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Fixed costs and long-lived investments

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ABSTRACT

Fixed costs models are difficult to analyze because they feature non-degenerate, time-varying distributions of capital across firms. If investments are sufficiently long-lived however then the cross-sectional distribution of capital holdings has virtually no bearing on the equilibrium and the aggregate behavior of fixed-cost models is essentially identical to neoclassical models. The findings are due to a near infinite elasticity of investment timing for long-lived investments – a feature shared by fixed-cost models and neoclassical models. “Irrelevance results” found in numerical studies of fixed-cost models are not parametric special cases but instead are fundamental properties of models with long-lived investment goods.

1. Introduction

Conventional neoclassical investment models typically assume that capital adjustment costs rise smoothly with investment and thus predict that firms should make frequent, small adjustments to their capital stocks. Microeconomic evidence, however, shows that firms make infrequent, large adjustments to their capital stocks (see, e.g., Doms and Dunne, 1998). Motivated by the micro-evidence, researchers have developed models emphasizing *fixed* adjustment costs. To avoid paying the fixed cost too often, firms wait to adjust their capital, and when they do adjust, they make large adjustments. While these models generate the observed firm-level investment behavior, it is not clear that the aggregate equilibrium behavior of fixed cost models differs significantly from the equilibrium behavior generated by neoclassical investment models. Prominent numerical studies of calibrated DSGE models with fixed adjustment costs suggest that there are only minor differences between the two (e.g., Thomas, 2002; Veracierto, 2002; Khan and Thomas, 2008). The cause of these “irrelevance results” is often attributed to consumption smoothing forces present in general equilibrium. The irrelevance results have been contested by other researchers on the grounds that they hold only for some parameter values and are not a general feature of equilibrium models with fixed costs.

The aggregate behavior of models with fixed adjustment costs is important for several reasons. Much of our existing understanding of investment is based on neoclassical models that abstract from fixed adjustment costs. Because the earlier models contrast sharply with the microeconomic evidence, researchers are justifiably concerned that predictions or policy conclusions based on these models may be misleading. On the other hand, if the aggregate behavior of the two modeling frameworks is similar, then the apparent failure of neoclassical models at the micro level does not mean that we need to

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abandon the neoclassical framework entirely to analyze aggregate investment. Indeed, there may be reasons to prefer the neoclassical model. Unlike neoclassical investment models, fixed-cost models are analytically very cumbersome. Models with fixed costs typically have non-degenerate distributions of capital across firms. These distributions are time-varying objects which enter the models as additional state variables, making the models extremely difficult to analyze, particularly in equilibrium settings.

This paper analyzes the approximate equilibrium behavior of an investment model where fixed costs matter at the microeconomic level. The analysis shows that optimal investment behavior is characterized by an extremely high intertemporal elasticity of substitution for investment purchases. For sufficiently long-lived capital goods (capital goods with low rates of economic depreciation), the elasticity is nearly infinite. This property has a number of implications.

First, for long-lived investment goods, the underlying distribution of capital holdings across firms has little bearing on the equilibrium. Because they are willing to drastically change the timing of their investments, firms that are bunched up or spread out relative to the steady state distribution simply delay or accelerate the timing of their investments to avoid high prices or take advantage of low prices. The high intertemporal elasticity of investment demand effectively eliminates any role for the cross-sectional distribution to influence aggregate investment.

Second, the near infinite elasticity of investment timing is a property that fixed-cost models share with neoclassical investment models. Investment demand in both models can be approximated by a demand curve which is essentially flat. As a result, if investment supply were the same, the equilibrium paths in the two models would be almost identical. Put differently, the analysis suggests that the "irrelevance results" are fundamental properties of fixed-cost models – not artifacts of particular calibrations. Thus, at the aggregate level, investment and investment prices, particularly for long-lived capital goods, can often be accurately analyzed with traditional, neoclassical investment models. While neoclassical models cannot match the behavior of the firms at the microeconomic level, they provide an easy, reliable guide to aggregate behavior, policy analysis and empirical predictions.

