

Asymmetric twin scroll turbines are an important component for balancing EGR-based NO_x reduction with efficiency; however, they create complex, interdependent flow characteristics. Previously developed techniques in engine modeling are here extended to account for this class of turbines, and a control-oriented, mean value model of a heavy duty diesel engine equipped with EGR, WG, and an asymmetric turbine is presented. This model is developed component-wise for the engine, and special attention is given to the flows of the turbine. Two

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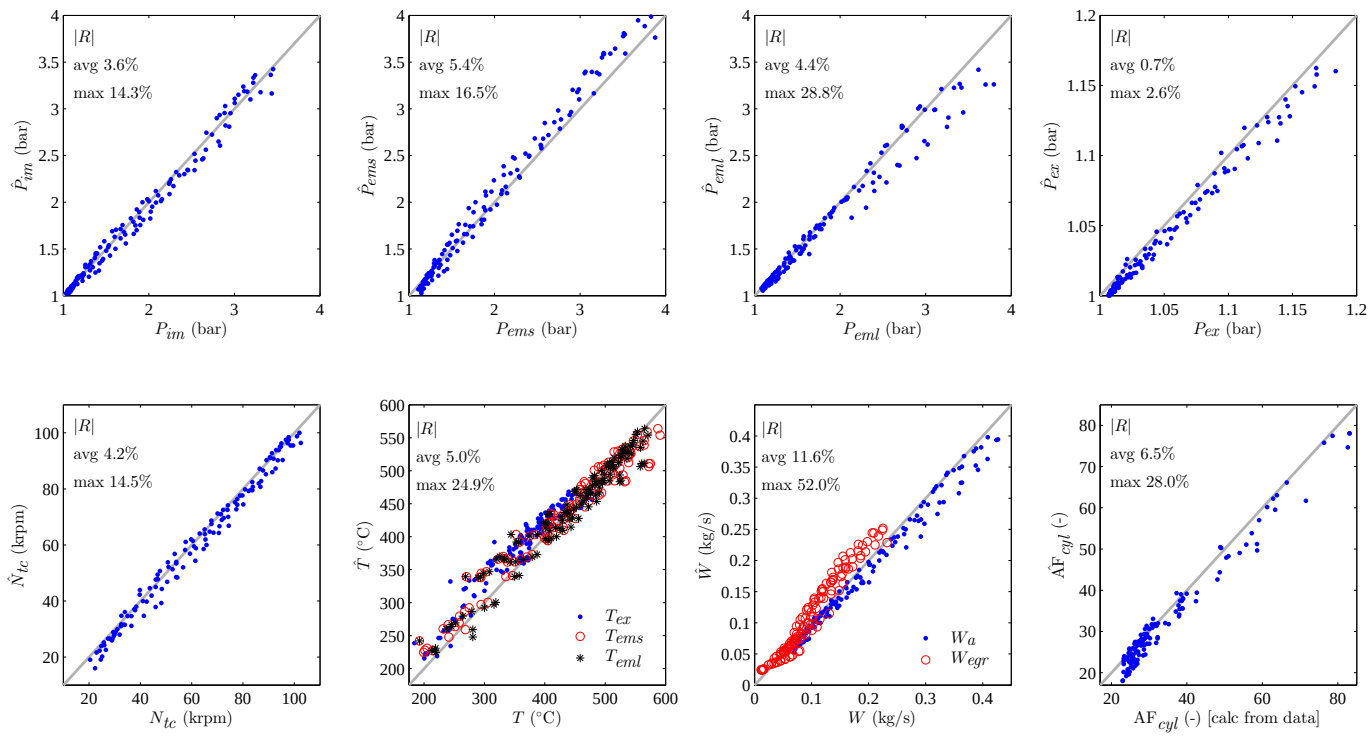

FIGURE 13. COMPARISON OF EXPERIMENTAL DATA TO THE MODEL PREDICTIONS. FOR THE MEASURED VARIABLE X THE MODEL PREDICTION IS DENOTED BY $\hat{X}$.

parameters are then systematically tuned in the composite model to ensure a good fit to data for the final model. The developed model shows good agreement with steady state measurements. This dynamic model is amenable to model-based control and diagnosis, which is the subject of future work.

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