

THE MEASUREMENT OF WIND VELOCITY BY SONIC MEANS

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ABSTRACT

The theory of the measurement of wind velocity by sonic means is presented. Two methods are discussed for determination of the rectangular components of the wind vector by use of the sonic principle. The response characteristics and application of sonic anemometers are discussed. A sonic anemometer for the measurement of the horizontal wind vector is described.

1. Introduction

Considerable attention has been directed in recent years to accurate determination of the wind vector. It has been long recognized that conventional wind instruments possess uncertainties which become significantly large when the wind is not steady. It is the purpose of this article to consider the theory and description of an absolute anemometer which utilizes the propagation characteristics of wind-borne sound waves.¹

The speed of propagation of a sound wave is related to the absolute temperature and water-vapor content of the atmosphere through which it is traveling by the following equation (Barret and Suomi,

1949):

$$c = 20.067 \left[T \left(1 + 0.319 \frac{e}{p} \right) \right]^{\frac{1}{2}}, \quad (1)$$

where c is the velocity of sound in meters per second, T the temperature in degrees Kelvin, e the water-vapor pressure, and p is the static pressure, the latter both expressed in the same units.

Sound energy radiated into a still, uniform atmosphere will, in general, propagate as a spherical wave concentric with its source, provided that the source is small compared to a wave length (fig. 1, left). If the atmosphere is in uniform motion (fig. 1, right), the emitted wave propagates as a spherical wave whose center translates at a velocity equal to that of the wind (Milne-Thomson, 1950). The physical explanation for this phenomenon is that sound is a compressional wave which is propagated by virtue of the

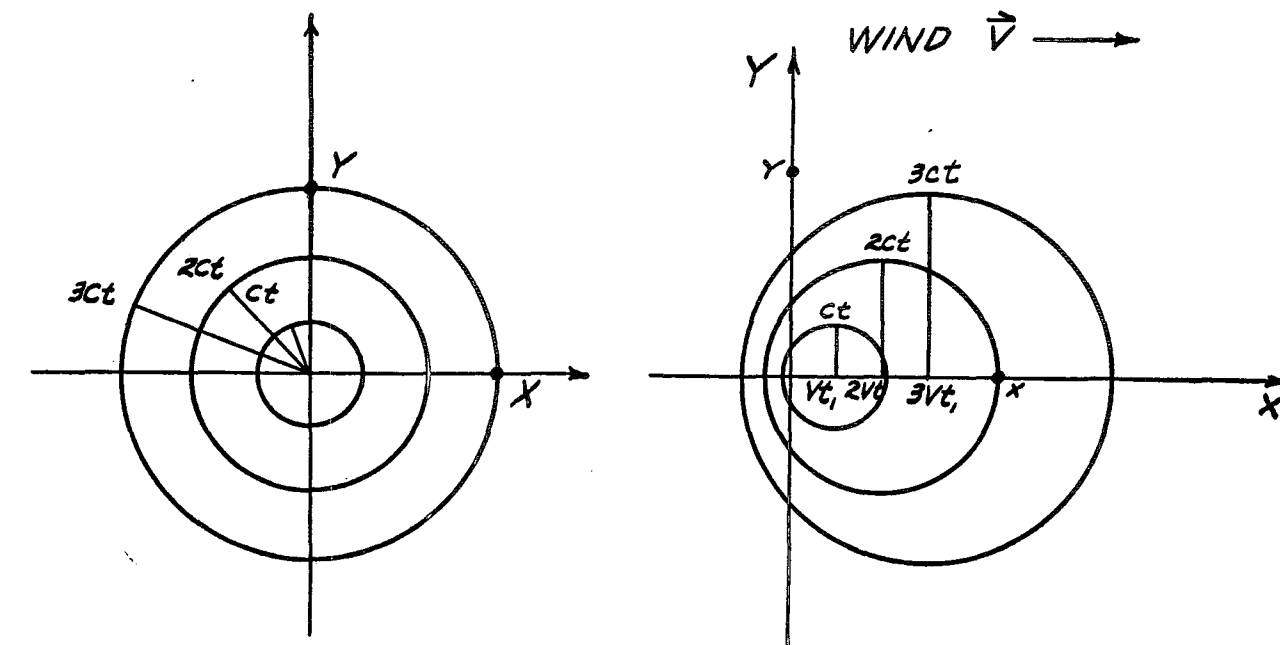


FIG. 1. Propagation of sound wave in stationary medium (left) and moving medium (right).