The similarity between neoclassical models and fixed-cost models does not rely exclusively on consumption smoothing. Because neoclassical models and fixed-cost models both have high intertemporal elasticities for investment timing, anything that causes the effective price of new capital goods to increase with total investment will make the models difficult to distinguish with aggregate data. An increasing quadratic adjustment cost in a neoclassical framework and an upward-sloping supply curve in the fixed-cost model will result in the same equilibrium paths provided that the elasticity of the marginal cost of investment is the same in each case. In one-good DSGE models, consumption smoothing motives naturally imply increasing marginal costs of investment and result in the striking similarities between dynamic models with and without fixed costs.

Before proceeding, I should make clear that the key property of the model which generates the irrelevance results – the infinite elasticity of investment demand – is a feature of the *models* and may not be a feature of *reality*. In the model, firms are extremely willing to change the timing of their investments in response to small price changes. In reality, firms might not pay much attention to small price changes and may be unwilling to change the timing of their investment plans.

2. Background and related literature

In micro data, plant level investment is characterized by long periods of relative inaction punctuated by episodes of high investment. Doms and Dunne (1998) show that most U.S. manufacturing plants experience at least one year in which their capital stock rises by at least 50 percent and for many establishments, half of all plant-level investment spending over a 17-year horizon is concentrated in the three years surrounding the year with the plant's greatest investment. Cooper et al. (1999) show that each year, roughly 1 out of every 5 manufacturing plants experiences an "investment spike," defined as an increase in plant-level capital of at least 20 percent. Moreover, aggregate variation in investment spikes accounts for the bulk of the variation in U.S. manufacturing investment. Gourio and Kashyap (2007) show that aggregate variation in investment spikes is primarily driven by changes in the number of firms experiencing spikes rather than changes in the average size of spikes.[1] As a whole, the evidence from the micro-data is in stark contrast to the predictions of standard neoclassical investment models with convex adjustment costs (e.g., Abel, 1982; Hayashi, 1982 and Summers, 1981).

Investment models with fixed costs can rationalize the lumpy investment seen in the data. Firms invest infrequently to avoid paying the fixed cost. Unlike earlier convex models however, models with fixed costs are difficult to solve even in partial equilibrium and are often completely intractable in general equilibrium. The difficulty in solving these models arises because not all firms have the same level of capital at any point in time. Some firms have old, outdated capital and are likely to adjust in the near term. Other firms have recently adjusted and will not purchase new capital for quite some time. The distribution of capital across firms changes whenever shocks or policies disturb the market. To solve the model, one must keep track of an endogenous, time-varying distribution of capital.

Because the position and dynamics of the distribution of capital can potentially influence the equilibrium, the distribution often plays a prominent role in the questions posed by the literature on fixed costs. For example, if there were an unusually large number of firms with relatively old capital, one might expect to observe unusually high investment

[1] Doms and Dunne (1998) and Cooper et al. (1999) base their findings on data from the Longitudinal Research Database (LRD), which includes most U.S. manufacturing plants. Gourio and Kashyap (2007) use both LRD and Chilean data on manufacturing plants. See also Cooper and Haltiwanger (2006).

and high capital prices in the near term as these firms update their capital. On the other hand, if most firms recently adjusted, then investment and prices should be unusually low. Policies might also have different effects in each case. If many firms are close to the point at which they would invest, a tax subsidy might have a considerable impact on investment. In contrast, if there are few firms with low capital stocks, the same subsidy might have little effect.[2]

While many researchers have analyzed models of investment with heterogeneous agents and fixed costs, most of the well-known results in this area come from models of individual firms taking prices as given.[3] Caballero and Engel (1999) and Adda and Cooper (2000) assume that all supply curves are perfectly elastic. This is tantamount to working in a partial equilibrium framework since, with perfectly flat supply curves, investment decisions of other firms have no influence on equilibrium prices and thus the complexity that arises from changes in the distribution is suppressed.

