NERS544: (Introduction to) Monte Carlo Methods

Assignment 7: **NERS Models** 3-week assignment, weight = 3 Fall 2016

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Simple models of nuclear and radiological interactions

Do the first problem and one of the remaining two for full credit. If you attempt a third problem, you wall be given extra credit.

P3.1, 80% An electron radiotherapy beam irradiates (end-on, normal to the flat surface, uniform in extent) a circular end of right circular cylinder with radius 11, and depth 11. The radius of the beam is 1. Divide the cylinder into 22 equally thick slabs normal to the beam. The scattering of the electron beam takes places as follows. After every transport distance of length 0.1, the polar angle of scattering (relative to the electron's direction) is given by the distribution:

$$p(\Theta)\sin(\Theta)\mathrm{d}\Theta = \frac{(2+a)a}{2}\frac{\sin(\Theta)\mathrm{d}\Theta}{(1-\cos(\Theta)+a)^2} \ ,$$

where a is a function of the total pathlength traveled

$$a(l) = \frac{a_0}{(1-l/10)^3} \ .$$

When the electron reaches a total pathlength of 10, it falls to zero energy and no longer moves.

Tally the average pathlength traveled (per incident electron) in each slab as a function of a_0 which may range from 10^{-6} (a megavoltage beam) to $a_0 = 1$ (a few keV beam). Use a logarithmic grid of the form:

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a_0 = logspace(-6,0,19)
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Owing to the set up of this problem, no electrons should escape the target except through backscattering. (This should be very small for the smallest a_0 and substantial for the largest a_0.) There also should be no electrons showing up in the most downstream slabs.

Tallies Plot the average pathlength and estimated error for each of the 19 values of a_0.

Plot the backscatter coefficient (number of electrons that escape through the from surface divided by the number of histories) along with estimated statistics as a function of a_0.

What to turn in email your code and a .pdf or a .doc that includes a discussion as well as all the 20 figures described above.

