Path Planning for Time-Optimal Information Collection

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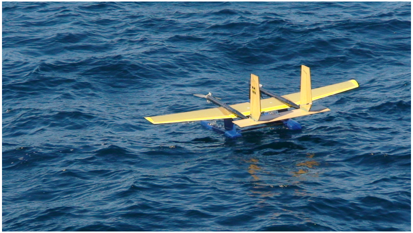
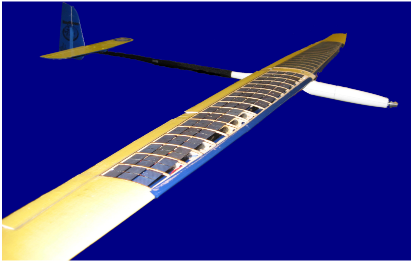
Motivation





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Many Intelligence, Surveillance and Reconnaissance (ISR) Missions require autonomous vehicles to collect information.

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Enhance autonomous information collection through:

1. Model the coupling between information collection and vehicle kinematics

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Many Intelligence, Surveillance and Reconnaissance (ISR) Missions require autonomous vehicles to collect information.

- Time is limited
- Many targets exist

Enhance autonomous information collection through:

1. Model the coupling between information collection and vehicle kinematics
2. Exploit this interaction to form optimal flight paths

Related Work

- *A Mathematical Theory of Communication*, Shannon, 1948
- *Optimal Path Planning for Unmanned Combat Aerial Vehicles to Defeat Radar Tracking*, Kabamba, Meerkov and Zeitz, 2006

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- *Path Planning for Cooperative Time-Optimal Information Collection*, Klesh, Girard and Kabamba, ACC 2008, Accepted
- *Real Time Path Planning for Time-Optimal Exploration*, Klesh, Girard and Kabamba, AIAA GNC 2008, Accepted
- *A Path Planning Framework for Autonomous Exploration*, Klesh, Girard and Kabamba, CDC 2008, Submitted
- *Autonomous Exploration and Remote Sensing*, Klesh, Girard and Kabamba, CDC 2008 Invited Session, Submitted

Unanswered Questions

Problem

How can we improve the rate of information return from ISR missions?

Original Contributions

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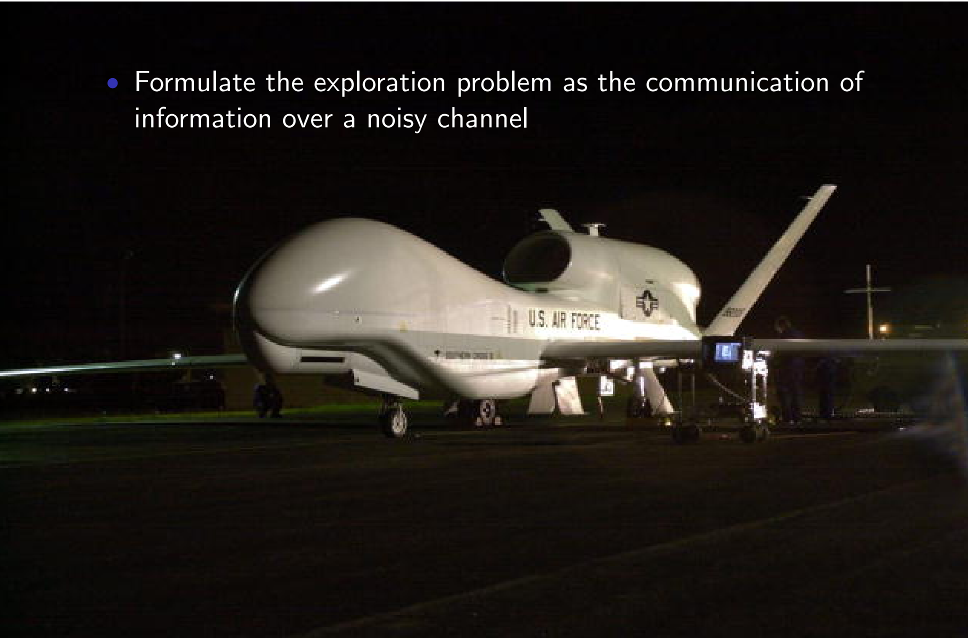
- Formulate the exploration problem as the communication of information over a noisy channel



