2016 American Control Conference (ACC)
Boston Marriott Copley Place
July 6-8, 2016. Boston, MA, USA

Estimating State-of-Charge Imbalance of Batteries Using Force Measurements

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***Abstract*— This paper addresses the problem of estimating SOC-imbalance between two battery cells connected in series. Particularly, the effectiveness of using force measurements for the SOC-imbalance detection against pack/total voltage measurements is studied. SOC imbalance estimation during charging using pack voltage measurement was previously demonstrated for the $LiFePO_4$/graphite battery chemistry. However, the Li-ion battery with $LiNiMnCoO_2$/graphite, which is of great interest in a hybrid electric vehicle application, exhibits an almost linear relation between SOC and voltage when battery SOC is greater than 15%. This characteristic makes SOC imbalance estimation using pack voltage challenging. The use of other novel measurands, related to volumetric change of the electrode materials during battery operations, make the problem feasible. To estimate SOCs of two batteries connected in series, a Moving Horizon Estimation (MHE) technique is applied and three different measurement sets are considered: (1) total voltage, (2) force, and (3) both voltage and force. Simulations results show that, for the batteries of interest, the inclusion of force measurements significantly improves the estimation of SOC-imbalance.**

I. Introduction

Battery cells are typically packaged in a constrained space. However, a lithium ion battery exhibits a change in its volume induced by lithium intercalation and de-intercalation [1]. Soft foam spacers may be placed between cells in the pack to accommodate this battery swelling. In this case a change in gap between adjacent cells (compressing the foam) can be measured using strain or displacement sensors [2], [3]. In the case of rigid packaging, with cells between two end-plates at fixed displacement as shown in Figure 1, a load cell may be used to estimate the bulk (or average) SOC of the cells [2] from the force exerted during swelling of the batteries.

In a pack, capacity variations among battery cells and their state-of-charge (SOC) imbalance are inevitable. Differences in capacity and self discharge rates resulting from variance in the manufacturing process as well as localized degradation by inhomogeneity of the thermal distribution inside the pack lead to imbalance. Battery SOC should be accurately monitored and balanced in real-time, or battery cells could be irreparably damaged due to overcharge and overdischarge. Thus, many approaches to SOC estimation have been proposed in literature [2], [4]–[9]. These techniques require voltage sensing of each individual cell, which is costly. Among these works, authors in [9] proposed an interesting technique to estimate SOC imbalance between two battery cells using pack/total voltage measurement, reducing the number of required measurements. Experimental results, conducted with two $LiFePO_4$/graphite chemistry cells, showed that a nonlinear observer, Newton Observer, could effectively identify the SOC imbalance between the two cells from a single pack voltage measurement. However, the algorithm relies on the strong nonlinearity between battery terminal voltage and SOC, specifically the presence of non-zero first and second derivatives with respect to SOC, and therefore is only effective in bounded regions of SOC, specifically high and low SOCs.

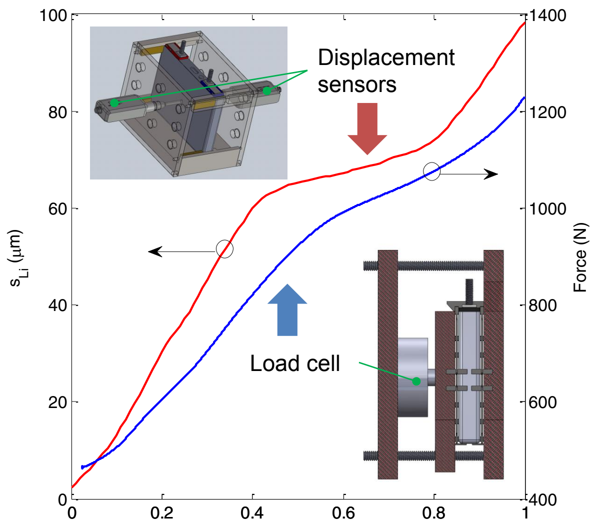

Fig. 1. Measured free swelling and force of the battery during constant current operation at 0.2C and 0.3C respectively. Schematic of a fixture for the free swelling measurements is adopted from [1].