Example output

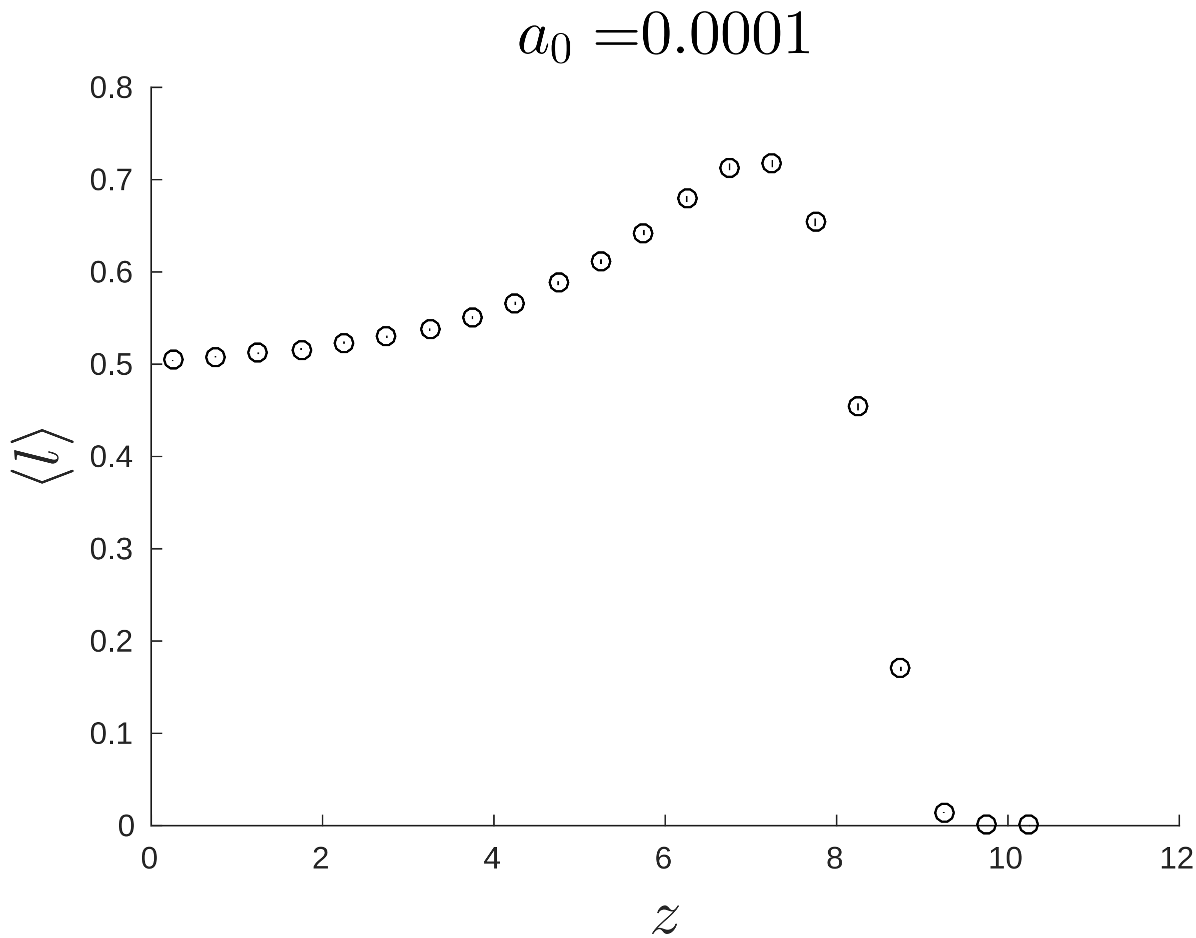

Figure 1: This is one of the 19 required plots: the relative tracklength distribution for $a_0 = 0.0001$. Note that the error bars should be shown. In this case, for 20000 histories, they are so small that they are within the plot symbol.

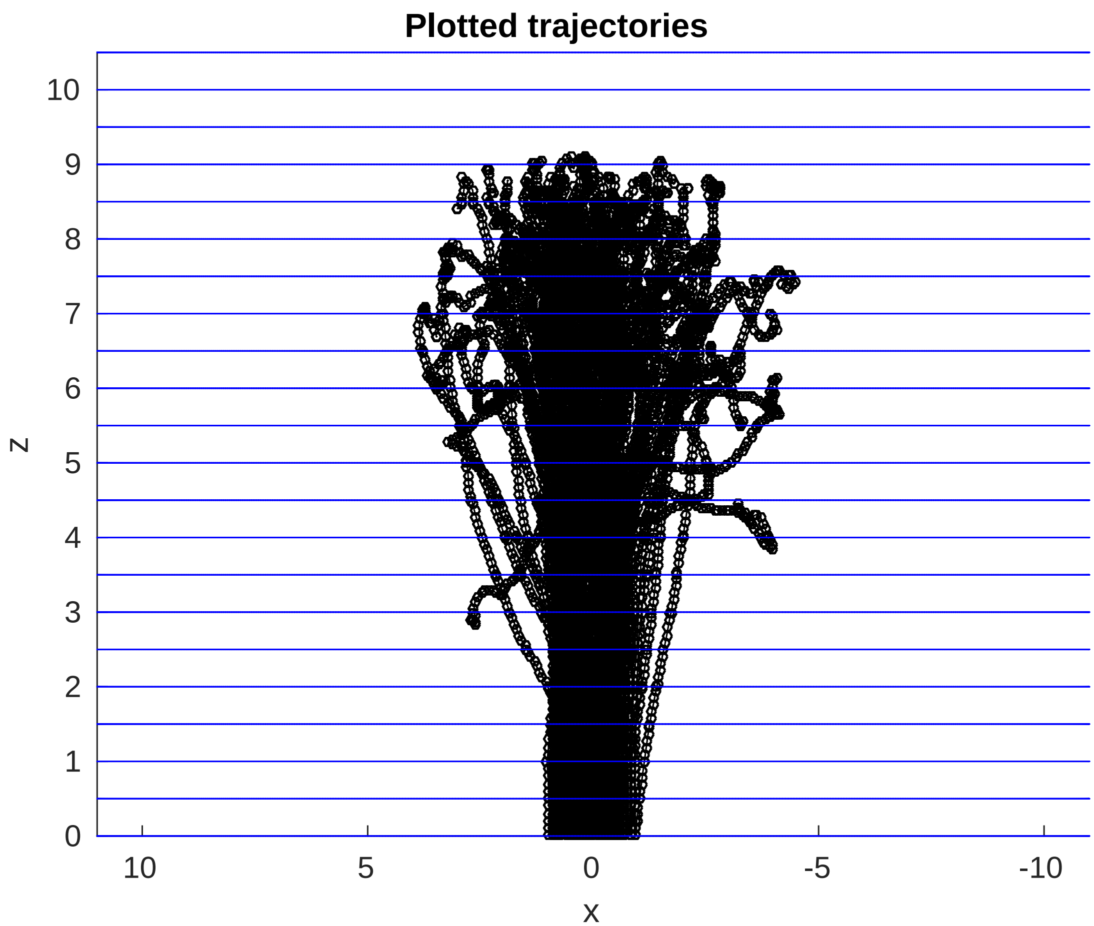

Figure 2: This is an optional plot useful for debugging. This shows the tracks of the first 100 particles for $a_0 = 0.0001$.