Original Contributions

- Formulate the exploration problem as the communication of information over a noisy channel
- Identify characteristics for information optimal flight paths

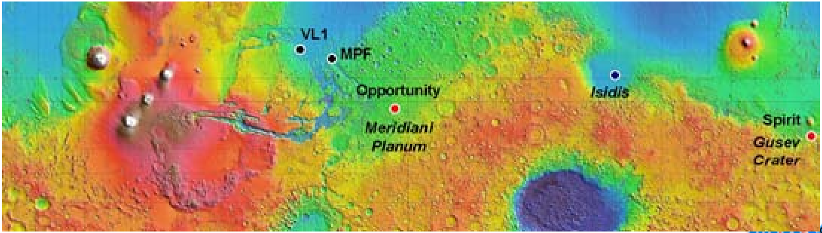
Outline

The Exploration Problem

Exploration

The collection of information about objects of interest with known locations, where information is understood in the classical sense of Shannon as *selection from a set*.

We do not seek to classify the object based on our acquired knowledge, merely collect information in a time-optimal way.



Communication and Exploration

Object of Interest $\longrightarrow$

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Object of Interest $\longrightarrow$ Sensing Process $\longrightarrow$

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Object of Interest	$\longrightarrow$	Sensing Process	$\longrightarrow$	Signal	$\longrightarrow$	Sensor
Transmitter	$\longrightarrow$	Noisy Channel	$\longrightarrow$	Information	$\longrightarrow$	Receiver

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Exploration

Exploration can be viewed as a communication process where each object of interest is a transmitter, the explorer is the receiver, the sensing processes are noisy communication channels, and the sensor signals carry information that allows the explorer to identify the objects of interest.

Information Collection Model

The rate of information received by the sensor:

$$\boxed{\dot{I} = W \log_2(1 + \text{SNR})} \tag{1}$$

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The signal-to-noise ratio of a signal with an active sensor:

$$\boxed{\text{SNR} = \frac{K^4}{((X-A)^2+(Y-B)^2)^2}} \quad (2)$$

Information Collection Model

The rate of information received by the sensor:

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The signal-to-noise ratio of a signal with an active sensor:

$$\boxed{\text{SNR} = \frac{K^4}{((X-A)^2+(Y-B)^2)^2}} \tag{2}$$

The rate of information received about a given object:

$$\boxed{\dot{I} = W \log_2\left(1 + \frac{K^4}{((X-A)^2+(Y-B)^2)^2}\right)} \tag{3}$$

where (A, B) is the Cartesian location of the object of interest.

Time-Optimal Exploration

Problem Statement

Find a flight path that minimizes the total flight time for a UAV to collect a specified amount of information about each object of interest in a given area.

$$\boxed{\min_{\psi(\cdot)} t_f, \text{ subject to } I(t_f) \geq 1} \tag{4}$$

Necessary Conditions

States:

X Rate:

$$\boxed{\dot{x}_i = v\cos(\psi_i), 1 \le i \le n}$$

Y Rate:

$$\boxed{\dot{y}_i = v\sin(\psi_i), 1 \le i \le n}$$

Information Rate:

$$\boxed{\dot{I}_j = \sum_{i=1}^{n} w \log_2\left(1 + \frac{k_j^4}{((x_i - a_j)^2 + (y_i - b_j)^2)^2}\right), 1 \le j \le m}$$

Optimality Condition:

$$\boxed{0 = v\lambda_{y_i}\cos(\psi_i) - v\lambda_{x_i}\sin(\psi_i), 1 \le i \le n}$$

Necessary Conditions

Costates:

X Costate:

$$\boxed{\dot{\lambda}_{x_i} = \sum_{j=1}^{m} \frac{4k_j^4 w(x_i - a_j)\lambda_{I_j}}{((x_i - a_j)^2 + (y_i - b_j)^2)^3 \Delta_j}, 1 \leq i \leq n}$$

Y Costate:

