Dynamical Modeling of the Grand Piano Action

Brent Gillespie

Center for Computer Research in Music and Acoustics (CCRMA)
Dept. of Mechanical Engineering, Stanford University, Stanford, CA 94305
email: brent@ccrma.stanford.edu

ABSTRACT

The grand piano action is modeled as a set of four rigid bodies using Kane's method. Computerized symbol manipulation is utilized to streamline the formulation of the equations of motion so that several models can be considered, each of increasing detail. Various methods for checking the dynamical model thus derived are explored. A computer animation driven by simulation of the equations of motion is compared to a high-speed video recording of the piano action moving under a known force at the key. For quantitative evaluation, the velocities and angular velocities of each of the bodies are extracted from the video recording by means of digitization techniques. The aspects of the model of particular interest for emulation by a controlled system, namely, the mechanical impedance at the key and the velocity with which the hammer strikes the string, can be studied in the equations of motion and compared to empirical data.

1. INTRODUCTION

Pianos are judged not only on the basis of their tone, but also feel or 'touch'. The 'dynamics' of the multi-body piano action determine the 'touch' or force history which one feels at the keyboard in response to a given gesture input. Behind these 'dynamics' lie the mechanical properties of the piano action, which are governed by the principles of newtonian mechanics. With tools from the field of applied mechanics, we can build dynamical models whose behaviors approximate those of the piano action. Use of good engineering approximations can be expected to lead to models which are not overly complex, yet descriptive enough to capture the salient properties.

Once such a dynamical model has been devised and proven, a few interesting applications are possible. First, the model allows the testing of piano action (or similar keyboard) designs without having to build prototypes. Secondly, piano simulators can be created. The possibility of synthesis by electronic instruments of not just the sound but also the touch-response of a piano is now within reach. Touch response can be emulated by a keyboard which in fact lacks a piano action but has instead actuators or programmable passive devices and an accompanying control system. A few designs have already been prototyped [Cadoz 1990, Baker 1988].

Analytical investigations into the dynamics of the piano action do exist in the literature [Pfeiffer 1967]. Excellent empirical investigations with a view to characterizing the touch response have also appeared [Askenfelt 1991] However, for the purposes of creating a simulator, an explicit model is desired. This paper documents the development of a dynamical model which fully reflects the effects of inertia properties and changing kinematic constraints on the mechanical impedance for one stroke of a key from rest to hammer/string contact. The various sets of equations of motion which govern this behavior are formulated using Kane's method. Simulations are run and preliminary empirical data taken in order to verify the model.

The remainder of this paper begins with a brief discussion of the piano action behavior as it pertains to touch response. Next, a dynamical model is described and used to formulate the equations of motion. Simulation results are presented and finally, experiments designed to verify the model are discussed.

2. MOTION DESCRIPTION

Touch response can be described in terms of the mechanical impedance (frequency generalized resistance to force) which the piano presents to the player. Mechanical impedance is a function of the inertia, damping, and compliance properties, and also the geometry and interconnection of each of the piano action elements. In this paper, only the behavior resulting from motion beginning at key rest to hammer/string contact will be considered. The functions of the repetition lever and damper are left out. Even this brief motion must be broken into three distinct phases for the purposes of modeling because three sets of kinematic constraints operate. During the first phase, which we will call acceleration, the jack has not yet risen to contact the regulation button (see Figure 1). Therefore, the jack does not rotate with respect to the whippen. The second phase, called let-off, reigns while there is contact between the jack and regulation button, and ends when the interaction forces between the jack and hammer at the knuckle are no longer compressive. The third phase is characterized by free flight of the hammer and further motion of the key, whippen and jack until the key hits the key rail. A model capable of describing the effects of these changing kinematic constraints is desired so that the trigger-like feel of the let-off resistance can be rendered.