For other battery chemistry, however, the voltage as a function of SOC does not have a strong nonlinearity when SOC is greater than 0.15, as can be seen from Fig. 2 for cells with $LiNiMnCoO_2$/graphite electrodes. Although the first derivative of the open-circuit voltage increases when SOC is greater than 0.97, this change is much smaller than that of a $LiFePO_4$ cell shown in [9], which poses a problem for using total voltage measurements for detecting SOC

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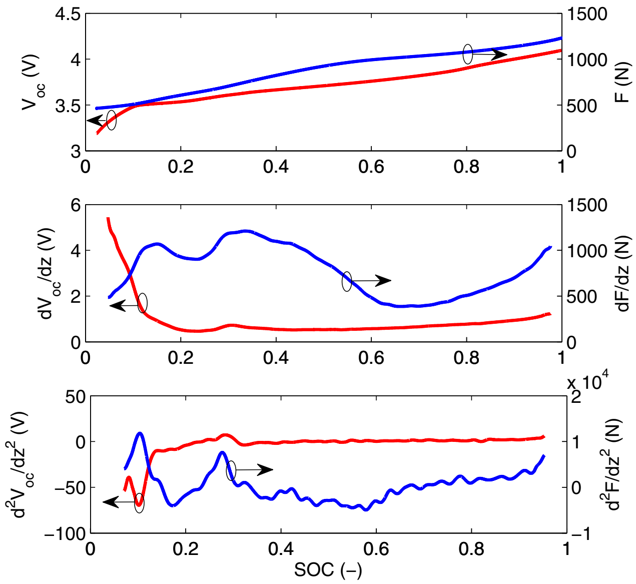

Fig. 2. Open-circuit voltage and constrained force of Li-ion battery under a low rate (C/5) constant current charge and their derivatives with respect to battery SOC. The battery consists of NMC cathode and graphite anode.

imbalance. Fortunately other novel measurands related to volume changes of the electrode materials during lithium intercalation [10] have stronger nonlinearities as shown in Fig. 2.

In this work, we investigate the use of a force measurement for estimating SOC imbalance between two battery cells, similarly to [9]. For state estimation, a nonlinear Moving Horizon Estimation technique in [11] is considered because the trajectory of output measured over a time interval is required as discussed in [9]. Simulation results show that an approach relying on total voltage only cannot estimate SOC imbalance while the inclusion of force measurement significantly improves the accuracy.

The remainder of the paper is organized as following: Section II details dynamic behaviors of the Li-ion battery under consideration. Specifically, the measurement of (1) lithium intercalation-induced swelling of the battery without constraint and (2) force in a constrained fixture are described. In Section III, a general idea of Moving Horizon Estimation scheme and its application to SOC imbalance estimation are explained. Section IV the effectiveness of using force measurements for estimating SOC imbalance between two battery cells is shown through simulation. Finally, Section V concludes the paper with a summary of presented work and with a discussion on future extensions.

II. Battery Modeling

A. Battery dynamics

A 5 Ah prismatic $LiNiMnCoO_2$/graphite Li-ion battery is considered in this study. The battery cell, was extracted from a 2013 Ford Fusion HEV battery pack. An OCV-R type battery model is used to describe the dynamics of the battery and expressed in discrete time domain with a sampling period Δt by the following equations:

$$\begin{aligned} z_{k+1} &= z_k - I_k \Delta t / C_{\text{batt}}, \\ V_k &= g(z_k) - I_k R_k, \end{aligned} \tag{1}$$

for $k = 0, 1, \ldots$, where z is the state-of-charge of a battery cell, I is the current and C_{batt} is the battery capacity. The terminal voltage V is determined from the nonlinear function $g(z_k)$ and the voltage drop because of the Ohmic resistance R. The nonlinear function $g(z_k)$ is obtained under a constant charging current operation at low rate.