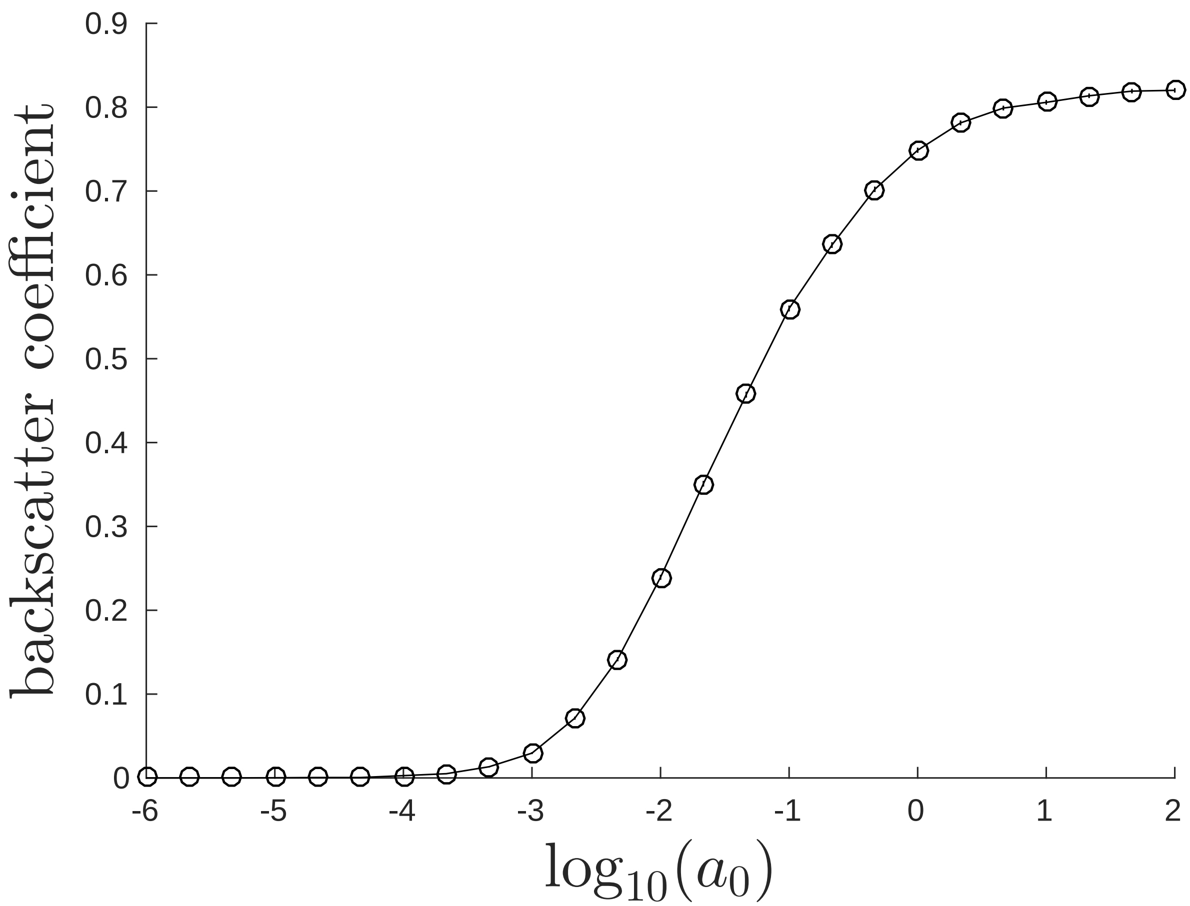

Figure 3: This is another mandatory plot of the backscatter factor. Note that the error bars should be shown. In this case, for 20000 histories for each a_0, they are so small that they are within the plot symbol. To create this plot I went past the specifications of the assignment to extend a_0 to 100, so that I could show the asymptotic behavior for large a_0.

