The properties of perfect gases

Atkins, Chapter 1

Gases have V(olume), p(ressure), T(emperature) and n (amount) as observables that can fully describe their state

Avogadro's principle: [1811]

$V \propto n$ **or** $V_m = \frac{V}{n}$ **= constant (if T and p don't change)**

Boyle's law: [1661]

$p \propto \frac{1}{V}$ **at constant T**

Charles's law: [1787] (+ Gay-Lussac 1802)

$V = V_{0^\circ C} + BT$ **[T in ^{o}C]**

Summary: The perfect gas equation of state

$$pV = nRT$$

Table 1.1 *The gas constant in various units*

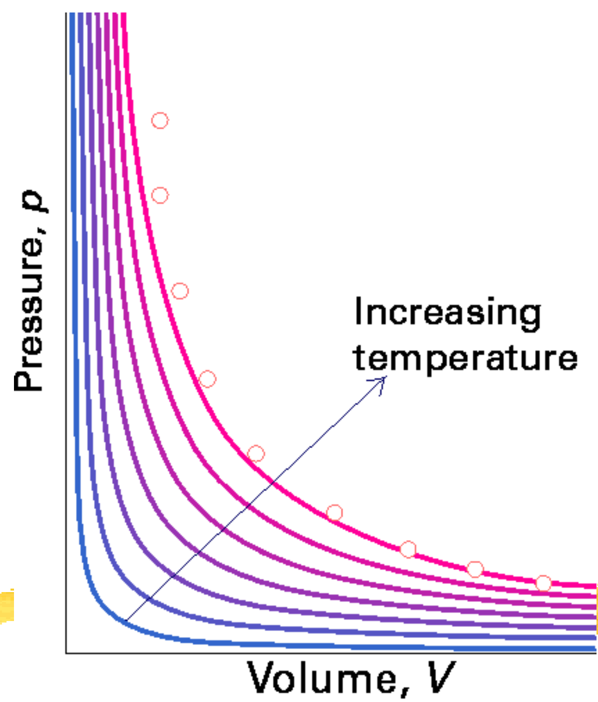
R =	8.314 51 J K^{-1} mol^{-1}
	8.314 51 kPa L K^{-1} mol^{-1}
	8.205 78 × 10^{-2} L atm K^{-1} mol^{-1}
	62.364 L Torr K^{-1} mol^{-1}
	1.987 22 cal K^{-1} mol^{-1}

Avogadro:

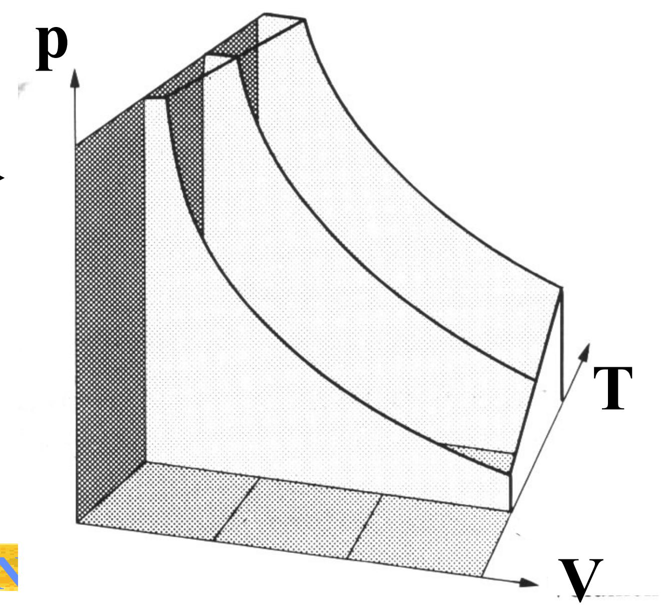
$$\frac{V}{n} = \frac{RT}{p} = 24.465 \text{ l/mol}$$