The constrained force experienced by the battery F is modeled by a nonlinear spring,

$$F_k = \alpha(z_k)\big(s_0 + s_{Li}(z_k)\big) + \beta(z_k)\big(s_0 + s_{Li}(z_k)\big)^3. \tag{2}$$

The terms s_0 and s_{Li} represent the change in free length due to the initial compression of the cells and the lithium intercalation induced swelling respectively. The coefficients of the nonlinear spring equation are denoted by α and β. These stiffness parameters are a function of the battery SOC as shown in Fig. 3.

The battery cell is a flat-wound jelly-roll type. This type of battery exhibits swelling s_{Li} because of electrode expansion in the direction perpendicular to its largest face [1]. The free swelling of the cell is measured during a 0.2 C-rate (1A) discharge with high-precision contact-type displacement sensors with 1μm accuracy and 0.1μm resolution (Keyence GT2-H12KL, Japan). The fixture holds the sensors at the center of the surface as shown in Fig. 1. The net displacement at the center of the battery is measured with respect to the battery SOC. Battery temperature, measured using a thermocouple, remained within 0.1°C of the 25°C ambient temperature during the battery operation and hence data at low discharge rate allows direct correlations to be made between swelling and Li-ion intercalation in a cell sandwich without significant influence by thermal expansion.

Figure 1 shows a schematic of a fixture to measure the force generated by expansion of the constrained battery cells during charging. Two garolite end plates are used to clamp the batteries together using connecting bolts with lock nuts to prevent the fixture from loosening. Rigid plastic spacers between are used to maintain passageways for passing air over the cell to provide convective cooling. The force measurement is made using a 500 lb (LC305-500) Omega load cell sensor (strain gauge type). The entire battery fixture is cycled inside a thermal environmental chamber for temperature regulation. The measured force as a function of SOC is shown in Fig. 2(a).

Based on two experimental data sets including free swelling versus SOC and force versus SOC shown in Figs. 1 and 2(a) respectively, the relation between force and swelling of the battery can be identified. Because of the structure of the battery cell, a jelly-roll encapsulated in an aluminum case, its force-displacement relationship is modeled as a

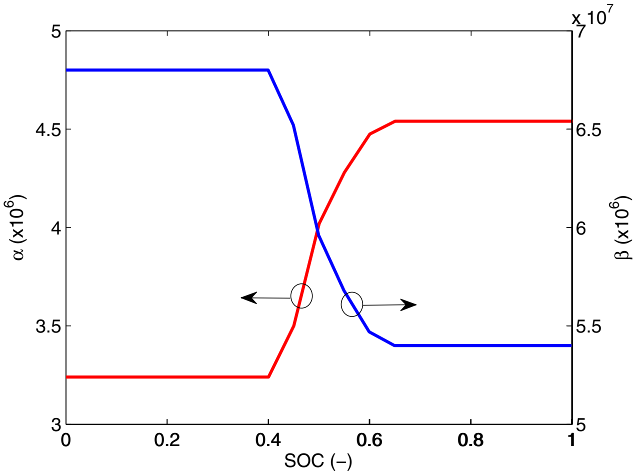

Fig. 3. Nonlinear spring constants, α and β, as a function of SOC

nonlinear spring. More details about the derivation of governing equations and parameter identification for the 1-D force model can be found in [12]. Figure 3 shows the identified nonlinear spring rates of the battery cell, α and β which depend on battery state of charge. The increase in α with SOC is expected due to stiffening of the graphite layers during lithium intercalation [13].