P3.2, 20% An ellipsoidal nucleus has a surface approximated by the shape:

$$x^2 + y^2 + (z/a)^2 = 1$$

where a is an eccentricity factor. When $|a| > 1$ it looks like a "cigar" with rounded ends, when $|a| < 1$ it looks like a pancake, and when $|a| = 1$ it is a sphere. In the limit $a \longrightarrow \infty$, the ellipsoid becomes a cylinder. In the limit $a \longrightarrow 0$, the ellipsoid becomes a circular disk.

A proton approaching a neutral atom does not see much of the positive nuclear charge until it penetrates the electron cloud and feels its repulsive force. We shall model this interaction as a contact interaction with complete reflection at the surface, according to the law of reflection discussed in class. The purpose of this study is to examine the angular distributions of the reflected protons assuming that they started out moving along the positive z axis.

What to turn in

Via email: Your code and a report describing your findings on the angular distributions for at least $a = [0.1, 0.5, 1, 2, 10]$, or more, with enough range in a to explain the limiting cases. Your solution should be obtained as a result of simulation, not analytic development. For the case of a sphere, the distribution is known to be isotropic. You are not expected to provide the theoretical distributions. Students from my NERS 312 class were taught how to do this and you may include that if you wish.

Example output

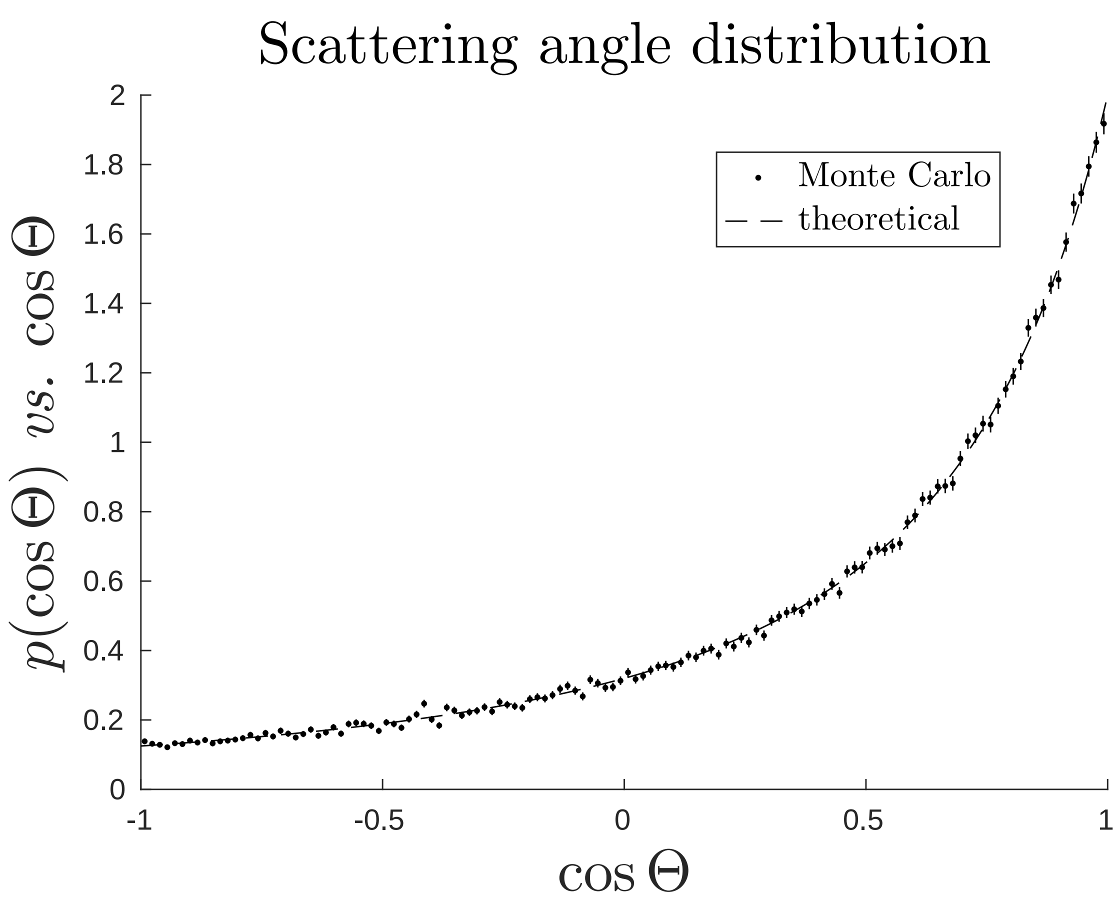

Figure 4: This is probability of the scattering angle Θ *vs.* $\cos\Theta$. 128 mesh points, 128000 histories, eccentricity factor, $a = 2$. Note that the error bars should be shown.

