Extremum Seeking Control for Soft Landing of an Electromechanical Valve Actuator [*]

Katherine S. Peterson [a], Anna G. Stefanopoulou [a],

[a] *Mechanical Engineering*
University of Michigan
Ann Arbor, MI

Abstract

Many electromagnetic actuators suffer from high velocity impacts. One such actuator is the electromechanical valve actuator, recently receiving attention for enabling variable valve timing in internal combustion engines. Impacts experienced by the actuator are excessively loud and create unnecessary wear. This paper presents an extremum seeking controller designed to reduce the magnitude of these impacts. Based on a measure of the sound intensity at impact, the controller tunes a nonlinear feedback to achieve impact velocities of less than 0.1 m/s while maintaining transition times of less than 4.0 ms. The control strategy is implemented with an eddy current sensor, to measure the valve position, and a microphone.

Key words: Electromagnetic Devices; Nonlinear Control; Extremum Seeking Control.

1 Introduction

Electromechanical (EM) actuators enable reliable control, and are quickly replacing hydraulic and pneumatic actuators due to their compactness and flexibility. Electromagnets bridge the gap between electrical and mechanical systems by inducing current in a coil of wire to give rise to a magnetic force. This force is then used to affect the motion of a physical component within the actuator or system being controlled. An advantage of using EM actuators is that the applied force is non-contacting and the actuator dynamics are often significantly faster then the dynamics of the system being controlled. EM actuators are found in a wide range of applications including those with transitional (linear) motion, large induced forces, and contacts between the activating and actuating components. EM actuators provide critical functionality in emerging technological areas such as bio-engineering and telecommunication routing devices. In bio-engineering an EM actuator controls intraventricular balloons to simulate a beating heart [4] and implantable drug delivery system [3]. Optical switches [1] used in controlling telecommunication traffic are ac-

tivated using EM or electrostatic actuation. In mature technological areas such as process and automotive industry EM actuation enables new designs. For example, electromagnets have been introduced for flow and distributed mixing control via throttling during material processing [5].

Of particular interest are electromechanical valve actuators (EVA) for use in achieving variable valve timing (VVT) in automotive engines. By de-coupling the valve timing from the piston motion, VVT allows greater flexibility in engine operation, potentially leading to improved fuel economy (18% to 23% [13]), emissions (12% to 15% [16]), and torque (20% [6]).

Impacts experienced by the moving components of the EVA create unnecessary wear and are excessively loud. Wear from repeated collisions could lead to actuator failure, valve breakage, or compromise sealing of the cylinders. Even if these materials issues could be addressed, the sound intensity produced by high velocity impacts is unacceptable.

The EVA, shown in Fig. 1, consists of an armature mounted between two opposing sets of springs and electromagnets. The pre-load of each spring is set such that they are both always in compression and the armature will come to rest midway between each electromagnet when no coil voltage is applied.
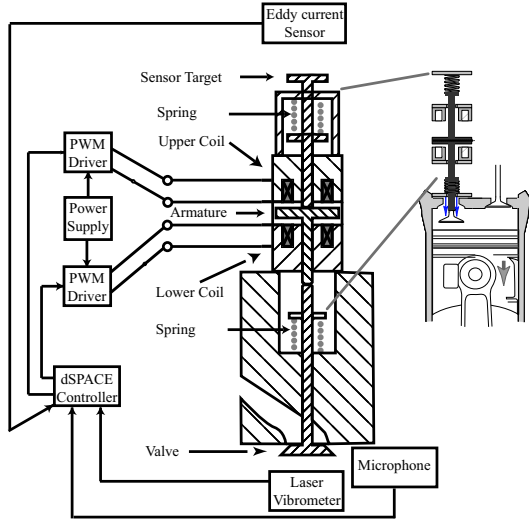
Fig. 1. Electromechanical valve actuator and experimental setup.

Valve motion is controlled via the voltage applied to the upper and lower magnetic coils as follows. Initially the armature is held against the upper magnetic coil causing the valve to rest in the closed position, creating an imbalance in the net spring force acting on the armature. When the voltage applied to the upper magnetic coil is reduced to zero, the potential energy in the springs drives the armature across the gap. A catching voltage is then applied to the lower magnetic coil to capture and hold the armature, thus opening the valve. The reverse process is used to close the valve.

