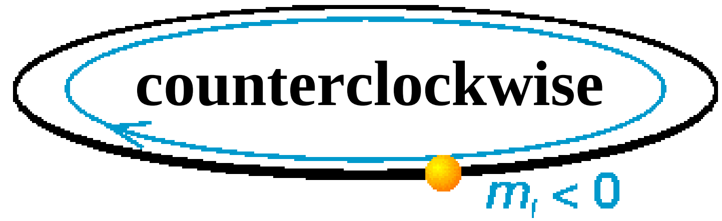
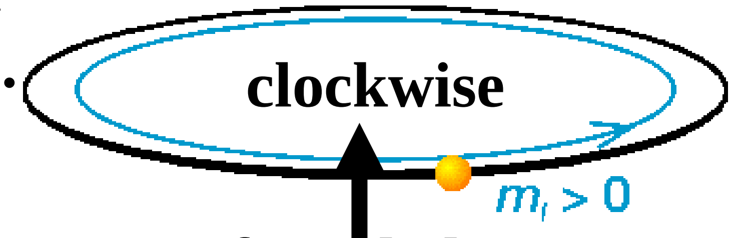
The Energy Levels of a Rotating Particle

$$\boxed{E_n = \frac{n^2\hbar^2}{2I} \quad \rightarrow \quad E_{m_l} = \frac{m_l^{\,2}\hbar^2}{2I}}$$

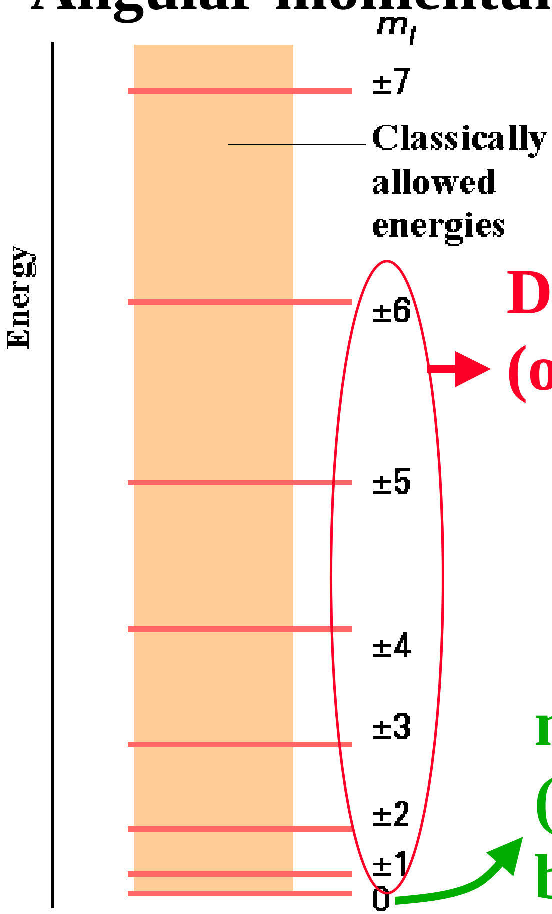


Angular momentum quantum number

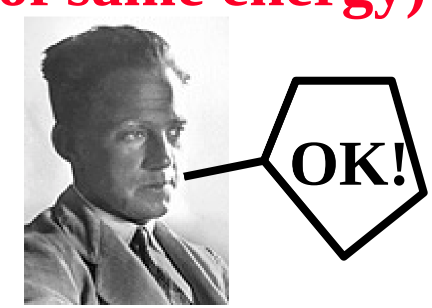
$m_l = 0, \pm1, \pm2, \ldots$



from below



Doubly degenerate states (of same energy)



non-degenerate state (zero energy, but uncertain between 0º and 360º)

Also the angular momentum is quantized

$$J = pr = \frac{hr}{\lambda} \quad \textbf{and} \quad \lambda = \frac{2\pi r}{m_l}$$