Boyle:

$$p = \frac{nRT}{V}$$



in 3D

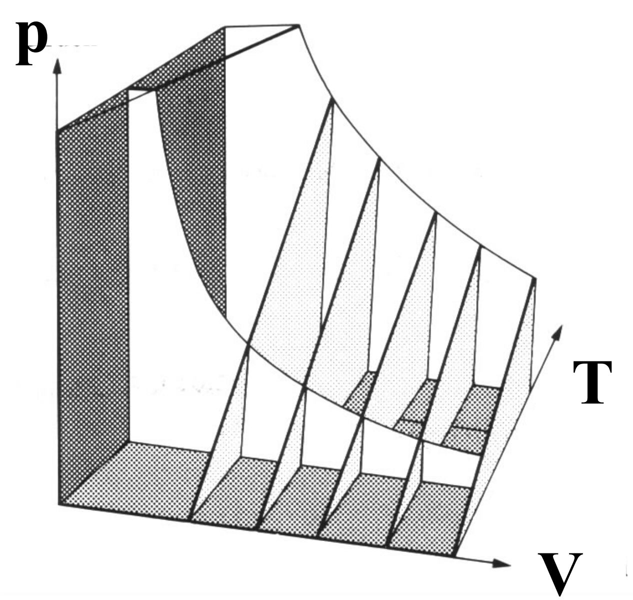


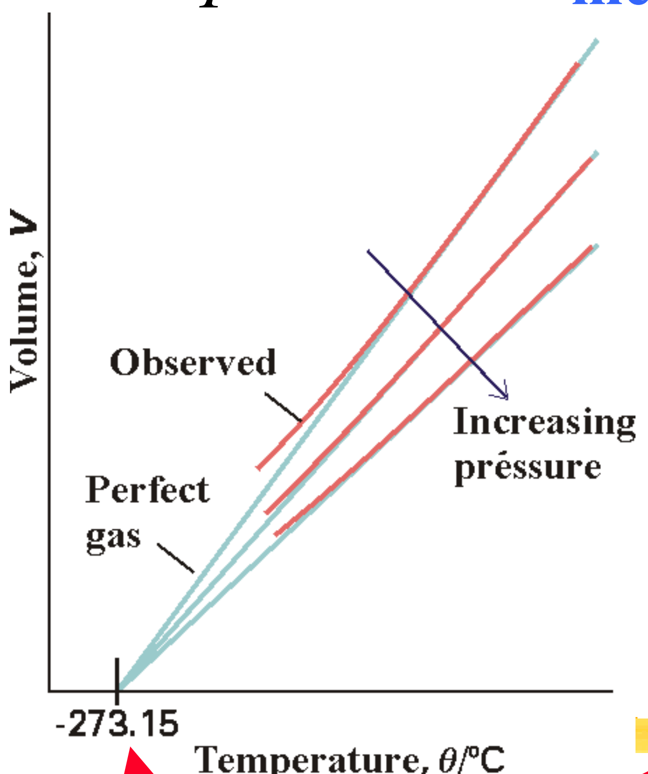
The perfect gas temperature: The Kelvin scale

Charles: $V = V_{0°C} + BT$ **[T in °C]**

$$V = \frac{nR}{p} T$$ ← **Only holds when T is measured in K(elvin)**

in 3D:



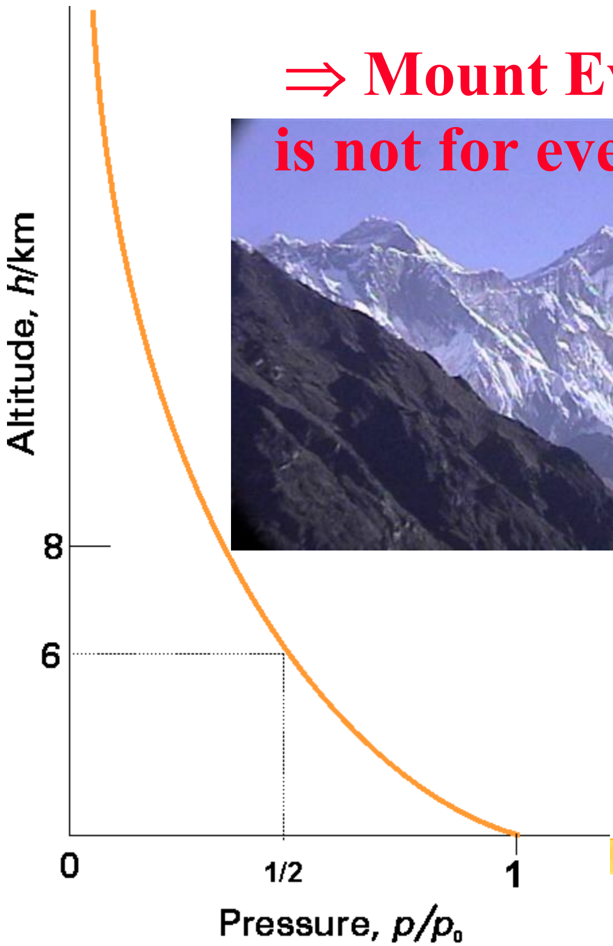


For all gases: Extrapolation to V = 0 yields same T = -273.15 °C = absolute zero = 0 K