3. MODEL DESCRIPTION

Figure 2 shows a schematic representation of the grand piano action to be used for reference in the following discussion. Four bodies comprise the model: The key **K**, the whippen **W**, the jack **J**, and the hammer **H**. (See Figure 2a.) In the present analysis, the function of the repetition lever and the damper lever are not taken into account because they do not play a role during the motion being considered. Let **N** denote a newtonian reference frame. Because only planar motion needs be considered, a point is sufficient to determine an axis of rotation. (See Figure 2c.) Body **K** can rotate in **N** about point $\mathbf{P_1}$ fixed in both **N** and **K**. Similarly, body **W** can rotate in **N** about point $\mathbf{P_2}$ fixed in **N** and **W**. Body **H** can rotate in **N** about point $\mathbf{P_4}$ fixed in **N** and **H** . Finally, body **J** is connected to **W** in such a way that it can rotate about point $\mathbf{P_3}$ fixed in **J** and **W**. Points **H*** and **K*** are the mass centers of bodies **H** and **K**. Moments of inertia for **H** and **K** are also included. Parameters L_1,..L_{20} (not shown) designate pertinent dimensions. To characterize the instantaneous configuration of the action, generalized coordinates q_1,...q_7 are employed. (See Figure 2b.) The radian measures of four angles q_1, q_2, q_4, and q_6 are used to locate each of **K**, **W**, **H**, and **J** with respect to the horizontal. Displacement q_3 locates a frictionless slider $\mathbf{S_1}$ which connects **K** to **W**. Displacement q_5 locates a frictionless slider $\mathbf{S_2}$ which connects **J** to **H**. Displacement q_7 locates a frictionless slider $\mathbf{S_3}$ which connects **J** to a horizontal line **RB** fixed in **N** corresponding to the regulation button. All of the sliders $\mathbf{S_1}$, $\mathbf{S_2}$, and $\mathbf{S_3}$ are existent or active, however, only during one phase of the motion of the action: during let-off. Table 1 shows the three phases of motion and the action of each of the sliders to further clarify the manner in which the kinematic constraints evolve.

Readers familiar with the design of the piano action will already be aware of several assumptions made in the above model construction. First, each action element has been modeled as a rigid body. Inertial and weight effects are accounted for, but damping and compliance effects are

Motion Phase	Sub-model Name	Active Sliders	Degrees of Freedon	Number of closed kinematic chains
I	Acceleration	S1 S2 x	1	2
II	Let-off	S1 S2 S3	1	3
IIIa	Hammer Flight	x x x	1	0
IIIb	Key Flight	S1 x S3	1	2

Table 1.

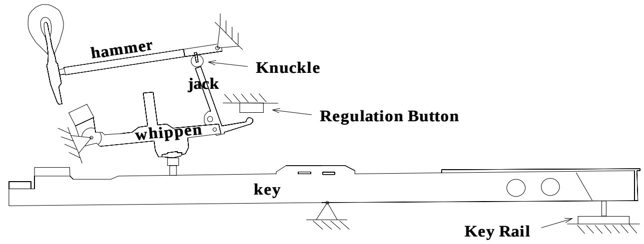


Figure 1.