¹ Reference is made to a sonic anemometer used at the Great Plains turbulence field program (Davidson and Lettau, 1953). Also, a sonic anemometer is described in a Cook Research Laboratory report, Report PR-41-4, May-June-July 1954.

elastic properties of the atmosphere. Consequently, any motion of the atmosphere is superimposed upon the propagation velocity of the wave.

2. Theory of the measurement

The propagation of a spherical wave in a homogeneous, uniform moving medium can be expressed in rectangular coordinates as follows (it is assumed that the wave is initiated at zero time from the point $X = Y = Z = 0$):

$$(X - V_x t)^2 + (Y - V_y t)^2 + (Z - V_z t)^2 = (ct)^2, \quad (2)$$

where (X, Y, Z) is a point on the constant-phase surface, (V_x, V_y, V_z) are the rectangular velocity components of the wind, t is the time required for the constant-phase surface to reach point (X, Y, Z) , and c is the velocity of sound with respect to the medium. Equation (2) can be expressed as a quadratic in terms of t , as follows:

$$t^2(V_x^2 + V_y^2 + V_z^2 - c^2) - 2t(XV_x + YV_y + ZV_z) + (X^2 + Y^2 + Z^2) = 0. \quad (3)$$

The time, t_x , for the surface to reach point $(X, 0, 0)$ is

$$t_x = \frac{X[V_x - (c^2 - V_y^2 - V_z^2)^{1/2}]}{V_x^2 + V_y^2 + V_z^2 - c^2}. \quad (4)$$

Similar equations can be obtained for t_y and t_z , the times required for the wave front to reach point $(0, Y, 0)$ and $(0, 0, Z)$, respectively.

From (4) it is seen that the time required for the wave front to reach a point on the x -axis is a function not only of the x -component of velocity, V_x , but of the other components as well. It should be noted that the value of t_x will vary from that in a still atmosphere even with a wind which has no x -component of velocity.

The difference, Δt_x , between the times required for the wave front to reach the point $(X, 0, 0)$ and the point $(-X, 0, 0)$ is obtained from (4):

$$\Delta t_x = \frac{2XV_x}{c^2 - (V_x^2 + V_y^2 + V_z^2)}. \quad (5)$$

Δt_x is thus seen to be a function of the x -component of the wind and also of the total wind speed. It is important to note that, in the absence of an x -component of wind, Δt_x is zero.

In the case of a continuously emitting sound source, (4) and (5) may be expressed in terms of the phase, N , and the frequency of oscillation, f , of the sound wave, since

$$N = ft. \quad (6)$$

Equation (4) becomes

$$N_x = \frac{fX[V_x - (c^2 - V_y^2 - V_z^2)^{1/2}]}{V_x^2 + V_y^2 + V_z^2 - c^2}. \quad (7)$$

Equation (5) becomes

$$\Delta N_x = \frac{2fXV_x}{c^2 - (V_x^2 + V_y^2 + V_z^2)}. \quad (8)$$

Equations (7) or (4), and (8) or (5), suggest two methods for the determination of wind velocity by sonic means.

Method I.—Measurements of the component phases N_x, N_y and N_z , or the transit times t_x, t_y and t_z , may be obtained by microphones located at $(X, 0, 0)$, $(0, Y, 0)$ and $(0, 0, Z)$. These quantities may be related to the wind components by three equations similar to (7) or three equations similar to (4). To obtain the individual wind components, it is necessary to solve the equations simultaneously, since each quantity is a function of the three wind components.

From a practical standpoint, it would be desirable if each component phase or time delay were mainly a function of a single wind component, as in that case the computation problem would be minimized. As an example, if N_x were considered a function only of the x -component of the wind, (7) written for the x -component would become

$$N_x = fZ/(c + V_x). \quad (9)$$

The component of the wind which is determined from (9) will be in error because of the above simplification. An equation relating this error, ΔV_{zc} , to the speed of the horizontal wind is obtained by equating N_x in (9) to N_x in (7) written for the x -component:

$$\Delta V_{zc} = c - [c^2 - (V_x^2 + V_y^2)]^{1/2}. \quad (10)$$

A similar error equation is obtained if transit-time equations are employed.