Because obtaining analytical results for models with fixed costs in equilibrium settings is difficult, much of the progress in this area has been made with numerical studies of specific dynamic models.[4] Using numerical techniques, Thomas (2002) and Veracierto (2002) found that calibrated DSGE models with fixed costs behaved almost identically to conventional DSGE models without fixed costs. Thomas (2002) and Khan and Thomas (2008) argue that the "irrelevance results" depend on consumption smoothing motives of the representative household. Gourio and Kashyap (2007) and Bachmann, et al. (2013) have challenged these results on the grounds that they hold only for certain parameter values and are not general properties of models with fixed adjustment costs. Recent papers by Fiori (2012) and Berger and Vavra (2012) show that in environments with high rates of substitution across different types of durable capital goods, the responsiveness of investment varies cyclically and along some dimensions has a better match with investment data. While Fiori extends the framework in Thomas (2002), and Berger and Vavra build on Bachmann et al. (2013), both papers emphasize the potential consequences of multi-sector models with fixed adjustment costs.

While numerical analysis has advanced rapidly in recent years, numerical techniques are limited to solving and cataloging particular cases. Furthermore, the techniques required are still quite cumbersome and the underlying economic forces at play are often obscured. The main objective of this paper is to shed light on these forces.

3. Model

The model is in continuous time.[5] The demand side consists of a continuum of firms that maximize discounted profits net of investment costs. Firms discount the future at the rate r. Each firm owns a stock of capital k, which depreciates exponentially at the rate δ. Because I focus on long-lived investment goods, I assume the annual depreciation rate is less than five percent. Flow profits are $A(t)k(t)^{\alpha}$, where $0<\alpha<1$ and $A(t)$ is a shock to the profitability of capital. When a firm adjusts its capital, say from k to k', it incurs two costs. The first is a fixed cost $F>0$, which is paid whenever investment at the firm is non-zero. The second is a cost per-unit of investment $p(t)\cdot[k'-k]$. To make matters simple, assume that when the firm adjusts, it must adjust to a fixed level of capital $\overline{k}$. As a result, the firm's problem is to decide simply when to adjust. (The assumption of a constant reset level is innocuous and is relaxed in Section 4.4. See also the discussion in Section 4.5.) If the firm does not adjust, then $\dot{k}=-\delta k$. If the firm adjusts at time T, its capital jumps from $k(T)$ to $\overline{k}$ and the firm pays $p(T)\cdot\left[\overline{k}-k(T)\right]+F$.

To focus attention on the demand side of the model, the supply side is intentionally kept as simple as possible. Aggregate investment $I(t)$ is the sum of firm-level investment. The flow supply of aggregate investment is governed by a supply curve $p(t)=S(I(t),Z(t))$ where $p(t)$ is the market price of new investment goods, $Z(t)$ is an investment supply shock (Z could be a vector) and $\partial S/\partial I>0$.

Below, I establish key properties of firm behavior in the steady state. I then use these properties to approximate the behavior of the model away from the steady state. I use numerical techniques to confirm and illustrate the approximate analytical results.

3.1. The optimal timing of investment in the steady state

Normalize the steady state price p and steady state productivity A to be 1. Let $\overline{V}$ denote the steady state value of having $\overline{k}$ units of capital and behaving optimally from then on. The optimization problem of a typical firm is to choose a time to

[2] Adda and Cooper (2000) present a dynamic analysis of a French automobile scrapping subsidy with implications exactly in this spirit.

[3] See, among others, Abel and Eberly (1994), Bertola and Caballero (1990), Cabellero (1993), Caballero and Leahy (1996), Caballero and Engel (1999), Cooper and Haltiwanger (1993), Cooper et al. (1999), Dixit and Pindyck (1994) and Eberly (1994). For studies that confine attention to steady states, see, Caplin and Spulber (1987), Hendel and Lizzeri (1999, 2002), House and Leahy (2004), House and Ozdenoren (2008), House and Zhang (2012) and Stolyarov (2002), Caplin and Leahy (1991, 1997) assume that firm-level investment does not react to endogenous changes in the distribution of firms.

