# NERS544: (Introduction to) Monte Carlo Methods

## Assignment 4: **Lewis Theory** Two week assignment, single weightFall 2016

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**Due:** Thursday Oct. 20, 2016 before class

Submit your answers to 1.(a) on paper and 1.(b) as a Matlab script M-file via email.

1. 10,000 particles start out in an infinite, unbounded medium with initial position vector $\vec{x}_0 = 0$ and initial direction vector $\vec{u}_0 = \hat{z}$. The interaction probability is:

   $p(s_i)\ \mathrm{d}s_i = \Sigma_0 \exp(-\Sigma_0 s_i)\ \mathrm{d}s_i$

   where $s_i$ is a measure of the pathlength of the particle along its current direction of motion starting from its current position to the $i$'th interaction point. $\Sigma_0$ is a constant in this case. Having interacted, the particle scatters isotropically and can scatter again, repeatedly, according to the same scattering law. This repeated scattering and transport results in a probability distribution function of the form

   $F(\vec{x}, \vec{u}, s)\ \mathrm{d}\vec{x}\ \mathrm{d}\vec{u}\ \mathrm{d}s$

   which you will develop stochastically through a Monte Carlo program. The variable $s$ is a measure of total pathlength traveled, *i.e.* $s = \sum_i^{N(s)} s_i$. Note that the number of interactions $N(s)$ is a probabilistic quantity.

   (a) Consider the following moments:
   - $\langle x \rangle \pm \sigma_{\langle x \rangle}$ (*i.e.* average $x$)
   - $\langle y \rangle \pm \sigma_{\langle y \rangle}$ (*i.e.* average $y$)
   - $\langle z \rangle \pm \sigma_{\langle z \rangle}$ (*i.e.* average $z$)
   - $\langle x\vec{u} \cdot \hat{x} \rangle \pm \sigma_{\langle x\vec{u} \cdot \hat{x} \rangle}$ (*i.e.* average correlation of $x$ with the $x$-axis direction cosine)
   - $\langle y\vec{u} \cdot \hat{y} \rangle \pm \sigma_{\langle y\vec{u} \cdot \hat{y} \rangle}$ (*i.e.* average correlation of $y$ with the $y$-axis direction cosine)
   - $\langle z\vec{u} \cdot \hat{z} \rangle \pm \sigma_{\langle z\vec{u} \cdot \hat{z} \rangle}$ (*i.e.* average correlation of $z$ with the $z$-axis direction cosine)
   - $\langle xy \rangle \pm \sigma_{\langle xy \rangle}$ (*i.e.* average correlation of $x$ and $y$)
   - $\langle xz \rangle \pm \sigma_{\langle xz \rangle}$ (*i.e.* average correlation of $x$ and $z$)
   - $\langle yz \rangle \pm \sigma_{\langle yz \rangle}$ (*i.e.* average correlation of $y$ and $z$)
   - $\langle x^2 \rangle \pm \sigma_{\langle x^2 \rangle}$ (*i.e.* average $x^2$)
   - $\langle y^2 \rangle \pm \sigma_{\langle y^2 \rangle}$ (*i.e.* average $y^2$)
   - $\langle z^2 \rangle \pm \sigma_{\langle z^2 \rangle}$ (*i.e.* average $z^2$)

   Which moments would you expect to be zero? Why?

   Which moments would you expect to be equal to each other? Why?

   (b) For $\Sigma_0 = 1$ and $\lambda \equiv \Sigma_0 s = 0.001$, 0.002, 0.005 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, 200, 500, 1000, tally the above moments, their estimated errors, alongside their theoretical predictions.

   Plot the results divided by $\lambda^n$ as a function of $\log(\lambda)$. $n$ is some power of $n$ such that the correct small and large pathlength asymptotic forms (according to Equations (9.32)–(9.45)) divided by this $\lambda^n$, is a constant. (*e.g.* $\langle z^2 \rangle / \lambda^2$ would be expected to go to the constant 1 for small $\lambda$ and $\langle z^2 \rangle / \lambda$ would be expected to go to the constant 2/3 for large $\lambda$.) If you expect the moment to be zero (on average), set $n = 0$. Verify that both the $\lambda \longrightarrow 0$ and the $\lambda \longrightarrow \infty$ limits discussed in this chapter are reached.