Supplement 1A

Maxwell's equations

These four vector relations summarize the previously discovered experimental laws to describe all known electrical and magnetic phenomena. In these expressions, ρ is the electric charge density, $\mathbf{J}$, the current density, $\mathbf{E}$, the electric field and $\mathbf{B}$, the magnetic induction. Maxwell's equations in free space (in the absence of dielectric or magnetic media) can be written

$$\nabla \cdot \mathbf{D} = \rho \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \tag{3}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{4}$$

The two auxilliary fields $\mathbf{D}$, the electric displacement, and $\mathbf{H}$, the magnetic field are defined by *constitutive relations.* In free space

$$\mathbf{D} = \epsilon_0 \mathbf{E} \qquad \text{and} \qquad \mathbf{B} = \mu_0 \mathbf{H} \tag{5}$$

where ϵ_0 and μ_0, are the vacuum electric permittivity and magnetic permeability, respectively.

Eq. (1) states that an electric field diverges from a distribution of electric charge. This implies Coulomb's law. Eq. (2) implies the nonexistence of isolated magnetic poles–the magnetic equivalent of electric charges. The most elementary magnetic objects are *dipoles*, connected pairs of north and south poles which can *not* be isolated from one another. Eq. (3) is an expression of Faraday's law of electromagnetic induction, which shows how a circulating electric field can be produced by a time-varying magnetic field. Eq. (4) contains Ampère's law showing how a magnetic field is produced by an electric current. The second term on the right, which was added by

Maxwell himself, is, in a sense, reciprocal to Faraday's law, since it implies that a circulating magnetic field can also be produced by a time-varying electric field.

In the absence of charges and currents, Maxwell equations can be transformed into three-dimensional wave equations

$$\left\{\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\}\mathbf{E} = 0 \qquad \text{and} \qquad \left\{\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\}\mathbf{B} = 0 \tag{6}$$

where $c = 1/\sqrt{\epsilon_0\mu_0} = 2.9979 \times 10^8$ m/sec, representing the speed of light in vacuum. Possible solutions to Eqs (6) represent synchronized transverse electric and magnetic waves propagating at the speed c, as sketched in Figure 1.3.

Even in the classical theory, electromagnetic fields can carry energy and momentum. The energy density of an electromagnetic field in free space is given by

$$\rho_{\mathrm{E}} = \frac{1}{2}\left(\epsilon_0\, E^2 + \frac{B^2}{\mu_0}\right) \tag{7}$$

The energy flux or intensity (energy transported across unit area per unit time across unit area) is given by the Poynting vector

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} \tag{8}$$

It is significant that the energy density and intensity depend of the *square* of field quantities. We will exploit an analogous relationship in the interpretation of the wavefunction in quantum mechanics.

Maxwell's first equation is equivalent to Coulomb's law. In its simplest form, the force between two point charges q_1 and q_2 separated by a distance r is given by

$$F = \frac{1}{4\pi\epsilon_0}\,\frac{q_1\, q_2}{r^2} \tag{9}$$

The algebraic signs of q_1 and q_2 determine whether the force is attractive or repulsive. If q_1 and q_2 are like charges, they repel ($F > 0$), whereas opposite charges attract ($F < 0$). In our applications to atomic and molecular structure, it is clumsy and unnecessary to carry the constant $4\pi\epsilon_0$. We will instead write Coulomb's law in gaussian electromagnetic units, whereby

$$F = \frac{q_1 \, q_2}{r^2} \tag{10}$$

The potential energy of interaction between two charges is related to the force by $F = -dV/dr$ (more generally, $\mathbf{F} = -\nabla V$). Coulomb's law therefore implies

$$V(r) = \frac{q_1 \, q_2}{r} \tag{11}$$

which we will repeatedly use in applications to the quantum theory of atoms and molecules.