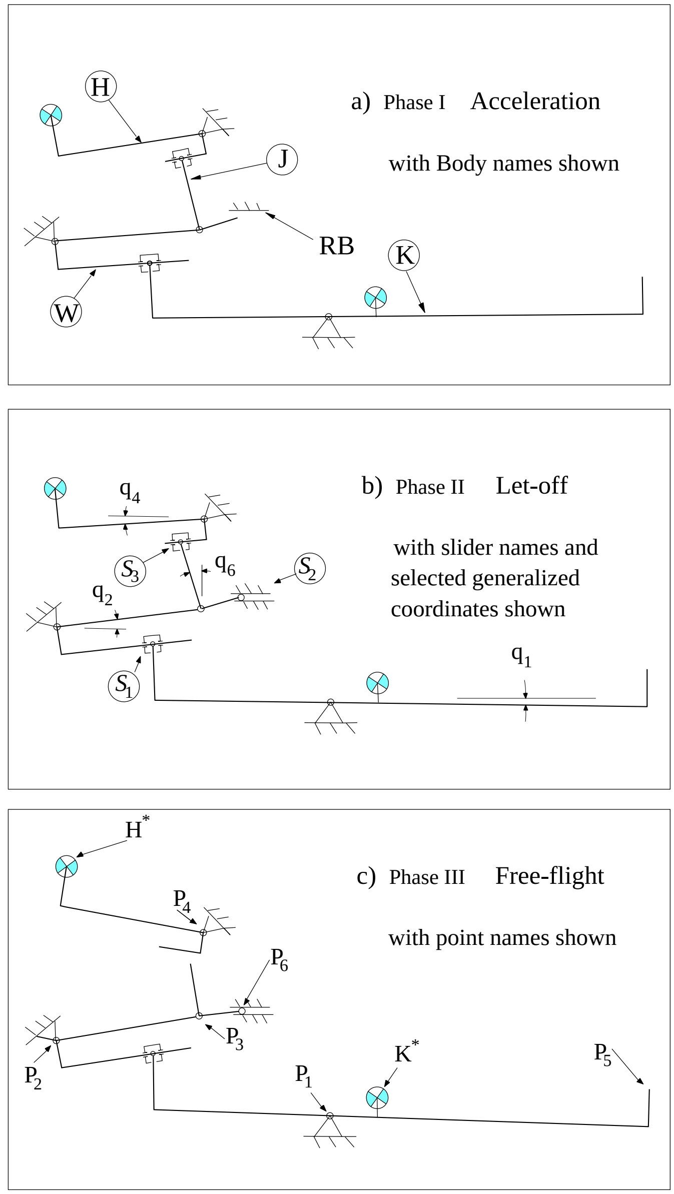


Figure 2.

Schematic diagram of the grand piano action in three configurations corresponding to the three phases of movement: a) acceleration, b) let-off, c) free-flight. The various symbols shown are generally applicable, but are separated over the three configurations for clarity.

not. Rotary friction forces acting at the revolute joints (pin and felt bushings) are neglected. Friction at each of the sliders is also neglected. The above assumptions seem reasonable except perhaps in the case of friction forces which develop during let-off between the jack and hammer at the knuckle. These forces contribute significantly to the 'letoff resistance' felt by a player. Other models not presented here do include these friction effects. Finally, the shape of the contact surfaces between the key and whippen, and jack and hammer are only approximately linear.

4. EQUATION FORMULATION and SIMULATION

Generalized speeds u_i (i=1,...7) are formed as a function of the generalized coordinates simply by letting $u_i = {}^{d}/_{dt}(q_i)$ (i=1,...7). Kinematic constraint equations are used to express u_i (i=2,...7) in terms of u_1. Expressions are found for the velocities of **K***, **H***, and $\mathbf{P_5}$, the point of application of a playing force on the key **K**. The partial velocities for each of these points are found, and are used together with the gravitational forces acting at **K*** and **H*** and the applied force at $\mathbf{P_5}$ to form the generalized active force F_1. Expressions for the accelerations of **K***, and **H*** are found and used with the partial velocities to form the generalized inertia force F_1*. Finally, the dynamical equation of motion is formulated: F_1+F_1*=0 [Kane 1985]. An auxiliary generalized speed u_8 is used in Model II to bring the interaction force between the jack and hammer into evidence. In order to determine the reaction forces to a specified velocity input at the key (the inverse-dynamics problem,) each of the models may be formulated as a zero-degree of freedom problem with controlled q_1 time-history. In this case, an auxiliary generalized speed must be used to bring the interaction force between the player and the key into evidence.

The equations of motion, the constraint equations, and expressions for the interaction forces are integrated numerically using a Kutta-Merson algorithm. Care must be taken to ensure that the initial conditions do indeed satisfy the applicable constraint equations. This is accomplished by solving the set of non-linear non-differential constraint equations numerically. A complete model simulation is in hand when simulations of the three sub-models are linked. When point $\mathbf{P_6}$ of body **J** rises to meet line **RB** (tested by a logical statement during simulation,) integration of sub-model I is stopped, and the final conditions are passed as the initial conditions to sub-model II. From then on, slider $\mathbf{S_2}$ constrains $\mathbf{P_6}$ to move along horizontal line **RB**. When the interaction force between **J** and **H** at slider $\mathbf{S_3}$ changes in sub-model II from compressive to tensile (again tested by a logical statement,) integration of sub-model II is stopped and the final conditions are passed as initial conditions to sub-models IIIa and IIIb. At this time, slider S3 disappears leaving two independent models.