$$\Rightarrow \boxed{J = m_l\hbar}$$

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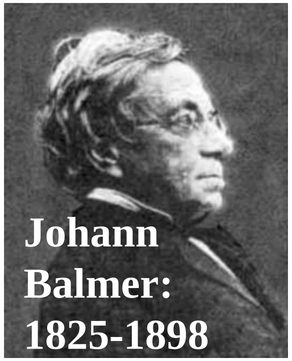
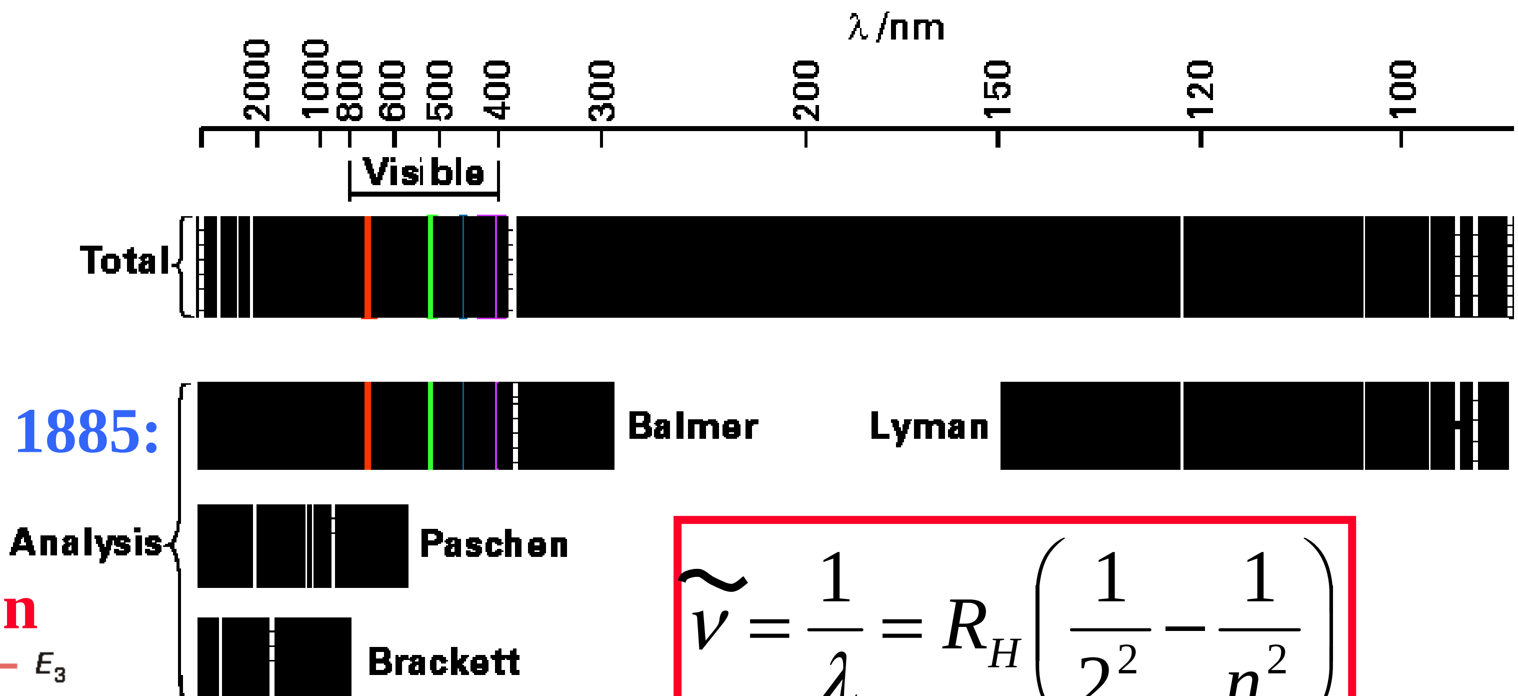
The Emission Spectrum of Hydrogen Atoms



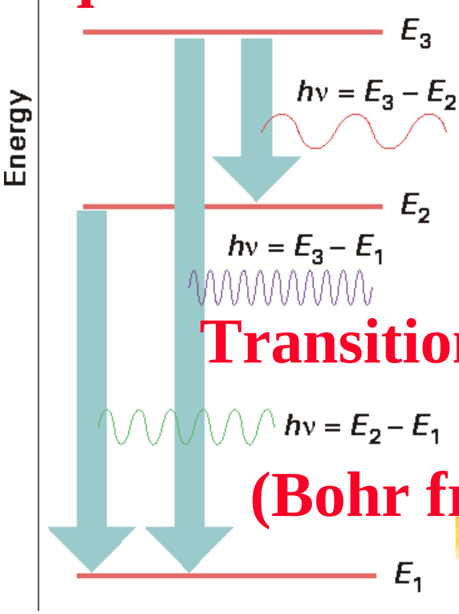
1885:



$$\boxed{\tilde{\nu} = \frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)}$$

Rydberg constant R_H = 109,677 cm^{-1}; n = 3, 4,...

quantization



In general: n_1 = 1, 2,…; n_2 = n_1+1, n_1+2,...