[4] Typically, obtaining analytical results requires strong assumptions. See Danziger (1999) for a closed-form analysis of a model with fixed costs. Gertler and Leahy (2008) adapt Danziger's approach to a more conventional model of price rigidity. Caplin and Leahy (2006) assume that idiosyncratic shocks smooth out the distribution over time, simplifying the solution. Miao and Wang (forthcoming-b, forthcoming-a) show that if all plants have constant returns to scale production functions then there is an exact aggregation result and a equivalence between a model with fixed costs and a Q-theory investment model. For numerical approaches see Krusell and Smith (1997, 1998); Rios-Rull (1999), Dotsey et al. (1999), Thomas (2002), Veracierto (2002); Khan and Thomas (2003) and King and Thomas (2006). For numerical approaches see Krusell and Smith (1997, 1998), Rios-Rull (1999), Dotsey et al. (1999), Thomas (2002), Veracierto (2002), Khan and Thomas (2003) and King and Thomas (2006).

[5] The basic structure of the model is inspired by Caplin and Leahy (2004, 2006).

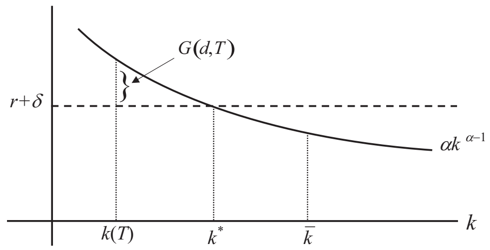

Fig. 1. Optimal behavior in the steady state and the gap $G(\delta,T)$

adjust τ to maximize

$$V(\tau)=\int_0^{\tau} e^{-rt}(e^{-\delta t}\overline{k})^{\alpha}dt-e^{-r\tau}[\overline{k}-e^{-\delta\tau}\overline{k}]-e^{-r\tau}F+e^{-r\tau}\overline{V}. \tag{1}$$

This is simply a continuous-time Bellman equation for a firm in the steady state. Let $T=\text{argmax}_{\tau}\{V(\tau)\}$ be the optimal time to adjust. The first order condition requires

$$(k(T))^{\alpha}-r[\overline{V}-F-\overline{k}]-(r+\delta)k(T)=0, \tag{2}$$

where $k(T)=e^{-\delta T}\overline{k}$. The first term is the gain the firm would get by using its existing capital stock more. The second term reflects the fact that waiting delays the payoff $\overline{V}-F-\overline{k}>0$. The last term captures the cost of delaying the resale and (because of depreciation) reducing the resale value. At the optimum, the firm is indifferent between adjusting and waiting. The second order condition requires

$$\delta k(T)[\alpha k(T)^{\alpha-1}-(r+\delta)]>0. \tag{3}$$

Condition (3) says that when the firm adjusts, the marginal product of capital $\alpha k(T)^{\alpha-1}$ must be strictly greater than the user cost of capital $r+\delta$. The difference between the marginal product and the user cost plays an important role in the analysis. Denote this difference as $G(\delta,T)=\alpha k(T)^{\alpha-1}-(r+\delta)$.

As mentioned above, the firm's reset level of capital $\overline{k}$ is fixed. This is done for simplicity and also because it makes the arguments stark. However, while the firms cannot adjust $\overline{k}$ freely, $\overline{k}$ is required to be optimal in the steady state. That is, if the firms existed in a completely stationary environment and could choose the reset level, they would choose $\overline{k}$.[6] It is relatively straightforward to show that this requires

$$\alpha\overline{k}^{\alpha-1}\left(\frac{1-e^{-(r+\alpha\delta)T}}{1-e^{-(r+\delta)T}}\right)\left(\frac{r+\delta}{r+\alpha\delta}\right)=r+\delta. \tag{4}$$

One can show that the marginal product of capital at $\overline{k}$ is less than the user cost $r+\delta$. Let k^* be the capital stock at which the standard user cost relation holds, so $\alpha[k^*]^{\alpha-1}=r+\delta$. Thus, $\overline{k}$ exceeds k^*, which in turn exceeds $k(T)$. Fig. 1 shows the relationship between $k(T)$, k^*, $\overline{k}$ and the gap $G(\delta,T)$.