A plot of this equation is presented in fig. 2. This equation may be applied to the other wind components by a cyclic change in subscripts. From an inspection of fig. 2, it may be concluded that this

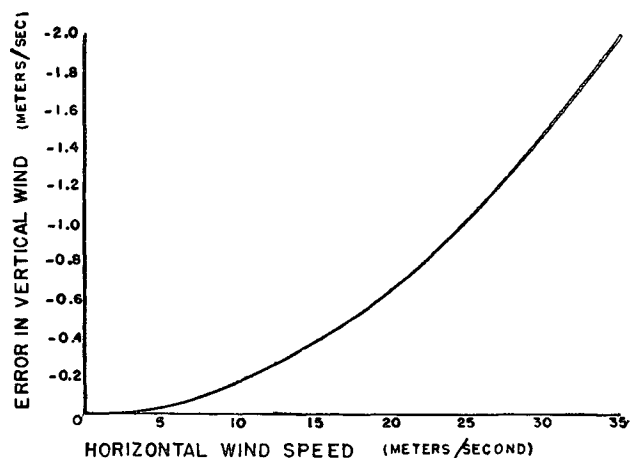


FIG. 2. Error in vertical wind speed as function of horizontal wind speed.

simplification of method I cannot be used if highly precise measurements are desired.

Method II.—Measurements of ΔN_x , ΔN_y and ΔN_z , or of Δt_x , Δt_y and Δt_z , may be obtained by six microphones placed equidistantly from the origin of the three coordinate axes. These quantities may be related to the component winds by means of (8) for ΔN_x and the two corresponding equations for ΔN_y and ΔN_z , or from (5) and the corresponding equation for Δt_y and Δt_z . It is necessary to solve each set of the equations simultaneously to determine the wind components, since each phase shift and time delay is a function of the total wind vector.

If the quadratic terms are neglected in (8), each phase shift becomes a function of a single wind component. The error introduced by this approximation may be determined by writing (8) in the following form:

$$V_x = \frac{\Delta N_x c^2}{2fX} \left[1 - \frac{V_x^2 + V_y^2 + V_z^2}{c^2} \right]. \quad (11)$$

Omission of the second term from the terms within the brackets introduces an error in the precision of measurement of V_x which depends upon the ratio of the square of the total wind speed to the square of the velocity of sound. For wind speeds less than 35 m/sec, the error in the measurement of a component velocity is less than 1 per cent of the true value. The same considerations apply to a determination of V_x from a measurement of Δt_x .

3. Characteristics of a sonic anemometer

Calibration.—The sonic anemometer is an absolute instrument. Its calibration may be predicted on the basis of the theory given above, provided the conditions under which the theory was derived are valid.

The principal conditions imposed by theory are that the sound source generate a spherical sound wave and that microphones measure the phase of the wave at a point.

A spherical wave will be generated by a sphere whose diameter pulsates periodically with time. An equivalent of a pulsating sphere can be constructed by arranging a group of speakers, whose diameters are small compared to the operating wavelength, about a point. If the speakers are phased correctly, the resulting waves will be closely spherical in shape.

The microphones which are used to measure the phase of the wave are not point pickups. The phase measured by a microphone of finite size will differ from the true phase because of the diffraction of the sound wave about the microphone and because of the averaging of the wave front by the diaphragm of the microphone. These effects are both proportional to the ratio of the wavelength, λ , to the diameter, d , of the micro-

phone and also to the radius of curvature of the wave front (Beranek, 1949). Phase measurements made with well designed microphones are usually not in error by more than 10 deg. This phase error is constant in value except for the slight variation due to the change in λ caused by wind velocity. Consequently this error will not appreciably affect the accuracy of measurement.

The reflection of sound from nearby objects, such as the frame and supporting members of the anemometer, introduces additional errors in the measurements of N_x and t_x due to the interference of the reflected and direct wave components. This error can be minimized by suitably locating the anemometer and by covering the frame and supporting members with an acoustically dead surface.

Response time.—The time, t_R , required for an instrument to respond from 0.1 to 0.9 of an applied step function is defined as the rise time. t_R for this instrument depends mainly upon two factors: the frequency band-width of the instrument, and the magnitude of the change of the velocity.