B. A 2-cell fixture model for simulation

The total force measured in the two battery cell fixture with rigid packaging between two end-plated of fixed displacement can be related to the free swelling of each individual cell by the following force balance:

$$\begin{aligned} F =&\alpha(z^{\mathrm{A}})(s_0 + s_{Li}(z^{\mathrm{A}}) + \Delta s) \\ &+ \beta(z^{\mathrm{A}})(s_0 + s_{Li}(z^{\mathrm{A}}) + \Delta s)^3 \\ =&\alpha(z^{\mathrm{B}})(s_0 + s_{Li}(z^{\mathrm{B}}) - \Delta s) \\ &+ \beta(z^{\mathrm{B}})(s_0 + s_{Li}(z^{\mathrm{B}}) - \Delta s)^3 \end{aligned} \quad (3)$$

where the variable Δs indicates the relative change in thickness of each battery due to a state of charge imbalance and can be computed from the force balance equation above. The superscripts A and B denote battery cell A and B in the fixture seen from Fig. 4.

When modeling the 2-cell fixture system, the following assumptions are made :

1) The capacity, open-circuit voltage and Ohmic resistance of the batteries are the same and known.
2) Temperature is regulated at a fixed value, e.g. room temperature.
3) The distance between two end-plates of the fixture is constant, and the battery stiffness functions α and β are known.

These assumptions are made to neglect the impact of battery aging and variability of materials and to remove the effect of thermal expansion for simplicity of analysis.

In the case where the two battery cells are SOC-balanced, i.e., $z^A = z^B$, then the thickness of the battery cells are

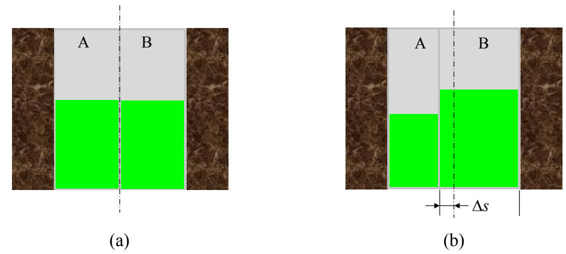

Fig. 4. Schematic of changes in displacement of two battery cells in the fixture at different SOC levels: (a) $z^A = z^B$; (b) $z^A < z^B$

the same i.e. $\Delta s = 0$ (–see Fig. 4(a)). When two batteries are operating at different SOCs, their thicknesses could be different as shown in Fig. 4(b). In the case when their spring constants are the same, Δs can be computed by

$$\Delta s = (s_{Li}(z^B) - s_{Li}(z^A))/2 \quad (4)$$

. Otherwise Δs should be solved from a cubic root or numerically from the force balance equation (3) and estimates of each batteries' SOC.

The problem of estimating SOC-imbalance can be formulated as finding initial SOC of each cell and Δs for every set of voltage, force and current measurements. The integration of current is used to compute a change in stored charge and a corresponding change in displacement associated with Li-ion intercalation. For the 2-cell fixture, given n measurements, there are $2n$ equations with $n+2$ unknowns to be solved to find the SOC. It is possible to identify SOC-imbalance in the ideal case, of noise-free force measurements, when $n \geq 2$ and F has 2^{nd} order non-zero derivatives[1]. However, this requires significantly higher computation effort compared to the case when spring constants of two battery cells are the same, i.e., $n+2$ vs. 2 variables to be estimated. The spring rates are found to be constant when SOC is greater than 0.65. Therefore, in the estimation problem, it is assumed that spring constants of two cells are the same when $z > 0.65$.

III. SOC IMBALANCE ESTIMATION

A. Moving Horizon Estimation

As discussed by authors in [9], the battery cell SOCs can be estimated from the trajectory of output measured over a time interval not a single time instant. Therefore, in this study, a trajectory-based nonlinear Moving Horizon Estimation (MHE) scheme is considered to identify SOC-imbalance between battery cells. A general idea of the MHE is summarized in this subsection. More details on the MHE can be found in [11].

Consider a nonlinear dynamic system expressed in discrete

[1]The observability of the system can be checked by constructing an observability matrix for nonlinear system based on Lie derivatives.

time domain:

$$x_{k+1} = f(x_k, u_k), \tag{5}$$
$$y_k = h(x_k). \tag{6}$$

where x is the vector of state variables, u is the vector of control variables and y is the vector of output variables or measurements.