A typical equation of motion involves (for each time step) approximately 400 each multiply and add operations and the evaluation of approximately 200 transcendental functions. Simulation in real time on a PC is not possible. However, computerized linearization and other proven simplification techniques can be exploited to speed up the integration process. Simulation output data for the generalized coordinates have been used to place drawings of the four bodies into successive frames which can be compiled into animations. Stick-figure animations (See Figure 3) and full 3-dimensional AutoCad animations have been produced. If the simulation does run in real time, the addition of an actuator whose torque command is driven synchronously by the appropriate generalized coordinate and the linking of simulation input variables to sensors on that actuator constitutes the creation of a simulator. Such schemes have been used to develop virtual piano actions and other virtual dynamical systems [Gillespie 1992].

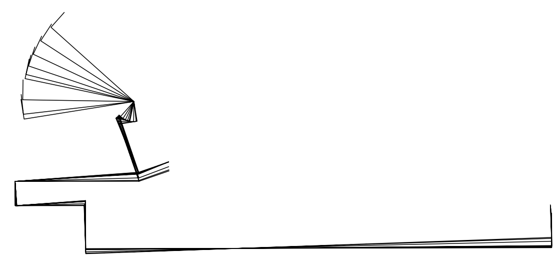
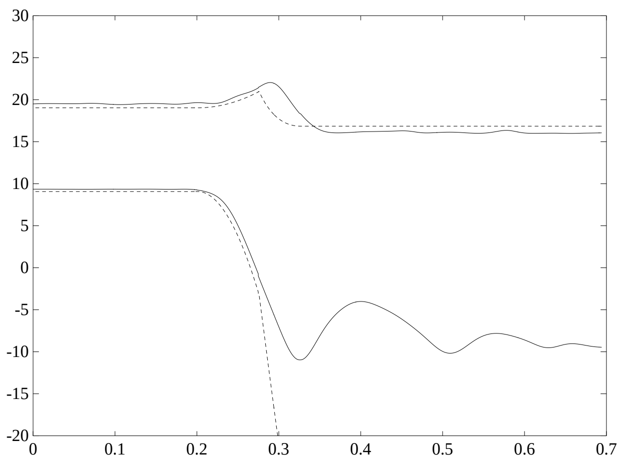


Figure 3
Sample stick-figure animation driven by simulation

5. EXPERIMENT DESCRIPTION

A one-key grand piano action was set into motion by a large linear motor coupled at the key (See Figure 4). Horizontal motions of the linear motor were coupled to vertical motions of the key by means of three bearing pulleys and a loop of Kevlar fiber. A strain gage-instrumented plastic flexure was fastened between the driving Kevlar fibers and the piano key to transduce and record the interaction forces. The linear motor was driven with a parabolic position trajectory identical to the trajectory used as the controlled variable during the inverse-dynamics simulation. Experiments in which a constant force was applied by releasing a weight were also performed. Position encoders recorded the resulting motions of the key, Kevlar loop, and damper. A high-speed video recording was made at 1000 frames per second of the motion of the piano action. Retro-reflective patches were attached to ten locations on the piano action in order to facilitate vision recognition by computer. Digitization and light patch centroid location determination from about 700 frames was performed by Jim Walton of 4-D Video. The digitized motions were used to deduce the generalized coordinate trajectories with direction cosine matrix transformations. The theoretical (simulated) and experimental generalized coordinate trajectories can be compared. Some preliminary results are presented in Figure 5. Such comparisons are serving to improve the model and further direct experiment design.



6. SUMMARY

The use of modern dynamical modeling techniques allows the development of accurate and descriptive models of the grand piano action mechanical impedance. This paper has documented a dynamical model which fully reflects the effects of inertia properties and changing kinematic constraints on the mechanical impedance for one stroke of a key from rest to hammer/string contact. This model captures the major characteristics of the piano action behavior, as verified by preliminary experiments. Such models can serve as the cornerstone in a haptic display device design. Model enhancements are the subject of continuing research at CCRMA.

