

HAPTIC DISPLAY OF SYSTEMS WITH CHANGING  
KINEMATIC CONSTRAINTS:  
THE VIRTUAL PIANO ACTION

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DOCTOR OF PHILOSOPHY

By  
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January 1996

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I certify that I have read this dissertation and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

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# Abstract

Skilled keyboardists enjoy very fine control over musical sounds at the piano but must make do without that fine control at the synthesizer keyboard. In contrast to the piano action, the typical synthesizer action is not subject to changing kinematic constraints and is therefore missing transient features in its mechanical impedance (touch-response). Keyboardists rely on such haptic features to develop and execute the aforementioned fine control. To remedy the inferior utility of synthesizer keyboards, this thesis develops and applies haptic interface technology. Each synthesizer key is motorized and a multibody, variable structure dynamical model of the piano action is simulated in real-time in a human-in-the-loop scheme to re-create the response and the varying mechanical impedance of the piano action. A combined simulation and experimental apparatus comprising a seven key motorized keyboard is described. For use as a haptic interface control engine, a detailed dynamical model of the piano action is developed using Kane's method. Computationally efficient submodels are constructed for the piano action in each of its constraint conditions. Simulation schemes based on a finite state machine are developed so that the submodels may be interactively sequenced together.

Limitations to the fidelity of haptic rendering invariably arise when the simulator is implemented as a sampled data controller. Restrictions must be placed on the mechanical impedance of the virtual object or exceptional computational power must be demanded of the interface controller lest meddlesome chatter arise between user and virtual object. This work notes that the destabilizing effects of sampled-data and computational delays can be fully compensated out if the entire coupled dynamical system is modeled: interface device and human limb. Such methods are fully explored for the virtual wall, the simplest virtual object subject to changing constraints. New algorithms are presented for the virtual wall which address the destabilizing effects of discrete control and discontinuous control, yet account for the coupled-in dynamics of the human. Specifically, the deleterious effects of the sample and hold operator and the asynchrony of constraint threshold crossings with sampling times are eliminated. Model-based prediction, digital control design techniques, and deadbeat control are employed.

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To my parents.

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