Define $\hat{x}_{k-N|k}, \ldots, \hat{x}_{k|k}$ as the estimates of $x_{k-N|k}, \ldots, x_{k|k}$ at time instant k, and $\bar{x}_{k-N}$ as the priori estimate of $\hat{x}_{k-N|k-1}$, that is,

$$\bar{x}_{k-N} = f(x_{k-N-1|k-1}, u_{k-N-1}). \tag{7}$$

The information vector $\mathcal{I}_k$ is defined as,

$$\mathcal{I}_k \triangleq [y_{k-N}, , \ldots, y_k, u_{k-N-1}, \ldots, u_{k-1}]. \tag{8}$$

Then, at each step, a nonlinear minimization problem is solved by considering the following cost function:

$$J = \sum_{j=k-N}^{k} ||y_j - h(\hat{x}_j)||^2 + \lambda||\hat{x}_{k-N|k} - \bar{x}_{k-N}||^2. \tag{9}$$

The cost function consists of two terms: the first term is used to minimize the summation of output errors while the second term is used to penalize the deviation from previous state estimates. The weighting factor λ is used to handle the trade-off between two terms.

B. Measurement-dependent Cost Function

In the problem of estimating SOC-imbalance, we consider the three cases and corresponding cost functions based on output measurements.

Case I: Voltage measurement only

$$J_1 = \sum_{j=k-N}^{k} \left(||V_j - \hat{V}_j^A - \hat{V}_j^B||^2\right) + \tag{10}$$
$$\lambda_1||\hat{x}_{k-N|k} - \bar{x}_{k-N}||^2. \tag{11}$$

where $x = [z^A, z^B]'$. The total voltage $V = V^A + V^B$ is used for the measurement because two batteries are connected in series.

Case II: Force measurement only

$$J_2 = \sum_{j=k-N}^{k} \left(||F_j - \hat{F}_j^A||^2 + ||F_j - \hat{F}_j^B||_2^2\right) + \tag{12}$$
$$\lambda_2||\hat{x}_{k-N|k} - \bar{x}_{k-N}||^2. \tag{13}$$

where $x = [z^A, z^B]'$ As mentioned in Section II, to address the general case, Δs over the time interval and the initial SOCs need to be included in the estimation, i.e., $x = [z_0^A, z_0^B, \Delta s_{j=k-N:k}]'$., here we consider the high state of charge case, where the spring constants are equal and Δs is given by Eq. 4. Unlike the *voltage only* case, the force applied to each cell is identical due to the structure and hence two force errors are used and equally weighted.

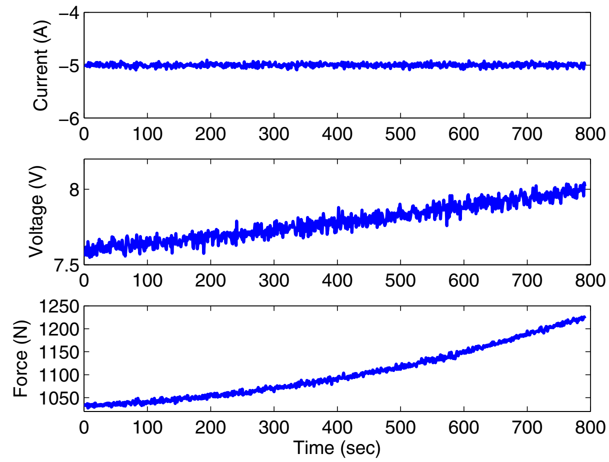

Fig. 5. Simulated total voltage and force during the charge process which are used for the SOC estimation.

Case III: Voltage & Force measurement

$$J_3 = \sum_{j=k-N}^{k} \left(||F_j - \hat{F}_j^A||^2 + ||F_j - \hat{F}_j^B||_2^2\right) + \tag{14}$$
$$\lambda_{31} \sum_{j=k-N}^{k} \left(||V_j - \hat{V}_j^A - \hat{V}_j^B||^2\right) + \tag{15}$$
$$\lambda_{32}||\hat{x}_{k-N|k} - \bar{x}_{k-N}||^2. \tag{16}$$

where $x = [z^A, z^B]'$. In this case, both voltage and force measurements are used; therefore, two weighting factors λ_{31} and λ_{32} are used.

