Math 732: Bridgeland stability
Winter 2021
Alex Perry

Bridgeland stability is a powerful tool for extracting geometry from homological algebra. In particular, it gives a framework for studying moduli spaces of objects in a triangulated category, such as the derived category of an algebraic variety. The subject was born as a mathematical interpretation of work in string theory, but has since impacted many areas, including classical algebraic geometry, derived categories of coherent sheaves, enumerative geometry, homological mirror symmetry, and symplectic geometry.

Plan. The goal of this course is to develop the theory of Bridgeland stability, and to explain some of its applications within algebraic geometry. More precisely, we plan to cover the following topics.

(1) *General theory*: We will define stability conditions on triangulated categories and prove Bridgeland's deformation theorem, which gives the space of stability conditions a natural complex manifold structure.

(2) *Examples*: We will construct stability conditions on curves, surfaces, and (depending on time) some higher-dimensional examples.

(3) *Moduli spaces*: We will discuss moduli spaces of Bridgeland stable objects and their behavior under variation of stability. We will describe the geometry of these spaces in more detail in the particularly nice case of K3 surfaces, as well as some applications (e.g. to the birational geometry of moduli spaces of sheaves).

Along the way, we will cover some background material on semistable sheaves and derived categories.

Time and place. Tuesdays and Thursdays 1-2:30 pm on Zoom. The Zoom link will be distributed by email.

Discord. There will be a Discord server for participants to discuss material related to the class; please use it professionally. The link will be distributed by email.

Course webpage. http://www-personal.umich.edu/~arper/stability/

Textbook. There is no required textbook for the class. I will post some relevant references on the course webpage.

Prerequisites. I will assume basic knowledge of scheme theory and coherent cohomology. Familiarity with derived categories will be very useful, but I will briefly review the key points about this theory in class.

Homework. There will not be graded homework, but sometimes I will assign background reading or leave as exercises some details in lecture.

Grading. Grades will be based on class attendance and participation, as well as a final paper. The final paper should be:

- an exposition of a topic related to Bridgeland stability;
- 5 to 10 pages in length;
- written in latex and submitted as a pdf;
- emailed to me by the exam date for the class (10:30 am on April 29).

There are many possible options for the topic of the paper, ranging from background results that we'll use as blackboxes in class to more advanced topics relying on the machinery we've built up. I'd be happy to discuss suggestions for topics halfway through the semester. Please send me your proposed topic for approval by March 31.