P3.3, 20% **An electron trapped in a nucleus** (from, for example, β^--decay) is repelled by the positive charge at the nuclear surface. We shall model this as an incomplete reflection by the interior of a unit cube. (OK, nuclei are not cubes, but I'd like you to practice making rectilinear solids.) We shall call the probability of reflection p_r. The electron can also scatter from individual protons in the nuclear material. We shall model this using in interaction probability distribution $p(l)\mathrm{d}l = (1/\lambda)\exp(-l/\lambda)\mathrm{d}l$. (OK, this is also a fiction, but I want you to practice attenuation-type interactions.) When the electron scatters, it does so isotropically. Eventually, the electron will escape by not being reflected inward. The particle is set in motion at a random point inside the nucleus, and in a random direction.

What to turn in

Via email: Your code and as a minimum, compute the average pathlength $\langle l\rangle \pm \sigma_{\langle l\rangle}$ as a function of p_r and λ for the special cases 1) $p_\mathrm{r} = 0$ and $0 < \lambda < \infty$ and 2) $0 \le p_\mathrm{r} < 1$ and $\lambda \to \infty$. (Graphical output please.) For extra credit, map out the $\langle l\rangle(p_\mathrm{r}, \lambda)$ surface.

Example output

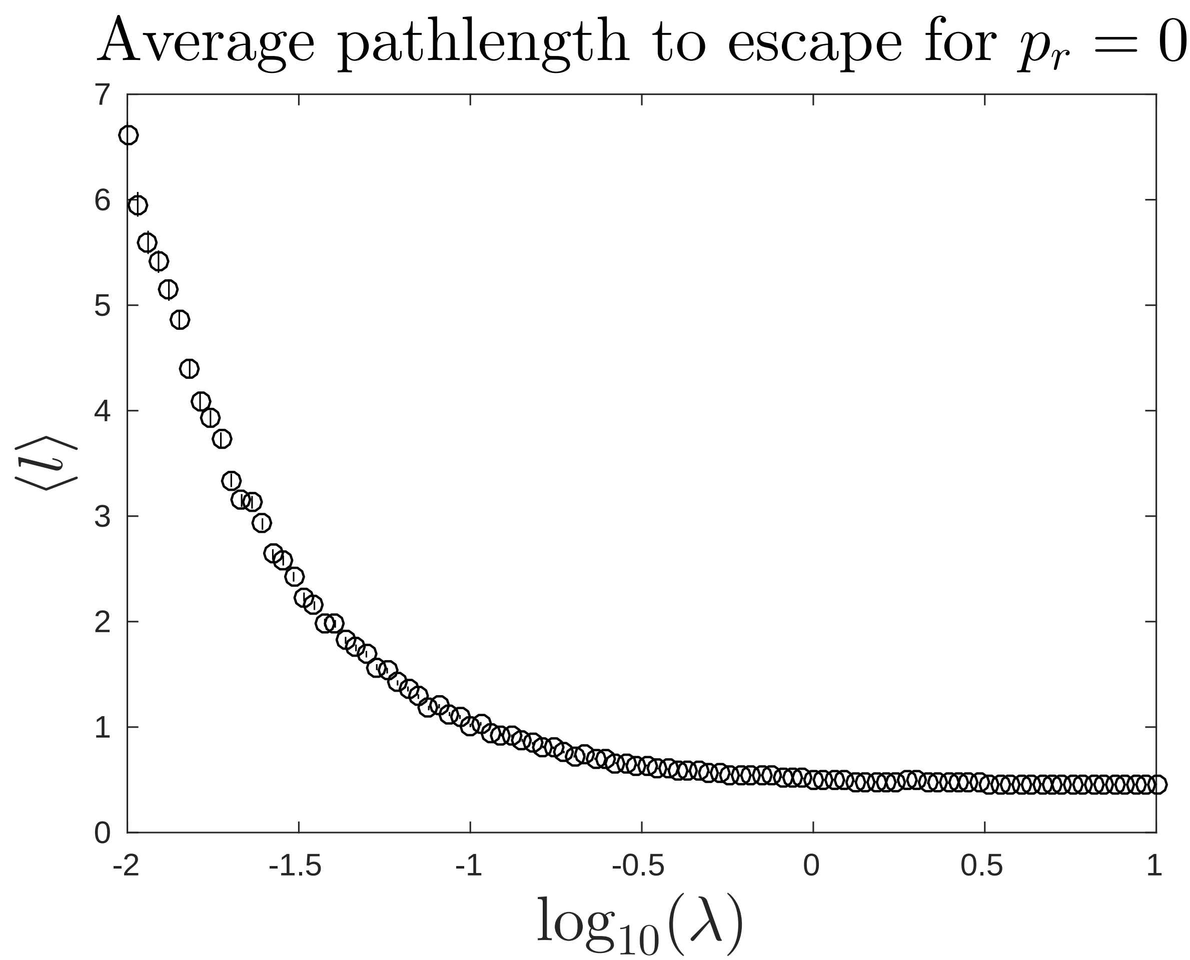

Figure 5: This is result for $\langle l\rangle \pm \sigma_{\langle l\rangle}$ for $p_r = 0$.