In the following section, a simulation is performed to investigate the usefulness of force measurements for estimating SOC-imbalance between two cells in a constrained fixture.

IV. Simulation Results and Discussion

To investigate the performance of the nonlinear MHE with three different measurements sets described afore, a simulation of constant current battery charging is performed. This setup is primarily meant to simulate charging batteries, for instance, plug-in hybrid electric vehicles or battery electric vehicles application. Two battery cells connected in series, with an initial compression of $s_0 = 100\ \mu$m, are charged at 1C current. The data is sampled at a 1 Hz rate. In practice, every measurement is contaminated by a noise and this noise is usually assumed to be Gaussian with zero mean and σ standard deviation. In this work, noise characteristics of current, voltage and force measurements are $\sigma_I = \sqrt{10^{-3}}$A, $\sigma_V = \sqrt{10^{-3}}$V and $\sigma_F = \sqrt{10}$N, respectively.

Before charging the batteries, their initial SOCs are set to 0.73 and 0.63, respectively. Two different sets of initial conditions (ICs) for the SOC estimators are considered as following:

$$\text{IC } ① : [\hat{z}^A, \hat{z}^B]' = [0.78, 0.58]',$$
$$\text{IC } ② : [\hat{z}^A, \hat{z}^B]' = [0.68, 0.68]'.$$

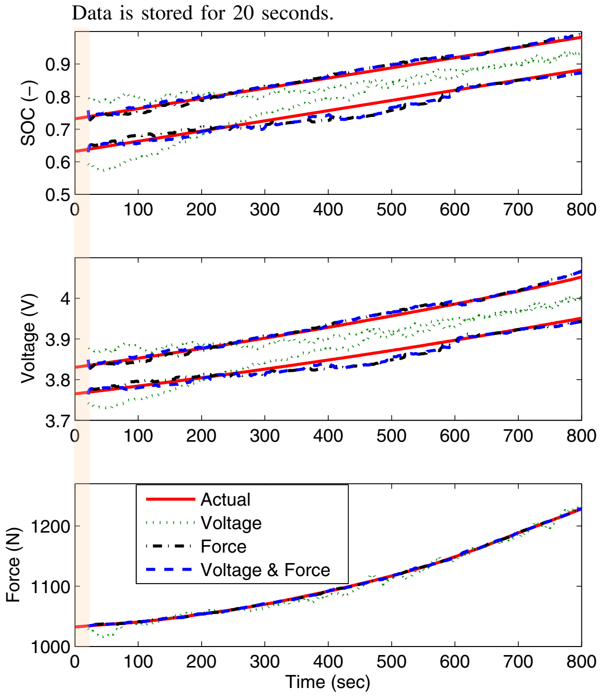

Fig. 6. Cell estimation results: SOC (top), Voltage (middle), and Force (bottom); initial guesses for SOC estimates are set as 0.78 and 0.58.