The limitations on the rise time due to the frequency response of a linear system have been discussed in the literature (Elmore and Sands, 1949). It is shown that one over the rise time is approximately equal to three times the band width of the system, provided certain conditions are met concerning the fall-off characteristic of the frequency-response curve. The band width of an instrument constructed with standard audio components could have an upper half power frequency of 10^4 cy/sec. This corresponds to a rise time of 3×10^{-5} sec.

It should be remembered that the sonic anemometer does not measure the velocity at a point. The rise time computed gives a measure of the response of the sonic anemometer if all the air contained in the volume described by the acoustical transmitters and receivers were to undergo an instantaneous change in velocity. A meaningful measure of the response of this instrument should include the delay time required for a step function of velocity to traverse the separation distance X between the acoustical transmitters and receivers, for it is only after this has occurred that the instrument realizes the occurrence of the change in velocity. This delay time is equal to $X/(V_x + \Delta V_x)$, where V_x is the velocity at $t = 0$, and ΔV_x is the change in velocity. With X equal to 1 m and V_x equal to zero, the delay time is equal to 1 sec for 1 m/sec change, and is equal to 0.1 sec for a 10 m/sec change. The delay time, which is usually much greater than the rise time, can be reduced only through miniaturization of the sensing head.

4. A sonic anemometer for measurement of horizontal wind

An instrument (fig. 3) which measures the horizontal wind vector has been constructed, based upon the principles discussed in method I.

Inspection of (9) shows that the phase shift between (0,0,0) and (X,0,0) depends directly upon the product fX for a given wind speed. The separation X , a measurement of the distance over which wind speed is averaged, is a parameter which depends upon the application of the instrument. The frequency of oscillation, f , is determined principally by the desired sensitivity of the instrument. A more complete discussion of these quantities will be found below.

The description of this instrument may be logically divided into two parts: the specification of the sensing head, and the specification of the associated electrical equipment.

Sensing head.—The frame of the sensing head is a rigidly constructed right isosceles triangle. A dynamic microphone is mounted at each of the base angles. The microphones are oriented so that each faces in the direction of the right angle. A sound source, consisting of a 3-in permanent-magnet speaker, is mounted at the right angle and located so that it radiates along the perpendicular bisector of the base. The legs of the triangle, which represent the averaging distance in (9) are 1 m in length. This distance was chosen so that the space-averaging properties of the instrument would approximate those of standard station anemometers.

Electrical equipment.—The functions of the electrical circuit in this instrument are to generate the electrical signal which derives the sound source and

to measure the phase shift which occurs at the microphone due to the wind.

A survey of the literature shows the existence of many types of phase detectors. An extremely simple and reliable phase detector was built with a 6BN6 gated-beam tube (Holman, 1953). This tube is constructed in such a manner that each of its control grids functions as an on-off switch for plate current whenever the grid potential falls below a critical value. Oscillating potentials applied to the grids produce a rectangular pulse of plate current whose height is constant, and whose width is proportional to the time the grids are coincidentally above the critical potential. The critical potential may be adjusted so that when the grid potentials are 180 deg out of phase the tube is turned off, while if the potentials are in phase, the tube conducts half the time. Use of a circuit similar to that given by Holman (1953) results in a sensitivity of 1.5 volts per degree phase shift, which is linear to within 5 per cent over a range from 20 to 160 deg. This allows 70 deg phase shift to be used for positive wind and 70 deg phase shift for negative wind components.

The generator must develop stable, sinusoidally varying voltage at a level sufficient to mask the ambient acoustical noise level at the microphone.

The operating frequency of the generator is determined from (9). Applying the condition that the maximum indicated speed, which is taken as 35 m/sec, be equivalent to the maximum allowable phase shift, which is 70 deg, we find the product fX to be 663 deg cycles meters per second. The averaging distance was fixed at 1 m, so that f must equal 663 cy/sec.

Generator stability must be such that random variation of frequency shall not contribute an uncertainty of measurement greater than the desired precision of measurement. Computation with a logarithmically differentiated form of (9) shows that a precision of 0.5 m/sec, which is a reasonable requirement for a station anemometer, demands a generator stability of 0.16 per cent.

A model 200c Hewlett-Packard audio oscillator has been found to meet the above requirements.