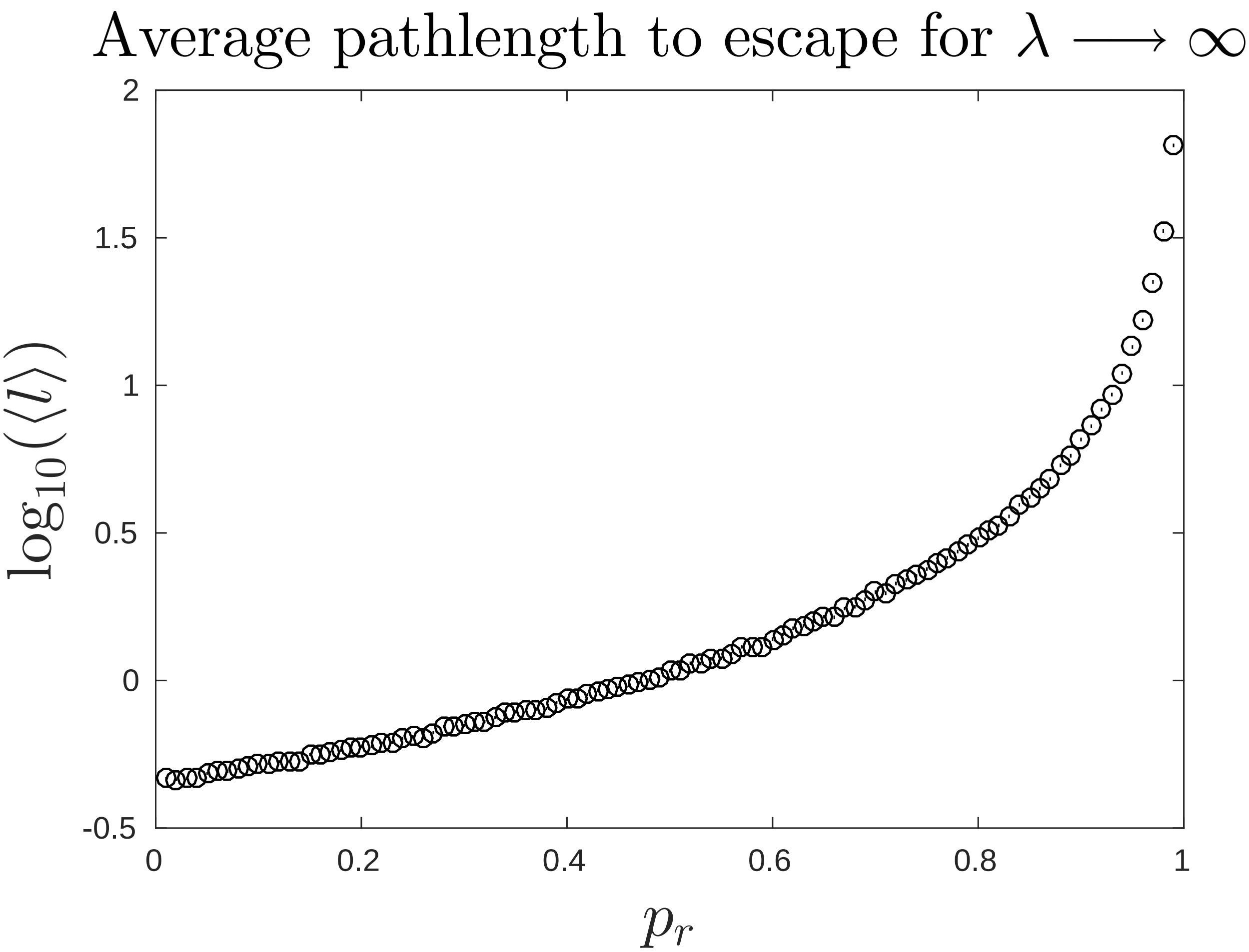

Figure 6: This is result for $\langle l \rangle \pm \sigma_{\langle l \rangle}$ for $\lambda \longrightarrow \infty$.