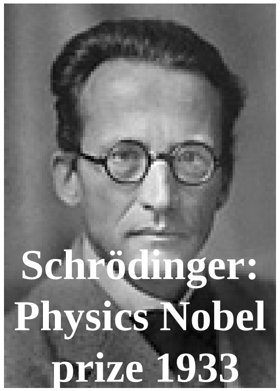
Transitions: photon emitted w/

$\Delta E = h\nu = hc\tilde{\nu}$

(Bohr frequency condition)

$$\boxed{\tilde{\nu} = R_H\left(\frac{1}{n_1^{\,2}} - \frac{1}{n_2^{\,2}}\right)}$$

What are the Energy Levels of a Hydrogen Atom? Schrödinger can help!

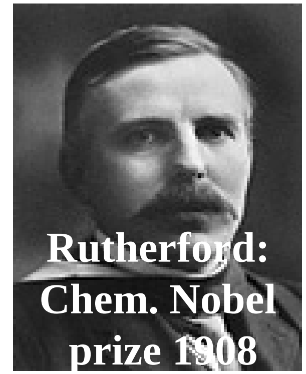

Schrödinger: Physics Nobel prize 1933

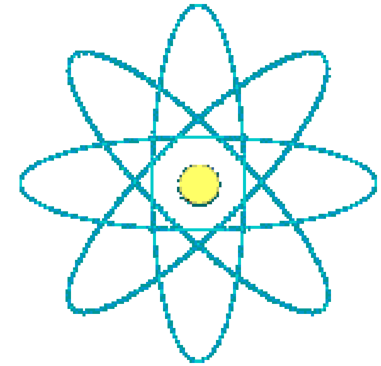
$$-\frac{\hbar^2}{2\mathrm{m}}\frac{d^2\Psi}{dx^2}+V\Psi=E\Psi$$

$$\boxed{V=-\frac{Ze^2}{4\pi\varepsilon_0 r}}$$

vacuum permittivity

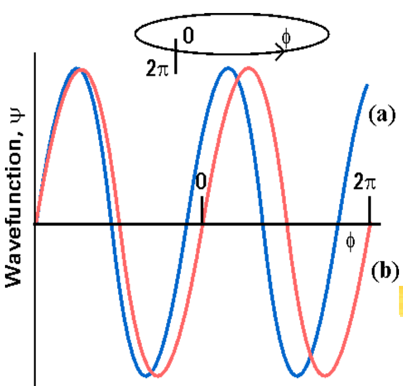

Coulomb: 1736-1806


Rutherford: Chem. Nobel prize 1908



Nuclear atomic model; nuclear charge +Ze, electronic charge -e

Boundary condition: $\Rightarrow$ **can be solved analytically:**



Reduced mass ≈ m_e

$$\boxed{E_n=-\frac{\mu e^4}{32\pi^2\varepsilon_0{}^2\hbar^2}\frac{Z^2}{n^2};\quad \mu=\frac{m_e m_n}{m_e+m_n}}$$

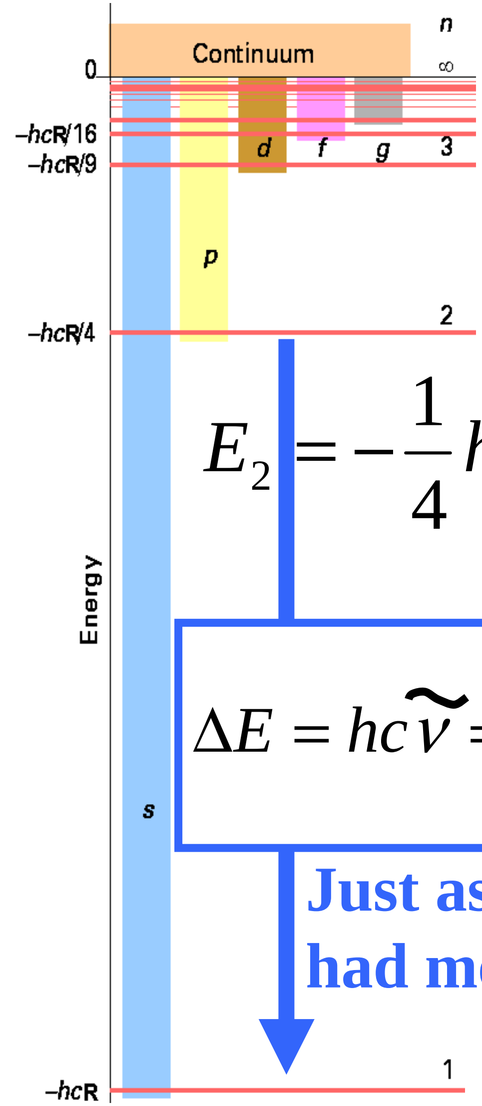
Quantum Mechanics explains Balmer's Experimental Observations

Rydberg constant!

$$E_n = -\frac{\mu e^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{Z^2}{n^2} = -hcR_H\frac{Z^2}{n^2}$$

Principal quantum number

All energies are negative (Coulomb attraction)

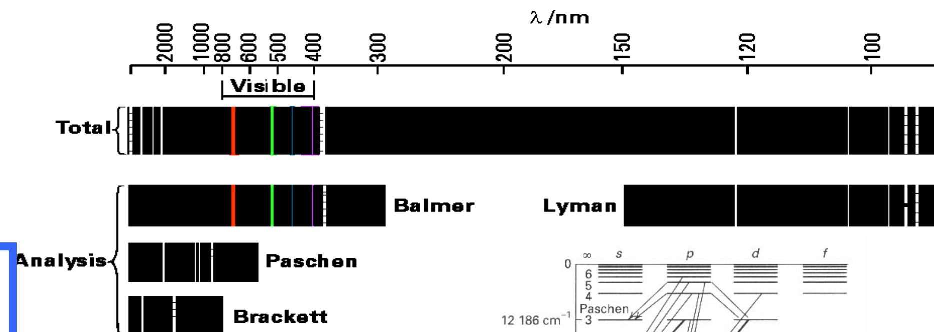


$$E_2 = -\frac{1}{4}hcR_H$$

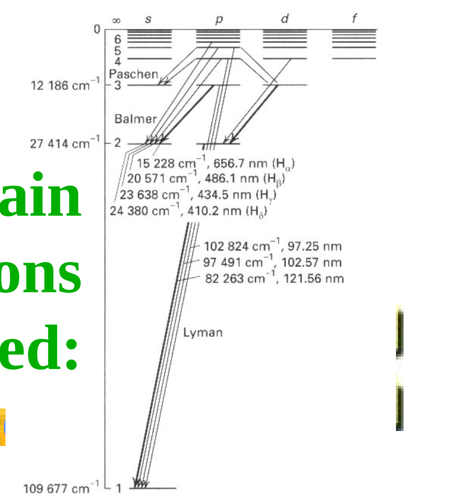
$$\boxed{\Delta E = hc\tilde{\nu} = hcR_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$$

Just as Balmer (Lyman) had measured!

$$for\ H\ atom: E_1 = -hcR_H$$



Only certain transitions allowed:



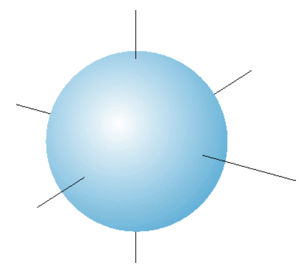
ils Walter:

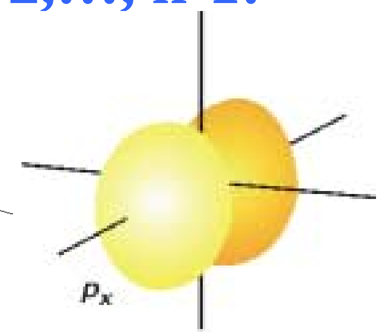
The Resulting Quantum Numbers Define the Atomic Orbitals

1.) n = Principal quantum number = 1, 2, 3,…∞ :
- the only determinant of the energy of hydrogen-like atoms
- defines the "size" of the orbital
- K, L, M, N,… shells

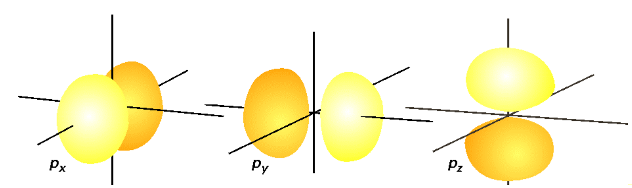
$$E_n = -\frac{hcR_H Z^2}{n^2}$$

2.) l = Angular momentum quantum number = 0, 1, 2,…, n-1:
- n different values
- defines the "shape" of the orbital
- s, p, d, f,… subshells, orbitals, electrons





3.) m_l = Magnetic quantum number = l, l-1, l-2,…, -l
- 2l+1 different values
- defines the "orientation" of the orbital



4.) m_s = Spin magnetic quantum number = +1/2, -1/2
- electron spins clockwise (+1/2, α) or counterclockwise (-1/2, β)