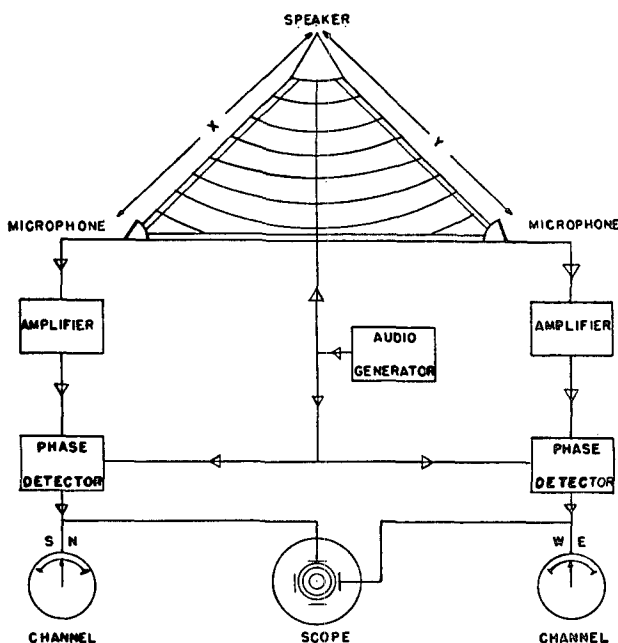


FIG. 3. Block diagram of station sonic anemometer.

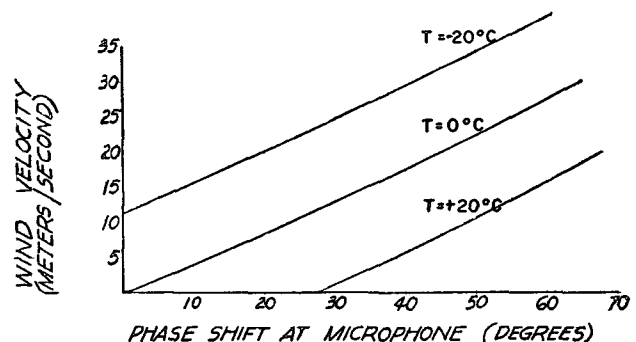


FIG. 4. Relation between component wind speed and component phase for station sonic anemometer.

Presentation of data.—Two forms of presentation are used with this instrument: a meter presentation and a cathode-ray tube presentation.

The output voltage of each phase detector was measured directly by means of a vacuum-tube voltmeter. The circuit constants were adjusted so that with zero component velocity the needle of the meter rests at the center of the scale. Displacements to the right or left of center correspond to positive or negative component wind.

A polar-coordinate presentation of the horizontal wind vector was obtained by connecting the output of the phase detectors to the horizontal and vertical deflection systems of a cathode-ray oscilloscope. This presentation enables the observer to see at a glance the endpoint of the actual wind vector.

Response characteristics.—Calibrations of component velocity *versus* phase, based on (9), are given in fig. 4 for three representative atmospheric temperatures. A linear interpolation may be applied for intermediate values of temperature. The error in the determination of a component velocity resulting from the use of this calibration curve may be approximately determined by reference to fig. 2.

A comparison of the indications of this sonic anemometer with those of a three-cup Gurley anemometer (Model 6040), both instruments roof mounted, was made during nearly steady winds. Over the test range of 4 to 12 m/sec, it was found that the maximum difference in indications was less than 10 per cent of the reading of the sonic anemometer. It is realized that this test is unsatisfactory, inasmuch as there was no control over the velocity of the wind and the uncertainty of the standard employed was not known. The information is presented as the only data available.

The frequency band-width of this instrument with cathode-ray tube presentation is 11,000 cy/sec, which corresponds to a rise time of 3×10^{-5} sec. The rise

time with meter presentation is approximately 0.25 sec.

The delay time, which is equal to the separation distance divided by the magnitude of $V + \Delta V$, is 1 sec for a 1 m/sec change in velocity and 0.1 sec for a 10 m/sec change when V is equal to zero.

5. Application of the sonic anemometer

It is expected that the sonic anemometer will be primarily used as a research instrument; the station anemometers in use today measure the horizontal wind vector with an accuracy which is sufficient for forecasting needs.

An instrument based upon method II will deliver electrical signals whose magnitudes are proportional to the velocity of the component winds with a precision of 1 per cent. These signals may be recorded directly or fed into an electrical computer to obtain the statistical properties relating to the wind components or other meteorological quantities.

Work has been initiated on the development of a sonic anemometer for the measurement of the three components of wind. This instrument will have a separation distance of 10 cm and is intended for use in turbulence studies.

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