ACKNOWLEDGEMENTS

I wish to thank Dr. Jim Walton of 4D Video, Sebastopol, CA, professor Tom Bowman for the loan of the SP2000 high speed video camera, Yamaha Corporation for donation of a one-key piano action and Delta Tau corporation for donation of a DSP motor control card.

REFERENCES

A. Askenfelt and E. V. Jansson, " From touch to string vibration II: The motion of the key and hammer,"
J. Acoust. Soc. Am, vol. 90 (5), pp. 2383-2393, Nov. 1991.

R. Baker, " Active touch keyboard,'United States Patent No. 4,899,631, 1988..

C. Cadoz, L. Lisowski, and J-L. Florens, " Modular feedback keyboard,"Proceedings of the ICMC, pp. 379-382, Glasgow, 1990.

B. Gillespie, " The touchback keyboard,"in these proceedings, 1992.

T. R. Kane, and D. A. Levinson, Dynamics: Theory and Applications, McGraw-Hill, New York, 1985, P. 158.

W. Pfeiffer, The Piano Key and Whippen, Verlag Das Musikinstrument, Frankfurt a. M, 1967.

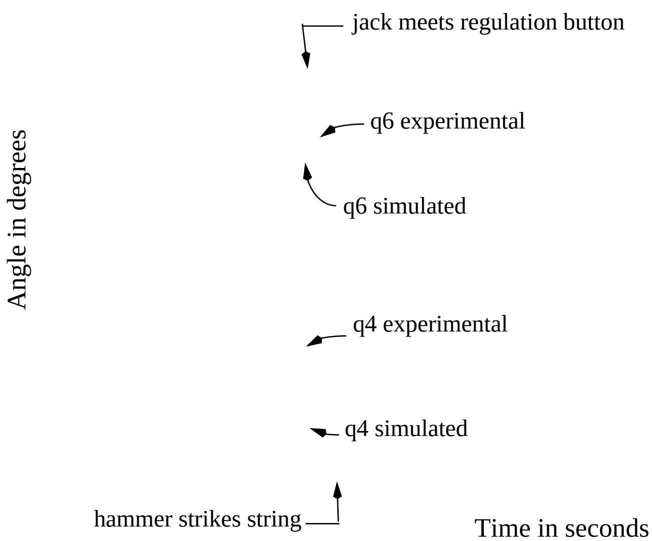


Figure 4
Generalized coordinates q4 and q6 vs. time
experimental and simulated.

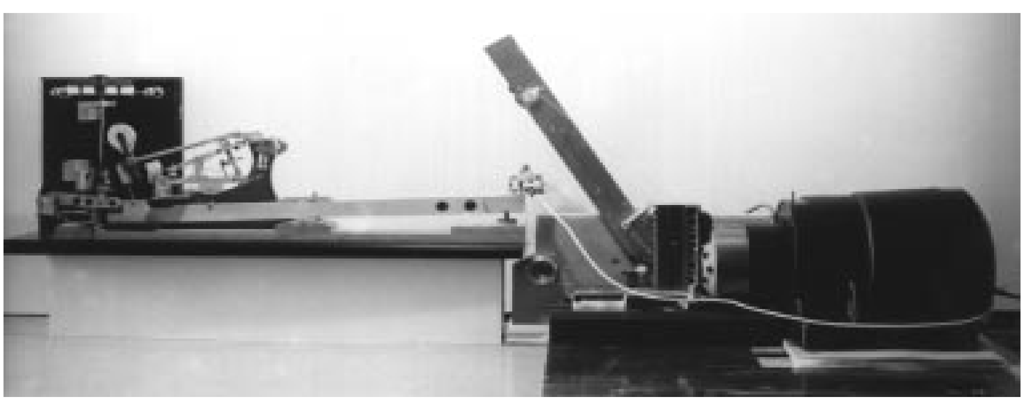


Figure 4.
Experimnetal Setup