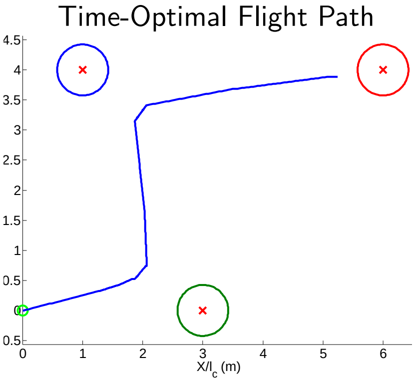
$$\boxed{\dot{\lambda}_{y_i} = \sum_{j=1}^{m} \frac{4k_j^4 w(y_i - b_j)\lambda_{I_j}}{((x_i - a_j)^2 + (y_i - b_j)^2)^3 \Delta_j}, 1 \leq i \leq n}$$

Information Costate:

$$\boxed{\dot{\lambda}_{I_j} = 0, 1 \leq j \leq m}$$

where $\Delta_j = (1 + \frac{k_j^4}{((x_i - a_j)^2 + (y_i - b_j)^2)^2}), 1 \leq j \leq m$

Single UAV



Assumptions:

- Constant Velocity
- Constant Altitude
- Instantaneous Turns
- Non-redundant additive information

Optimal Path Planning

Proposition 1

If the objects of interest are isolated, then the optimal flight paths consist of sequences of straight lines (far from the objects of interest) connected by short turns (near the objects of interest)

Corollary 1.1

If in addition to being isolated, the objects of interest are poorly visible, then the problem becomes a multi-vehicle TSP (MTSP).

Proposition 2

When the visibility of all the objects of interest approaches infinity, $t_f \longrightarrow 0$ and the lengths of paths traveled by the UAVs approach zero.

Generalized Exploration

Kinematics:

$$\boxed{\dot{\overline{\chi_i}} = \overline{f_i}(u_i), 1 \leq i \leq n}$$

Informatics:

$$\boxed{\dot{I_j} = \rho_j(\overline{\chi_1}, ..., \overline{\chi_n}, w_1, ..., w_n), 1 \leq j \leq m}$$

where the functions $\rho_j, 1 \leq j \leq m$ satisfy:

$$\boxed{\rho_j(\overline{\chi_1}, ..., \overline{\chi_n}, w_1, ..., w_n) = 0, \overline{\chi_1}, ..., \overline{\chi_n} \notin D_j, > 0, \text{ otherwise}}$$

Generalized Exploration

States:

X Rate:

$$\boxed{\dot{\overline{\chi_i}} = \overline{f}(u_i), 1 \leq i \leq n}$$

Information Rate:

$$\boxed{\dot{I_j} = \rho_j(\overline{\chi_1}, ..., \overline{\chi_n}, w_1, ..., w_n), 1 \leq j \leq m}$$

Optimality Condition:

$$\boxed{0 = \overline{\lambda}_{\overline{\chi_i}} \frac{\partial \overline{f}(u_i)}{\partial u_i}, 1 \leq i \leq n}$$

Generalized Exploration

Costates:

Position Costate:

$$\dot{\lambda}_{x_i} = -\sum_{j=1}^{m} \lambda_{I_j} \frac{\partial \rho(\overline{\chi_1}, ..., \overline{\chi_n}, w_1, ..., w_n)}{\partial \overline{\chi_i}}, 1 \leq i \leq n$$

Information Costate:

$$\dot{\lambda}_{I_j} = 0, 1 \leq j \leq m$$

Generalized Exploration

Control Rate Magnitude:

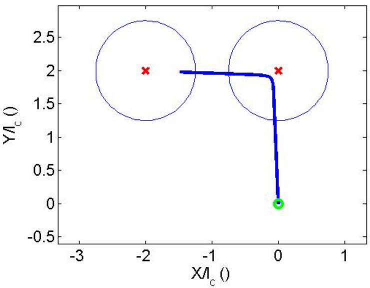
$$\dot{u}_i = \frac{\sum_{j=1}^{n} \rho_j(\overline{\chi_1}, \ldots, \overline{\chi_n}, w_1, \ldots, w_n) \frac{\partial \overline{f}(u_i)}{\partial u_i}}{\sum_{i=1}^{n} \overline{\lambda}_{\chi_i} \frac{d}{dt} \left(\frac{\partial \overline{f}(u_i)}{\partial u_i} \right)}$$