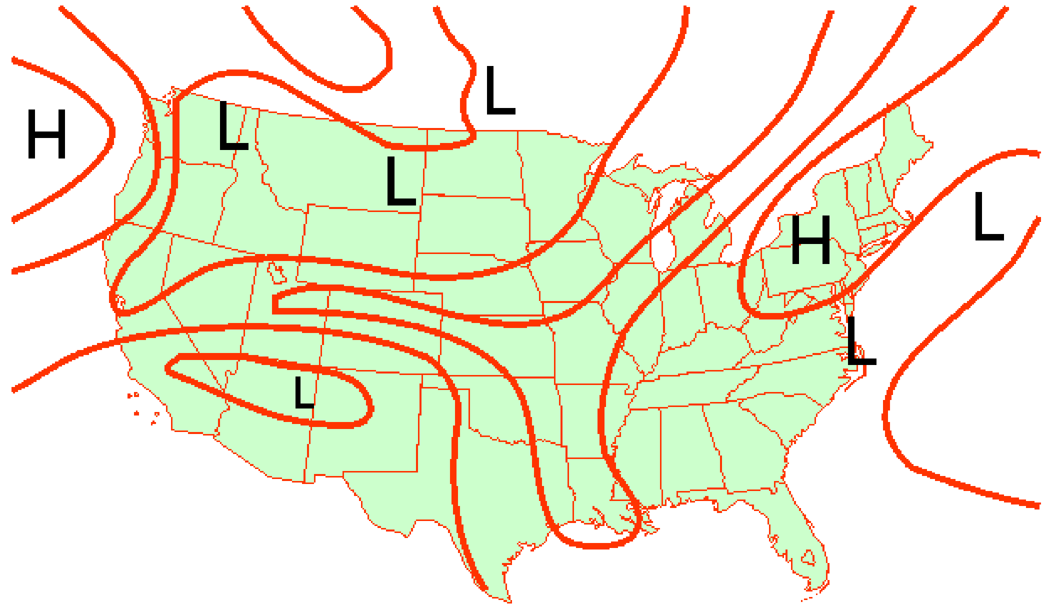
Applying the perfect gas law

change in conditions

$$\frac{pV}{T} = nR \quad \Rightarrow \quad \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$