3.2. The intertemporal elasticity of investment demand

Firms in the fixed-cost model are surprisingly willing to adjust the timing of their investments in response to transitory price changes. This high intertemporal elasticity of demand is the key property used to approximate the behavior of the model away from the steady state. It is also a property shared by neoclassical investment models. My analytical strategy is to quantify the elasticity of investment timing in the steady state and then use this information to approximate the behavior of the model away from the steady state.

To quantify the firms' willingness to adjust the timing of their investment, consider the following calculation: Suppose we wanted to compensate a firm sufficiently to induce it to change the timing of its investment by an amount Δ. Abusing notation somewhat, let $p(\Delta)$ denote the price at which the firm is indifferent between adjusting at time T and paying a price of 1 and adjusting at time $T+\Delta$ but paying $p(\Delta)$. That is, $p(\Delta)$ satisfies

$$\overline{V}-F-[\overline{k}-k(T)]=\int_0^{\Delta} e^{-rt}(e^{-\delta t}k(T))^{\alpha}dt+e^{-r\Delta}\{\overline{V}-F-p(\Delta)[\overline{k}-e^{-\delta\Delta}k(T)]\} \tag{5}$$

It is straight-forward to show that $p(0)=1$, $p'(0)=0$ and $p''(0)<0$ so the firm always requires a price cut to willingly invest at a time other than T. To a second order approximation,

$$p(\Delta)=1-\frac{1}{\zeta}\Delta^2 \tag{6}$$

[6] If the firm could choose both $\overline{k}$ and τ, the optimization problem would be $\max_{\overline{k},\tau}\{V(\overline{k},\tau)-\overline{k}\}$ where $V(\overline{k},\tau)=(1-e^{-r\tau})^{-1}\{\overline{k}^{\alpha}(1-e^{-(r+\alpha\delta)\tau})(r+\alpha\delta)^{-1}-e^{-r\tau}[F+\overline{k}(1-e^{-\delta\tau})]\}$ Condition (4) is the first order condition for $\overline{k}$.

where $\zeta = 2/p''(0)$. I refer to ζ as the firm's intertemporal price elasticity of adjustment. The measure ζ is similar to an intertemporal elasticity of investment demand since it describes the willingness on the part of firms to adjust the timing of their investments in response to a temporary price change. The following proposition provides an expression for the elasticity of investment timing in the fixed-cost model.

Proposition. *Let T, $\overline{k}$ and $\overline{V}$ be optimal and let $p(\Delta)$ satisfy (5). Then,*

(i) *The intertemporal price elasticity of adjustment is*

$$\zeta = -\left(\frac{2}{\delta}\right)\left(\frac{e^{\delta T}-1}{G(\delta,T)}\right) \tag{7}$$

where the gap $G(\delta,T)$ is given by

$$G(\delta,T) = (r+\delta)\left[e^{(1-\alpha)\delta T}\left(\frac{1-e^{-(r+\delta)T}}{1-e^{-(r+\alpha\delta)T}}\right)\left(\frac{r+\alpha\delta}{r+\delta}\right)-1\right] \tag{8}$$

(ii) *For small $(r+\delta)T$,*

$$G(\delta,T) \approx \delta(r+\delta)(1-\alpha)T \tag{9}$$

and

$$\zeta \approx -\frac{2}{\delta(r+\delta)(1-\alpha)} \tag{10}$$

Proof. See Appendix 1.