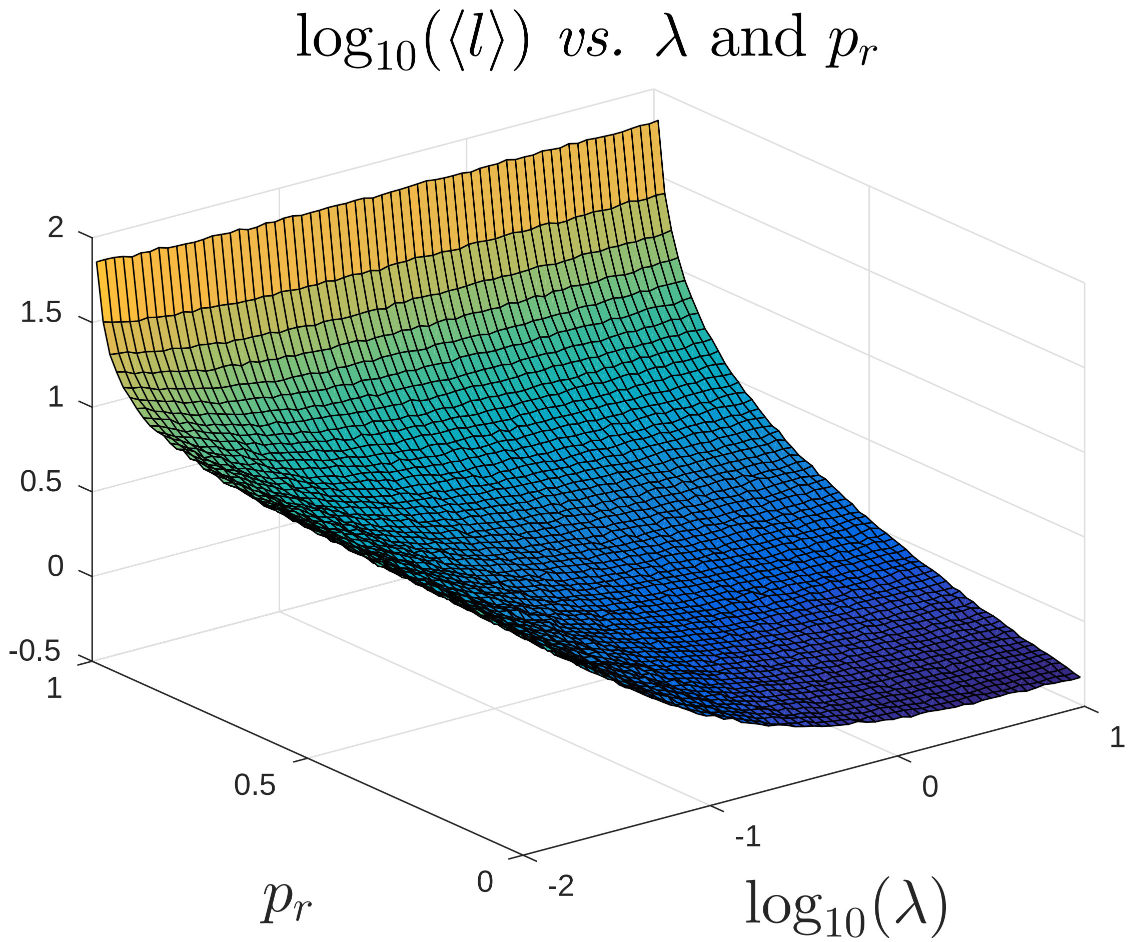

Figure 7: This is result for $\langle l \rangle \pm \sigma_{\langle l \rangle}$ *vs.* λ and p_r.