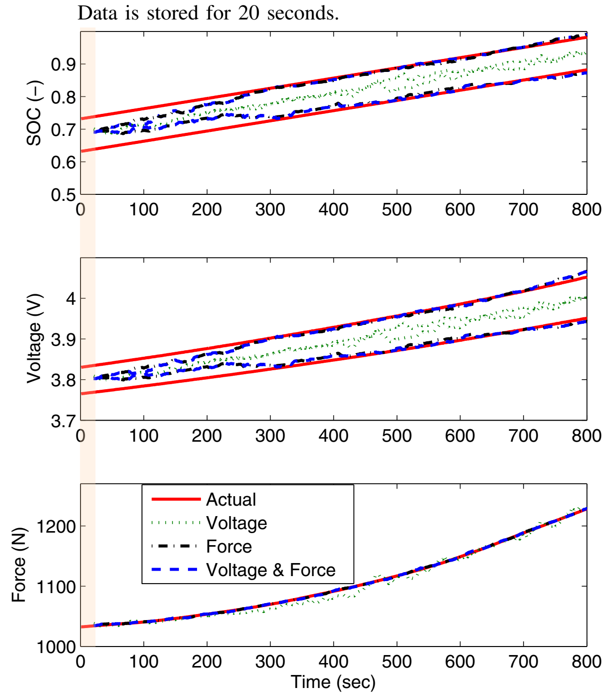

Fig. 7. Cell estimation results: SOC (top), Voltage (middle), and Force (bottom); initial guesses for SOC estimates are set as 0.68 and 0.68.

During the charge process, time evolutions of current, total voltage and force are stored and used for the SOC estimation as seen from Fig. 5. The number of samples N is tuned to be 20, or 20 s, through various simulations considering the accuracy and computation time. The weighting factor λ for each case is also tuned as following:

$$\lambda_1 = \lambda_{32} = 6.8 \times 10^4, \lambda_2 = 4 \times 10^{-1}, \lambda_{31} = 10^2.$$

The performance of three approaches to SOC-imbalance estimation are compared in Figs. 6 and 7 where SOC, voltage, and force of battery cells are depicted. Note that the actual battery SOCs are *measured* by the integration of current, Coulomb counting[2]. Clearly, the voltage-based approach fails to estimate SOCs of two cells whereas the approaches utilizing force measurements successfully estimate the individual SOC for both cells under unbalanced conditions. The estimation error in final SOC and root-mean-square errors during the battery charge process are summarized in Table I. Both force based methods show similar performance. As discussed with Fig. 2, the voltage-SOC relation has a weak nonlinearity, resulting in unobservable SOC conditions for both cells. One can also use the force measurement for bulk SOC estimation as in [14].

TABLE I
SOC-IMBALANCE ESTIMATION PERFORMANCE COMPARISON
(CELL A/CELL B)

SOC Error	IC	Case I (Voltage)	Case II (Force)	Case III (Voltage + Force)
Final	①	0.05/0.05	0.01/0.01	0.01/0.01
	②	0.05/0.05	0.01/0.01	0.01/0.01
RMS*	①	0.05/0.04	0.02/0.01	0.02/0.01
	②	0.05/0.05	0.01/0.01	0.01/0.00

∗RMS errors are calculated using data after t=300 seconds.

V. CONCLUSION

This paper addresses the estimation of SOC-imbalance for two battery cells connected in series. The effectiveness of using force measurements for SOC estimation compared to traditional voltage measurements is investigated. The $LiNiMnCO_2$/graphite battery has a weak nonlinearity between voltage and SOC resulting in an almost zero 2nd derivative with respect to SOC. This inhibits the use of pack/total voltage measurements for estimating individual SOCs and requires another novel measurand exhibiting a strong nonlinearity, such as force, for improved state observability during battery operation. A trajectory-based nonlinear estimator is designed by applying a Moving Horizon Estimation scheme. Three different measurement sets (Voltage /

[2]High accuracy laboratory grade current sensors required for accurate counting are too expensive for automotive systems and subject to errors over longer time periods due to sensor bias.

Force / Force+Voltage) are tested by simulation of battery charging for an electric vehicle. Utilizing force measurements significantly improves the accuracy of the SOC estimates compared to using total voltage measurements alone.

Future work will attempt to address the complications arising from thermal swelling of the battery during operation because of internal heat generation from the battery so the estimation scheme can be performed at high current rates.

ACKNOWLEDGMENT

The information, data, or work presented herein was funded in part by the Advanced Research Projects Agency-Energy (ARPA-E), U.S. Department of Energy, under Award Number DE-AR0000269. The authors would like to thank Shankar Mohan from University of Michigan, Dyche Anderson from Ford Motor Company and Dr. Aaron Knobloch from GE Global Research for their support and valuable discussions about this research.

DISCLAIMER

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