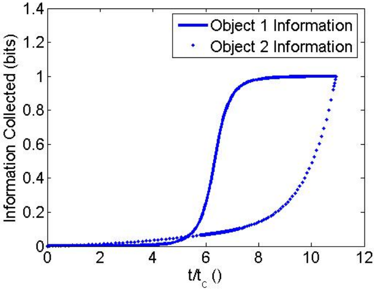
Generalized Exploration

UAV Control Rate Magnitude:

$$\boxed{\dot{\psi}_i = \frac{-4k_j^4 \lambda_{l_j} w_i (\cos(\psi_i)(b_j - y_i) + \sin(\psi_i)(a_j - x_i)) \cos(\psi_i)}{\lambda_{x_i} r^6}, 1 \leq i \leq n}$$

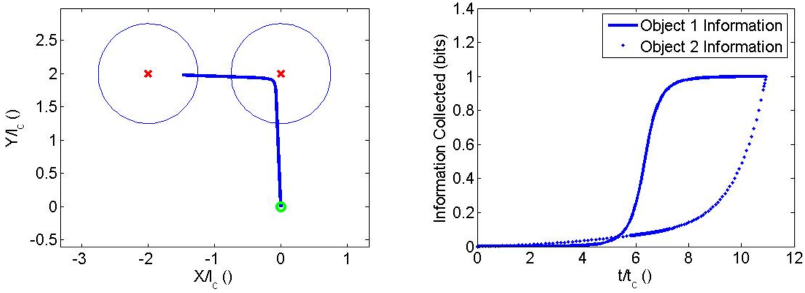
Information Constraints





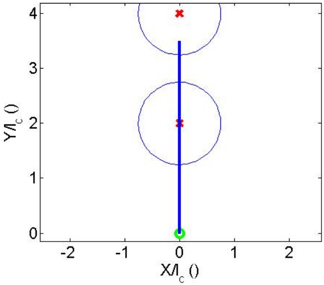
- Two constraints are active

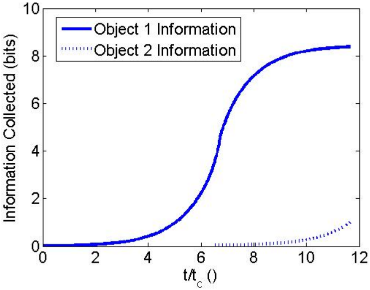
Information Constraints



- Two constraints are active
- These are critical objects

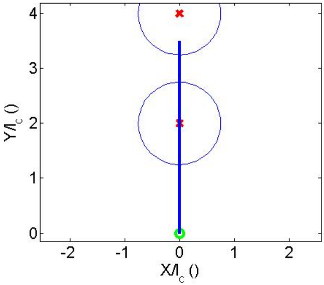
Information Constraints

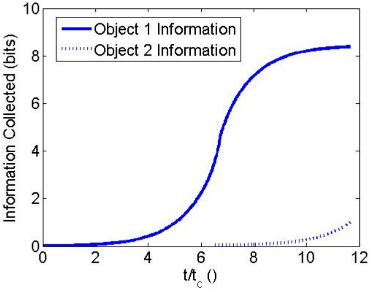




- One active constraint

Information Constraints





- One active constraint
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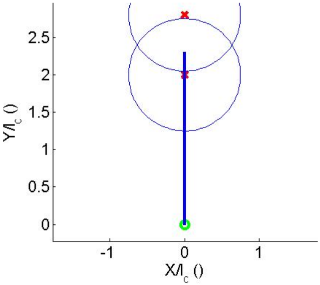
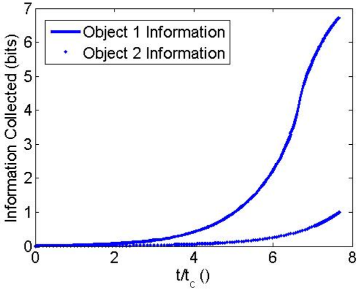
Information Constraints - Clustering





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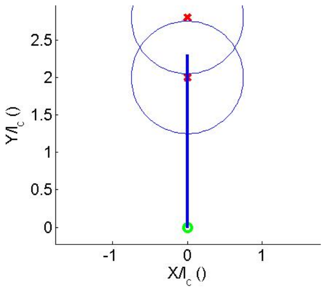