The intertemporal price elasticity of adjustment ζ governs the extent to which prices and investment change over time. If ζ is relatively small then firms want to make adjustments to their capital stock on schedule and will require large price cuts to induce them to delay or accelerate their investment. If ζ is high, then even small price differences can cause firms to make large changes in the timing of investment. The proposition suggests that ζ is indeed very high suggesting that firms are highly responsive to transitory changes in the price of investment goods. Using (6), the price change needed to induce a firm to change its investment timing by Δ is

$$dp = -\frac{1}{\zeta}\Delta^2 = -\frac{1}{2}\delta(r+\delta)(1-\alpha)\Delta^2 \tag{11}$$

For long-lived investment goods, this price change is extremely small. Mechanically, the magnitude of the price change is dominated by the presence of the term $\delta(r+\delta)$. For example, if $\delta = .04$, $r = .02$, and $\alpha = .5$, the price change required to induce a firm to change its timing by one year is roughly $dp = -0.0006$ or 6 basis points (6/100ths of one percent). Price changes of this magnitude are so small that they would not be detectable in data.[7] For quarterly data, a temporary 6 basis point price reduction would cause all firms planning on adjusting in the next four quarters, to adjust now, increasing investment by 300 percent. This corresponds to an elasticity of investment demand of 5000.

The calculations above demonstrate that firms will make sharp changes to the timing of their investments to take advantage of small predictable price changes. I use this observation to approximate the equilibrium out of the steady state. Because firms react so strongly to even small price differences, my approximation assumes that changes in the equilibrium price will be approximately unpredictable. Moreover, if I restrict attention to *transitory* shocks so the long-run price is near the steady state, I will simply approximate the equilibrium price with its steady state level. This is equivalent to approximating the solution with a flat demand curve at the steady state price. This approximation leads to several implications discussed below. Section 4 uses a numerical model to show that this approximation is surprisingly accurate.

3.3. Implications

3.3.1. Investment demand

The analysis above shows that slight transitory changes in prices cause firms to dramatically alter the timing of their investment decisions. For sufficiently long-lived investment projects and sufficiently patient firms (low δ and low r), the incentive to delay or accelerate investment in response to predictable price changes is very high. Thus, despite the apparent complexity of the fixed-cost model, characterizing its dynamic behavior is disarmingly simple. Investment demand is approximately summarized simply by a flat demand curve. The demand curve may shift up or down, but these shifts are unpredictable. If the shocks confronting the firm are short-lived, and thus have little impact on the long-run value of capital, the demand curve remains close to the steady state price. This continues to be true regardless of whether there are relatively

[7] Increasing r to 0.08 or reducing α to zero will increase the price change to 12 basis points. Adding growth acts like depreciation. With growth, the price change is $-(1/2)(\delta+g)(r+\delta+g)(1-\alpha)\Delta^2$. If g=0.02, the approximate price response would again be 12 basis points.

many or relatively few firms near the adjustment trigger. If there are many firms considering adjustment, the demand curve shifts to the right. If few firms are at near the adjustment margin, the demand curve shifts to the left. However, because demand is nearly horizontal, price and quantity are unaffected by these shifts and thus the equilibrium is approximately independent of the distribution.

The reader may be struck that the implied price path is so close to a partial equilibrium framework (in which prices are assumed to be fixed), but yet aggregate investment is not influenced by the micro-level heterogeneity. After all, researchers have previously found that the distribution of capital has effects in partial equilibrium settings. Of course, the two results are consistent. Because firms are so sensitive to price changes, even a seemingly small departure from perfectly constant prices will cause dramatic changes in the equilibrium. This is why partial equilibrium models produce results that are in stark contrast to equilibrium models and is why partial equilibrium analyses of fixed cost models give such misleading results.

3.3.2. Comparison with neoclassical investment models

The extremely high intertemporal elasticity of investment demand is a feature that the fixed-cost model shares with neoclassical investment models and it is why the two models, though very different at the micro-level, are so similar at the aggregate level. In neoclassical settings, firms equate the marginal cost of investment with the marginal benefit. Let $q(t)$ be the marginal benefit of additional capital. In a neoclassical framework,

$$q(t) = \int_t^{\infty} e^{-(r+\delta)s} MP^k(s) ds, \tag{12}$$

where $MP^k(s)$ is the marginal product of capital at time s. At an optimum, $q(t) = p(t)$.