Minimizing wear and noise, while allowing for operation at high engine speeds, requires that:

- the impact velocity between the armature and magnetic coil be less than 0.1 m/s. In the case of multiple impacts, the impact velocity is taken to be the largest one.
- the transition time, from when the valve is 98% open/closed to 98% closed/open, be less than 4.0 ms.

Achieving these requirements is referred to as "soft landing".

Control techniques that address soft landing can be divided into compensation either during a single valve event or based on past valve events, with many strategies using a combination of the two. A relationship between velocity and the ratio of current to the rate of change of current was derived by Butzmann [2]. By fixing this ratio, the authors are able to effectively control the impact velocity. The methodology itself is quite simple and does not require a position sensor, both of which are desirable for implementation. However, the lack of a position sensor may jeopardize robustness when unknown gas forces are applied to the valve. LQR observer based control is introduced by Hoffmann [8] based on

the valve position. The LQR controller is used primarily for stabilizing the system and an iterative learning controller is employed to achieve the desired performance. More recently a LQR controller capable of achieving the desired performance without iterative learning control has been presented by Peterson [15]. Excitation of the impacts between the armature and valve stem are considered by Tai [18]. The authors designed a PD controller which includes a notch filter to avoid excitation of these impacts. Later, they too develop a LQR controller for the EVA [19]. An H_∞/preview controller was introduced by Mianzo [14] by utilizing current control. Unfortunately the effectiveness of the current controller was never demonstrated. Many of the aforementioned controllers rely on observers to estimate the unmeasured states. Various observers based on pole placement [7], Kalman filters [15], and disturbance estimation [11] have been developed for the EVA.

Due to the repetitive nature of the EVA, many designs employ some form of cycle to cycle based compensation. An iterative learning controller for the EVA has been designed by Hoffmann [8] with experimental results given in his later work [7]. The desired performance is achieved through tracking of a predefined trajectory. Similar results have been verified by Tai [18] with a repetitive learning algorithm. These methods are hindered by the need for an accurate model of the system and are computationally intensive.

This paper presents an extremum seeking controller which adjusts a nonlinear feedback from cycle to cycle to reduce the impact velocity. A small microphone obtains a measure of the sound intensity which the controller then minimizes. The reduced sound intensity results in decreased impact velocities. The advantage of the extremum seeking controller is that it does not require a model of the system dynamics and its computational load is minimal.

2 Experimental Setup

The experimental setup, shown schematically in Fig. 1, consists of:

- a prototype 200 V electromechanical valve actuator,
- a 200 V power supply (Power-Ten Inc. Model Number:P83),
- two PWM brush type servo amplifiers (Advanced Motion Controls, Model Number:50A-DD),
- a dSpace 1103 processing board,
- an eddy current sensor (Magnetic Moments),
- a microphone (RadioShack), and
- a laser vibrometer (Polytec, Model Number:0VD 20).

The controller, which consists of a nonlinear feedback and an extremum seeking algorithm, is implemented

as follows: The displacement of the armature is determined by the eddy current sensor which detects changes in a magnetic field, generated by the sensor, caused by the motion of the sensor target attached to the rear of the armature. Based on the encoded nonlinear feedback, given in Sec. 4, and the measured displacement the dSpace processing board regulates the duty cycle of the PWM amplifiers to control the armature motion during a valve closing/opening event. The switching frequency of the PWM amplifiers is sufficiently fast that we assume that the effective voltage applied to the actuator is the duty cycle command multiplied by the supply voltage, which was set to 180 V. At the end of each valve closing/opening event a microphone is used to obtain a measure of the sound intensity of the impact. The extremum seeking controller, given in Sec. 5, then uses this measurement to tune the nonlinear feedback before the next valve closing/opening event to reduce the impact velocity. The experimental results are obtained using the laser vibrometer, which is capable of measuring both the position and velocity very accurately. The measurements obtained from the laser vibrometer are only used to determine the effectiveness of the controller and are not used in the feedback.

3 Electromechanical Valve Actuator Model

The EVA model derived by Wang [20] is summarized here. The dynamics of a valve opening event are essentially identical to those of a valve closing event and by employing the optimal release developed by Wang [20] the releasing coil has very little effect on the overall system dynamics. The EVA can thus be described considering only the effect of the magnetic coil used to capture the armature at the end of the valve transition. Selecting the following states,

i the current in the magnetic coil
z the distance between the armature and magnetic coil
v the velocity of the armature

the dynamics of the system are given by:

$$\frac{dz}{dt} = v \tag{1}$$

$$\frac{dv}{dt} = \frac{1}{m} \left(-F_{mag}\left(i, z\right) + k_s \left(l - z\right) - bv \right) \tag{2}$$

$$\frac{di}{dt} = \frac{V_c - ri + \chi_1\left(i, z\right) v}{\chi_2\left(z\right)}, \tag{3}$$

where m is the combined mass of the armature and valve in kg, F_{mag} is the magnetic force generated by the magnetic coil in N, k_s is the spring constant in N/m, l is half the total armature travel in m, b is the damping coefficient in kg/s, V_c is the applied voltage in V, r is the resistance of both the wiring and magnetic coil in Ω, $\chi_1 v$

is the back-EMF generated by the armature motion in V, and χ_2 is the inductance of the magnetic coil in H.

The nonlinear functions F_{mag}, χ_1, and χ_2 which describe the magnetic field properties are given by

$$\chi_1\left(i, z\right) = \frac{2k_a i}{\left(k_b + z\right)^2}, \quad \chi_2\left(z\right) = \frac{2k_a}{\left(k_b + z\right)} \tag{4}$$

$$F_{mag}\left(i, z\right) = \frac{k_a i^2}{\left(k_b + z\right)^2} \tag{5}$$

where k_a and k_b are constants determined experimentally. Eqns. (4) (5) neglect the higher order effects of flux saturation and leakage as they are difficult to accurately model. The model is only included here to provide an explanation for our choice of feedback given in Sec. 4. As we will see, the extremum seeking controller, which is the main focus of this paper, does not depend upon the system model.

4 Nonlinear Feedback

The controller consists of two different components: nonlinear feedback, discussed in this section, and an extremum seeking controller for self-tuning, discussed in Sec. 5. We first motivate the need for nonlinear feedback by examining the physics inherent to the EVA. The feedback itself is then presented, followed by experimental results.

Controlling the electromechanical valve actuator is challenging because of the changing magnetic/electric properties during the armature travel. At the beginning of the armature travel the magnetic force generated by the catching coil has very little influence on the armature. During this period it is undesirable to apply large voltage inputs to the system to control the armature motion. Doing so would not only increase power consumption, but large currents generated early in the transition may result in high impact velocities. A more practical solution is to let the potential energy stored in the springs do the work of moving the armature across the gap. However, if voltage is not applied until the armature is near the catching coil, it is difficult to generate the necessary current to capture the armature due to the back-EMF which becomes significant near the magnetic coil.

Both of these phenomena are captured in the system model. From Eqn. (5) we see that the magnetic force is inversely proportional to the square of the distance. Thus, at large distances the magnetic force is much less than the spring force and has little influence on the armature motion. The second effect requires more mathematical manipulation to visualize. Substitution of Eqn. (4)

into (3) yields:

$$\frac{di}{dt} = \frac{(V_c - ri)(k_b + z)}{2k_a} + \frac{iv}{(k_b + z)} \qquad (6)$$

Let us replace the term $k_b + z$ with the small parameter $\varepsilon = k_b + z$, as $k_b \ll 1$ and z is approaching zero. Eqn. (6) reduces to

$$\frac{di}{dt} = \frac{(V_c - ri)\varepsilon}{2k_a} + \frac{iv}{\varepsilon} \qquad (7)$$

As the armature approaches the magnetic coil (ε goes to zero) the velocity, which is negative, has a disproportionately larger effect (division by ε) than the command voltage (multiplication by ε) on the current and will tend to drive it toward zero.

Therefore small voltages should be applied while the armature is far from the magnetic coil and large voltages when the armature is near the magnetic coil. This methodology is the opposite of linear feedback, where the input is proportional to the error. Instead, use of the nonlinear feedback

$$V_c = \begin{cases} \frac{K_1}{\gamma + z}v + \frac{K_2}{\beta + z} & \text{if } z \leq 2l - 1\text{x}10^{-3} \\ 0 & \text{if } z > 2l - 1\text{x}10^{-3} \end{cases} \qquad (8)$$

is proposed. Here, the voltage is inversely proportional to the distance from the catching coil, thereby alleviating the affects of the changing magnetic/electrical characteristics of the system discussed in the preceding paragraph. The parameters K_1, K_2, γ, and β are used to tune the controller to achieve the desired performance.

The first term of the feedback, $\frac{K_1}{\gamma + z}v$, can be considered as a nonlinear damping gain. The value of K_1 is used to adjust the damping and γ is set to a small non zero value (≈ 0.5) to avoid division by zero. The second term, $\frac{K_2}{\beta + z}$, insures that large inputs are applied near $z = 0$ to overcome the effect of the back-EMF. The magnitude of the input at $z = 0$ is adjusted primarily by K_2. The value of β governs how the second term transitions for a small value at $z = 8$ to a large value at $z = 0$. If β is set too small the transition will occur too early resulting in a large impact velocity. If β is set too large the transition will occur too late and the electromagnet may fail to capture the armature. The feedback is only applied after the first millimeter of travel to ensure that the armature has been released before control is applied.

4.1 Experimental Results

The nonlinear feedback is implemented on the EVA using the setup described in Sec. 2. As the velocity measurement from the laser vibrometer is not used in the feedback, a Kalman filter is used to estimate the velocity which is accurate to within +/- 0.4 m/s [15].

Experimental results are given in Fig. 2 and Table 1. Again, note that the signal sent to the PWM drivers is a duty cycle command equal to the voltage calculated from Eqn. (8) divided by the supply voltage. A mean impact velocity of 0.16 m/s is achieved while avoiding actuator saturation except at the very end of the transition.
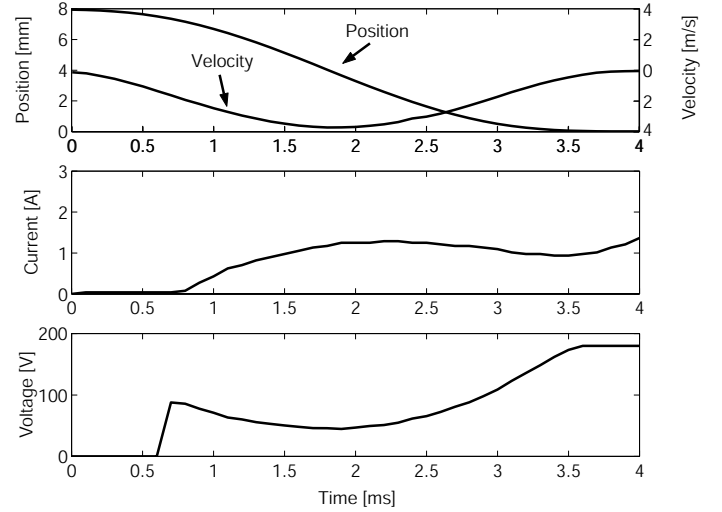

Fig. 2. Experimental results using the nonlinear controller.

	Transition Time	Impact Velocity
Mean	3.23 ms	0.16 m/s
σ	0.04 ms	0.08 m/s
Max	3.3 ms	0.32 m/s
Min	3.2 ms	0.05 m/s

Table 1
Statistical results for the nonlinear controller.

This result is typical of the impact velocities achieved in the literature, which range from approximately 0.05 m/s to 0.2 m/s [2,7,14,15,18,19], and compares favorably with the open loop results of 0.5 m/s discussed by Wang [20]. Smaller impacts are achievable using the feedback given in Eqn. (8) but the results become less consistent. The temperature, humidity, and other environmental factors in the laboratory in which these experiments were conducted were not tightly controlled and are largely influenced by the outside weather. These unknown factors appear to have a small influence on the performance of the system, and as such the results of Table 1 are based on a single set of values of K_1, K_2, γ, and β which achieved the best results over several days. In order to achieve better results without having to manually make slight adjustments to K_1, K_2, γ, and β an extremum seeking controller is implemented to automatically tune the nonlinear feedback in Sec. 5.

4

5 Cycle to Cycle Based Compensation

To account for unknown, but slowly varying, factors which affect the performance of the system an extremum seeking controller is employed to apply cycle to cycle based compensation to tune the feedback given in Eqn. (8). These factors may be a result of the changing outside temperature, humidity, or other uncontrollable environmental factors. Cycle to cycle based compensation is particularly well suited for the EVA as the valve events are repeated several thousand times a minute.

Using sinusoidal excitations [9], the extremum seeking controller selects the value of the parameter β from Eqn. (8) which minimizes a pre-defined cost function based on the sound intensity of the impact recorded by a small microphone. The parameter β was chosen as it was found to have the strongest influence on the impact velocity. By minimizing the cost function (and thus the sound intensity) the impact velocity is also minimized. The extremum seeking controller used here is intended for static nonlinearities, so the dynamics of the EVA are concealed from it by discretizing at a sampling rate equal to the rate of the valve events. An overview of the extremum seeking control algorithm is presented in Sec. 5.1. The adaptation of the general algorithm for use with the EVA is given in Sec. 5.2.

5.1 Extremum Seeking Control

A complete proof of the extremum seeking controller is provided by Krstic [12]. The proof is summarized here for completeness. Extremum seeking control relies on the theorem of averaging [10] which states that given a system of the form

$$\frac{dx}{dt} = \varepsilon f\left(t, x, \varepsilon\right),\ \varepsilon > 0 \tag{9}$$

where $f\left(t, x, \varepsilon\right)$ is sufficiently smooth and T-periodic in t, then

$$\left\|x\left(t, \varepsilon\right) - x_{av}\left(t, \varepsilon\right)\right\| = O\left(\varepsilon\right) \tag{10}$$

where

$$\frac{dx_{av}}{dt} = \varepsilon f_{av}\left(x\right)\ \text{and}\ f_{av}\left(x\right) = \frac{1}{T}\int\limits_{0}^{T} f\left(\tau, x, 0\right) d\tau. \tag{11}$$

Stated qualitatively, a time varying periodic system can be approximated by the time invariant system derived from integrating the original system over a single period. If the resulting time invariant system is stable about an equilibrium, the time varying periodic system converges to a periodic orbit about the same equilibrium.
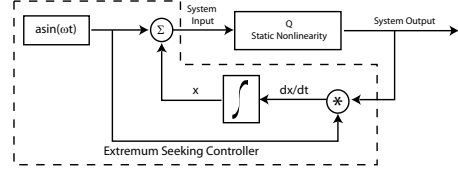

Fig. 3. Extremum seeking control as applied to a static nonlinearity.

The dynamics of the extremum seeking controller applied to the static nonlinearity Q, as shown in Fig. 3, are

$$\frac{dx}{dt} = Q\left[x + a\sin\left(\omega t\right)\right] a\sin\left(\omega t\right). \tag{12}$$

Let $\tau = \omega t$ and define $\widetilde{x}\left(\tau\right) = x\left(\frac{\tau}{\omega}\right)$, therefore

$$\frac{d\widetilde{x}}{d\tau} = \varepsilon Q\left[\widetilde{x} + a\sin\left(\tau\right)\right] a\sin\left(\tau\right) \tag{13}$$

where $\varepsilon = \dfrac{1}{\omega}$. Assuming Q is analytic,

$$
\begin{aligned}
Q\left[\widetilde{x} + a\sin\left(\tau\right)\right] = Q\left(\widetilde{x}\right) &+ \frac{\partial Q}{\partial \widetilde{x}}\left(\widetilde{x}\right) a\sin\left(\tau\right) \\
&+ \frac{1}{2!}\frac{\partial^2 Q}{\partial \widetilde{x}^2}\left(\widetilde{x}\right) a^2 \sin^2\left(\tau\right) + O\left(a^3\right).
\end{aligned}
$$

Substituting into Eqn. (13), and applying the theorem of averaging

$$\frac{d\widetilde{x}_{av}}{d\tau} = \frac{a^2}{\varepsilon}\left(\frac{1}{2\pi}\int\limits_{0}^{2\pi}\sin^2\left(\tau\right) d\tau\right)\frac{\partial Q}{\partial \widetilde{x}}\left(\widetilde{x}_{av}\right), \tag{14}$$

where the equilibrium points are the local extremum of Q (i.e $\frac{\partial Q}{\partial \widetilde{x}} = 0$) and are stable if and only if $\left.\frac{\partial^2 Q}{\partial \widetilde{x}^2}\right|_{\widetilde{x}=x_e} < 0$, which occurs when x_e is a local maximum of Q.

By the theorem of averaging, the system output converges to a periodic orbit about a local maximum. By the same argument, the output is driven to a periodic orbit about a local minimum when the integrator is multiplied by -1. An overview of extremum seeking methodologies can be found in the review by Stenby [17].

5.2 Extremum Seeking Control for Soft Landing

Before applying the extremum seeking controller to the EVA, the appropriate system input and output must be selected and the extremum seeking controller must be modified to treat the EVA as a static nonlinearity.

The parameter β of the feedback used in Eqn. (8) is selected as the input and the cost function

$$Q = (S_{des} - S_{meas}[k])^2 \qquad (15)$$

is selected as the output. Where S_{des} is the desired sound intensity and $S_{meas}[k]$ is the measured sound intensity of the armature impacting the magnetic coil at the kth iteration. S_{des} is set to a small non-zero value to prevent the controller from minimizing the sound intensity by avoiding impact. Due to uncertainty in when the impact occurs, S_{meas} is determined by summing the absolute value of the sampled signal from the microphone over a 1 ms interval which begins when the armature is less than 0.1 mm away from the magnetic coil.

While effective in a bench-top experiment, measuring the sound intensity of the impact in an internal combustion engine is impractical. By filtering the signal from the knock sensor, mounting load washers under the springs of the EVA, or through clever placement of accelerometers it should be possible to obtain a measure of the impact magnitude in order to define a similar cost function Q.

The proof given by Krstic [12] requires that Q be a static nonlinearity. Before applying the extremum seeking controller to the EVA we must first modify it by discretizing the extremum seeking controller at the same frequency as the valve events. Given that the valve opens every ΔT sec the extremum seeking controller is discretized at $\dfrac{1}{\Delta T}$ Hz. The dynamics of the discrete extremum seeking controller and the value of β at the kth iteration are

$$x[k+1] = x[k] + \Delta T \sin(\omega k \Delta T + \phi)(S_{des} - S_{meas}[k])^2$$
$$\beta[k] = x[k] + \sin(\omega k \Delta T + \phi)$$

where $\omega = \dfrac{\pi}{\Delta T}$ and $\phi = \dfrac{\pi}{2}$. The final controller is shown in Fig. 4.
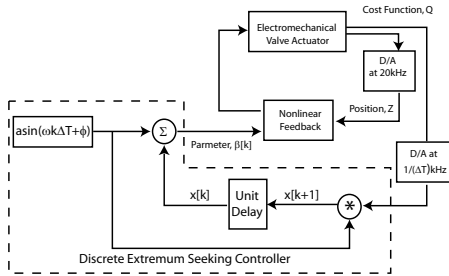

Fig. 4. Extremum seeking control as applied to the EVA.