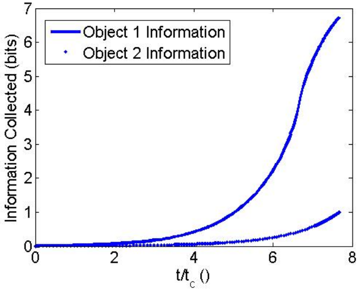

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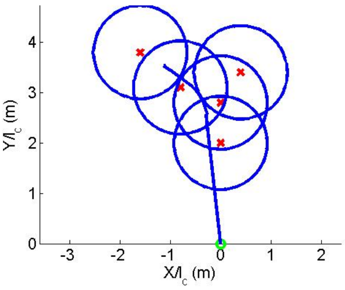
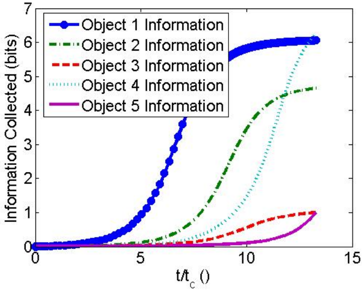
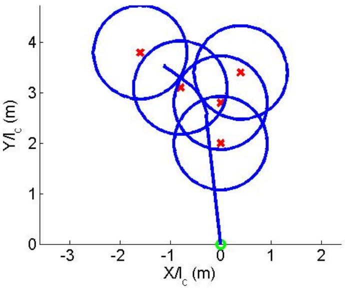
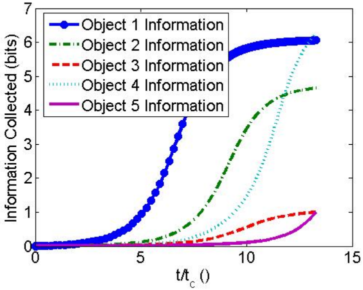




- Two active constraints

Information Constraints - Clustering

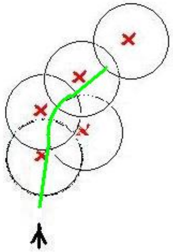




- Two active constraints
- Two critical objects

Clustered Objects

Proposed Procedure:



1. Connect the spheres of influence with a greedy path

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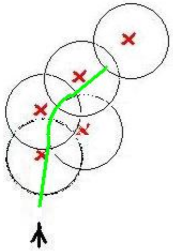

1. Connect the spheres of influence with a greedy path
2. Evaluate information gain

Clustered Objects

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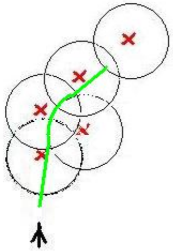

1. Connect the spheres of influence with a greedy path
2. Evaluate information gain
3. Those items with an information gain equal to 1 bit are critical objects

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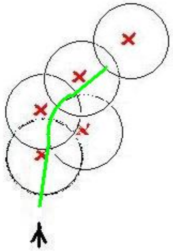
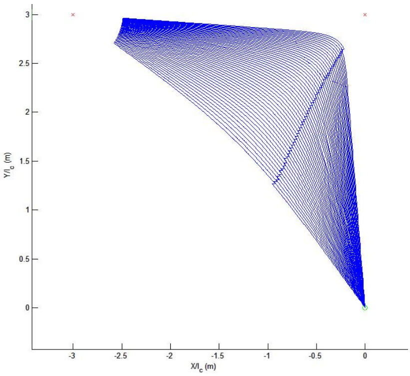


1. Connect the spheres of influence with a greedy path
2. Evaluate information gain
3. Those items with an information gain equal to 1 bit are critical objects
4. Optimize path for critical objects

Parametric Study

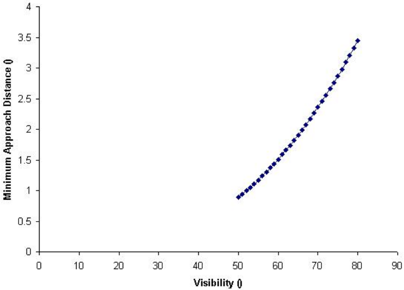


Parametric Study:

- Vary only K_1, the visibility of object 1
- Examine flight paths

Parametric Study

Radius of Closest Approach



Parametric Study:

- $f(K) \propto K^3$

Conclusions

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3. The exploration problem can be expressed as the communication of information over a noisy channel
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Conclusion

Autonomous exploration can be enhanced through optimal path planning

Future Work

- Investigate Exploration vs Exploitation
- Expand exploration formulation for cooperative vehicles

Thank you