The marginal value of capital $q(t)$ is a discounted sum of payoffs. For low δ and low r, there cannot be large predictable movements in $q(t)$. With sufficiently long-lived capital and sufficiently short-lived shocks, one can safely approximate $q(t)$ with its steady state value $\overline{q}$.[8] If the firm is patient and depreciation is slow, the integral places substantial weight on the future terms. Because transitory shocks influence only the first few terms in the integral, they have a negligible impact on the value of capital and so $q(t) = \overline{q}$. By affecting more terms in the integral, persistent, long-lasting shocks have a greater effect on $q(t)$. Even then, changes in $q(t)$ will be nearly unpredictable.

Note, the intertemporal price elasticity of adjustment ζ is not identical to the intertemporal elasticity of investment *demand*. In continuous time models, the intertemporal elasticity of investment demand is not well-defined. Any momentary change in price will typically entail $\partial I / \partial p = \infty$. In a discrete time model however, we can calculate the associated intertemporal demand elasticities. For a discrete-time neoclassical model, if $MP_t^k = \alpha k_t^{\alpha-1}$, then if the price falls for one period and then returns to the steady state, $\partial I_t^{NEO} / \partial p_t = -[(1-\alpha)\delta(r+\delta)(.25)^2]^{-1}(1+r)$ where the 0.25 term converts the annual rates r and δ to quarterly rates (the superscript NEO indicates that the calculation is for the neoclassical model). For the fixed adjustment cost model, suppose the price falls by enough to induce all firms that would normally adjust at $t-1$ and $t+1$ to adjust instead at date t. Using (10), we have $\partial I_t^{FC} / \partial p_t = (\partial I_t^{NEO} / \partial p_t)(1+r)$ (FC denotes the calculation for the fixed-cost model).

For long-lived capital goods, the investment demand curves implied by the neoclassical model and the fixed cost model are essentially the same – both are approximately flat lines at the steady state price. Thus, in the low-depreciation limit, differences in equilibrium outcomes reflect differences in supply. If supply is the same in the two models, then the equilibrium outcome in the fixed-cost model and the equilibrium outcome in the neoclassical model will be nearly identical.

3.3.3. The distribution of capital holdings

The distribution of capital holdings features prominently in the theoretical and empirical literature on fixed costs.[9] The analysis here suggests that, for sufficiently long-lived investments, variations in the distribution of capital across firms should have no independent influence on equilibrium investment or prices. In particular, if the only changes to the system are changes in the distribution, then equilibrium prices and investment should remain close to their steady state levels.

Firms that are bunched up or spread out relative to the steady state distribution either delay or accelerate the timing of investment to avoid paying high prices. Because the costs of changing this timing are so small, firms are perfectly willing to re-time their investments. In equilibrium, through a kind of intertemporal arbitrage, firms eliminate predictable price changes and the price remains close to its steady state even though the distribution of capital is not.

4. Numerical analysis and applications

Based on the analysis in Section 3, we should expect to observe the following in fixed-cost models for long-lived investments: (i) a temporary supply shock should have no noticeable effect on the price of new capital but should reduce equilibrium investment by the amount of the shock; (ii) a temporary demand shock (modeled as a temporary increase in A) should have virtually no influence on either prices or investment; (iii) different initial distributions of capital should have no

[8] Barsky et al. (2007) and House and Shapiro (2008) evaluate the accuracy of this approximation in neoclassical settings.

[9] Using LRD data, Caballero et al. (1995) argue that changes in the distribution explain changes in the responsiveness of investment to disturbances. Using BEA investment data Caballero and Engel (1999) argue that the distribution has predictive power for investment.

consequences for prices or investment; and (iv) for sufficiently transitory shocks, the aggregate behavior of the fixed-cost model should be very close to the aggregate behavior of a conventional neoclassical model.

Of course, the analysis in Section 3 is only approximate. Furthermore, the identical behavior of the neoclassical model and the fixed-cost model, and the irrelevance of the distribution are results we should expect only in the low-depreciation limit. Here, I use a numerical model to check the accuracy of the approximation away from the low-depreciation limit. The results suggest that the approximation is accurate even for modest depreciation rates. I begin by sketching out the broad features of the quantitative model used. