ALGEBRAIC $\mathcal{D}$-MODULES (MATH 731)

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The theory of $\mathcal{D}$-modules provides an algebraic approach to solutions of systems of linear PDEs. Since its conception in the early 70s, this theory has had major applications in many different areas of mathematics, including algebraic geometry, commutative algebra, representation theory and even symplectic geometry.

Goal. The primary goal of this course is to develop the algebraic theory of $\mathcal{D}$-modules on an algebraic variety over a field of characteristic 0. Our perspective will be sheaf-theoretic: our main aim is to explain why $\mathcal{D}$-modules provide a coherent counterpart of constructible sheaves. In particular, we plan to discuss the notion of holonomicity (which is key finiteness condition in this theory), the 6-functor formalism, and (aspects of) the Riemann-Hilbert correspondence.

Prerequisites. We will use the language of schemes and coherent cohomology, say as developed in chapters 2 and 3 of Hartshorne's *Algebraic Geometry* book. It will also be quite useful to acquire some familiarity with the basics of derived categories (e.g., at the level of chapter 10 in Weibel's *Homological Algebra* book).

Homework. Problem sets will be sporadically posted to the course webpage.

References. There is no required reference. Some useful references are:

(1) *$\mathcal{D}$-modules, perverse sheaves, and representation theory* by Hotta, Takeuchi and Tanisaki. The material in this course corresponds roughly to (a subset of) the first half of the book.
(2) *Algebraic theory of $\mathcal{D}$-modules* by Bernstein. The material in this course corresponds roughly to chapters 1 through 5 of these (quite condensed) notes.
(3) *Algebraic $\mathcal{D}$-modules* by Borel et al.

Besides the above, there are many other lecture notes one can find online (e.g., those by Braverman, Elliott, Ginzburg, Milicic, Schapira, Yun), and you are strongly encouraged to browse these contemporaneously with the lectures.

Grades. Grades in this class will be based on participation and a final project. The final project should be:

- an expository paper that is submitted by the exam date (10:30am on December 15),
- approximately 10-15 pages in length,
- on a subject related to $\mathcal{D}_X$-modules (e.g., an application that has not been covered in class),
- written in latex and submitted electronically as a pdf, and
- should read like a research paper (as opposed to rough notes).

I will be happy to discuss suggestions for topics halfway through the semester.

Time and location. M/W from 1:00am to 2:30pm on Zoom. The link will be distributed over email.

Some guidelines for the lectures.

- It is required to leave the camera on during the lectures (virtual backgrounds are fine). If this is not possible for some reason, please contact me.
- Questions during the lecture are encouraged (similarly to to a regular class).
- Taking notes during the lectures is encouraged. (In fact, besides the usual reasons to do so, I have found this practice especially helpful myself during the pandemic: it helps avoid distractions on the internet.)
- There will be a Discord server attached to this class. The purpose of this chat room is to provide a convenient location/format for informally discussing material related to the class with other participants in the class. Please use this chat room professionally. A link will be circulated over email.

Office hours. TBA

Website. `http://www-personal.umich.edu/~bhattb/teaching/mat731f20/`

Date: Fall 2020.