NERS544: (Introduction to) Monte Carlo Methods

Assignment 1: **Elementary probability Theory** Fall 2016

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Q1.1, 20% Which of the following are candidate probability distributions? For those that are not, explain. For those that are, determine the normalization constant N. Those that are proper probability distributions, accompanied by mathematical proof, which contain moments that do not exist?

1. $f(x) = N\exp(-\mu x) \quad ; \quad 0 \le x < \infty$ where μ is a positive, real constant
2. $f(x) = N\exp(-\mu x) \quad ; \quad 0 \le x < \Lambda/\mu$ where μ, Λ are positive, real constants
3. $f(x) = N\sin(x) \quad ; \quad 0 \le x < \pi$
4. $f(x) = N\sin(x) \quad ; \quad \le x < 2\pi$
5. $f(x) = N/\sqrt{x} \quad ; \quad 0 \le x < 1$
6. $f(x) = N/\sqrt{x} \quad ; \quad 1 \le x < \infty$
7. $f(x) = Nx/(x^2+a^2)^{3/2} \quad ; \quad 0 \le x < \infty$ where a is a real constant

Q1.2, 20% Verify that the following are true probability distributions:

1. $p(x) = \dfrac{1}{\pi}\dfrac{\gamma}{\gamma^2+(x-x_0)^2} \quad ; \quad \forall\ |x| < \infty$ where γ is a positive, real constant
2. $p(\mu) = \dfrac{a(2+a)}{2}\dfrac{1}{(1-\mu+a)^2} \quad ; \quad -1 \le \mu \le 1,$ where a is a positive, real constant
3. $p(\Theta) = 4a\dfrac{\Theta}{(\Theta^2+2a)^2} \quad ; \quad 0 \le \Theta < \infty$ where a is a positive, real constant

Q1.3, 20% Consider the probability distribution,

$$p(x) = (1/2)[\delta(x-a)+\delta(x-b)] \quad ; \quad \forall\ |x| < \infty \text{ where } a, b \text{ are arbitrary, real constants}$$

What are all the moments of this distribution?

Q1.4, 20% Consider the probability distribution,

$$p(x) = N[\Theta(x-a)-\Theta(x-b)] \quad ; \quad \forall\ |x| < \infty. \text{ where } a, b \text{ are arbitrary, real constants}$$

1. Can this be a proper pdf?
2. If so, what is N?
3. Does it matter what the relative values of a and b are?
4. What are all the moments of this distribution?

Q1.5, 20% Prove:

$$\mathrm{var}\{x \pm y\} = \mathrm{var}\{x\} + \mathrm{var}\{y\} \pm 2\ \mathrm{cov}\{x, y\}$$

Simplify in the case that x and y are independent.