To test the extremum seeking controller the parameter β is initialized at a non-optimal value. The evolution of β,

the impact velocity, and the cost function Q generated by the microphone over the next 70 iterations are shown in Fig. 5. Although the cost function Q and the impact velocity do not have a one to one correspondence, on average they both follow the same trend. Thus as we minimize Q the impact velocity is also minimized. An enlarged view of the impact velocity at three different iterations is given in Fig. 6.
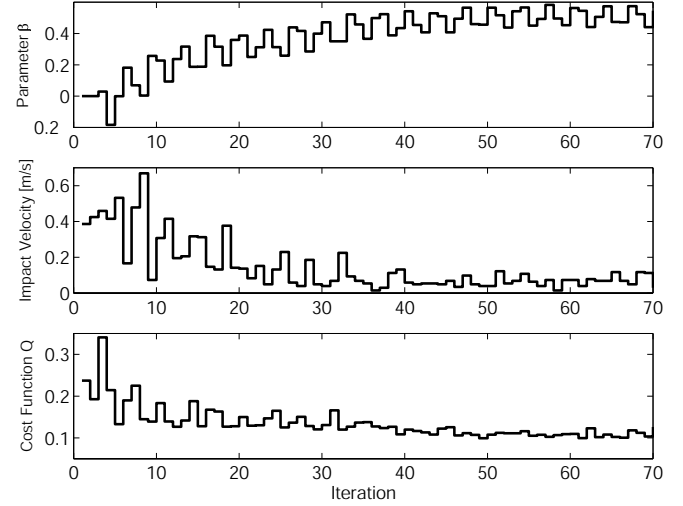

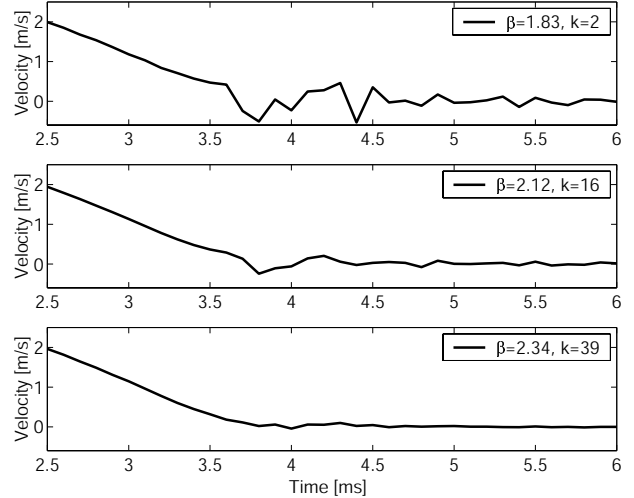Fig. 5. The value of β , the impact velocity, and the cost function Q at each iteration.

Fig. 6. Impact velocity at 2nd , 16th , and 39th iteration.

After approximately 40 iterations both β and the impact velocity converge to a periodic orbit about their final value, resulting in a reduction in the impact velocity from 0.4 m/s to approximately 0.07 m/s as shown in Table 2. The impact velocity and β continue to oscillate around their final mean value due to the sinusoidal excitation. The excitation amplitude should therefore be kept small to avoid large deviations from the final mean value.

	Impact Velocity
Mean	0.07 m/s
σ	0.02 m/s
Max	0.12 m/s
Min	0.02 m/s

Table 2
Statistical results after the 40th iteration.

The improvements made from Table 1 to Table 2 are a result of the fact that the extremum seeking controller can select the optimal value of β more accurately then what can be achieved manually in the presence of slowly varying unknown disturbances. The same results could be achieved without the extremum seeking controller if the operator were to spend time manually re-tuning the gains every so often. However, the extremum seeking controller is a far more practical solution for implementation in production automotive engines. Additionally, in comparison to other methods that apply cycle to cycle based compensation, such as the work done by Hoffmann on ILC [7], the extremum seeking controller is advantageous as it is computationally quite simple and only requires an additional microphone for implementation.

6 Summary and Conclusion

Large impacts experienced by the EVA prevent it from being used for VVT in IC engines. This paper presented the implementation of an extremum seeking controller to achieve practical impact velocities by exploiting the repetitive nature of the system. Experimental results demonstrate that transition times and impact velocities of less than 4.0 ms and 0.1 m/s respectively were achieved.

In future work we will investigate systematically the sources and effects of uncertainty to the system so we can better assess its robustness. Moreover, we will explore the improvements associated with a multivariable extremum seeking controller. Finally, we will concentrate on re-designing a nonlinear feedback controller with better repeatability and preformance.

Acknowledgements

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