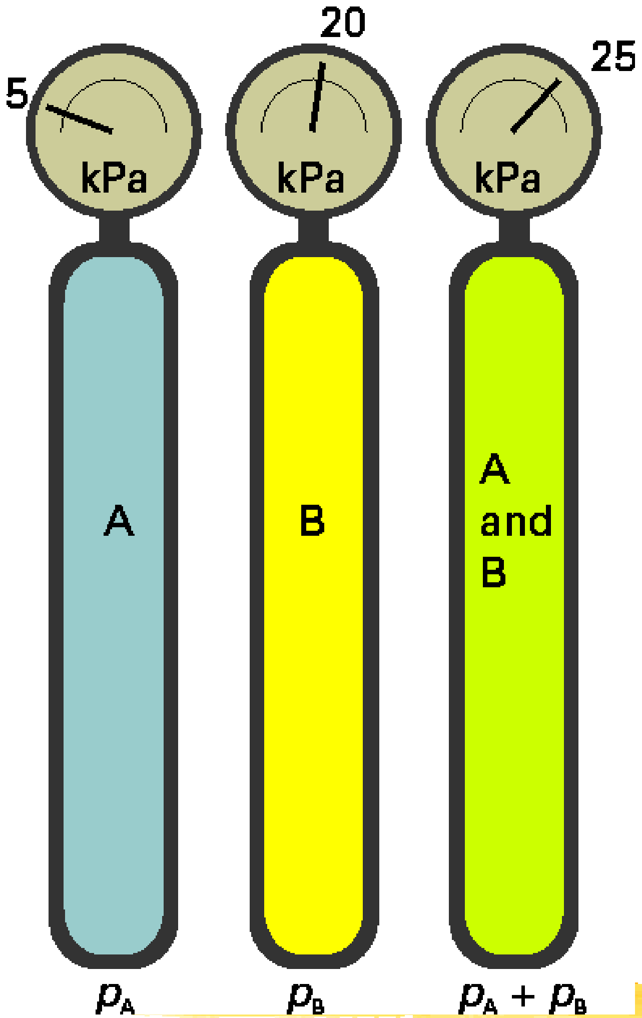
⇒ Mount Everest is not for everyone



Dalton's law

$$p = p_A + p_B$$

partial pressure



$$p = n\frac{RT}{V} = n_A\frac{RT}{V} + n_B\frac{RT}{V}$$

since $\mathbf{n_A + n_B = n}$

$$x_A = \frac{n_A}{n_A + n_B} = \frac{n_A}{n}$$

$$x_B = \frac{n_B}{n_A + n_B} = \frac{n_B}{n}$$

mole fractions $\mathbf{x_A + x_B = 1}$

$$p = x_A n\frac{RT}{V} + x_B n\frac{RT}{V} = x_A p + x_B p$$

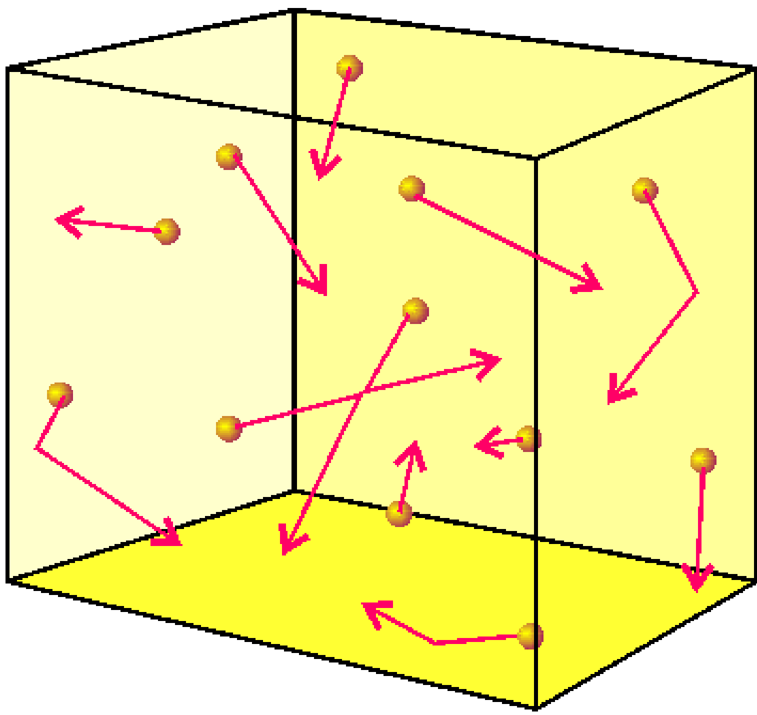
Description of a perfect gas by the kinetic model

Three assumptions

→ **A gas consists of molecules in ceaseless motion in 3D**

→ **The size of the molecules is small compared to their average distance traveled**

→ **The molecules do not interact**



⇒ Many molecules continuously collide with the wall and produce pressure

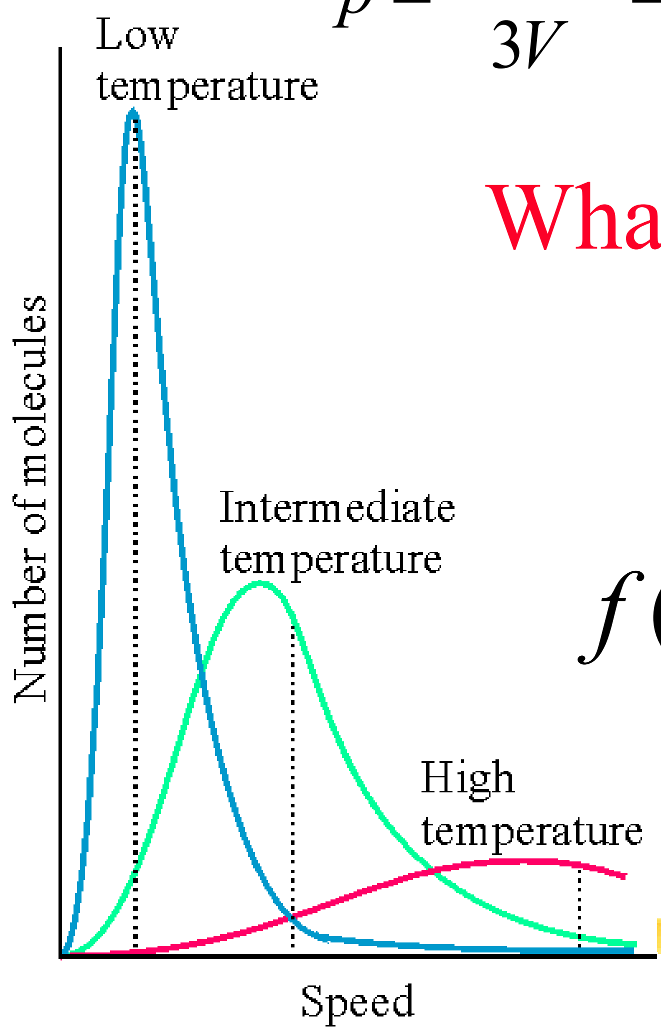
$$\Rightarrow \quad p = \frac{nMc^2}{3V}; \qquad c = \sqrt{\frac{s_1^{\ 2} + s_2^{\ 2} + \ldots + s_N^{\ 2}}{N}}$$

root-mean-square speed

The Maxwell distribution of speeds

perfect gas law

$$p = \frac{nMc^2}{3V} = \frac{nRT}{V} \quad \Rightarrow \quad c = \sqrt{\frac{3RT}{M}}$$ **an average speed**

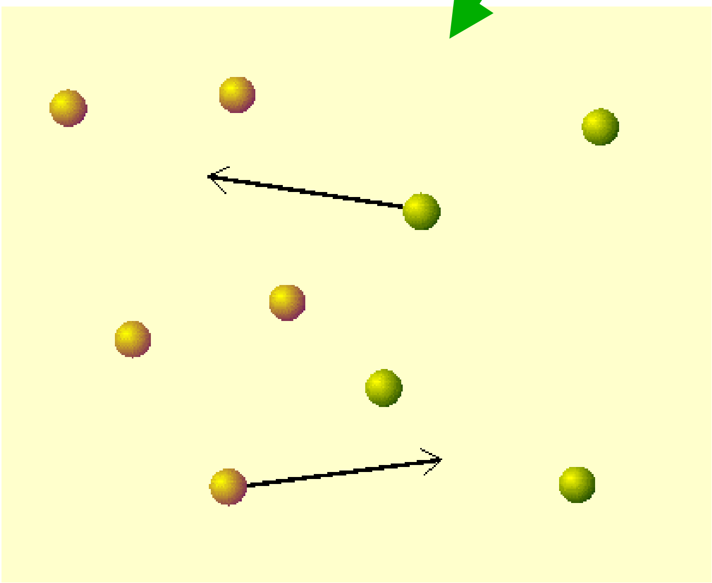


What are the individual speeds?

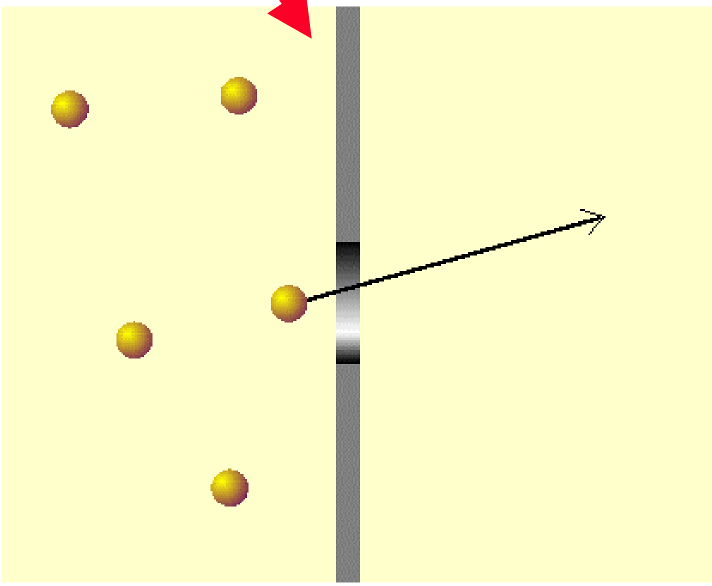
Maxwell [late 1800's]:

$$f(s) = 4\pi \sqrt[\frac{3}{2}]{\frac{M}{2\pi RT}} s^2 e^{-Ms^2/2RT} \Delta s$$

Diffusion and Effusion



Mingling of different substances, e.g., gases (different from convection!)



Escape of gas through a small hole

Graham's law [1833]: $rate \propto \sqrt{\frac{1}{M}}$

$$c = \sqrt{\frac{3RT}{M}}$$

Used for purifying $^{235}UF_6$ from $^{238}UF_6$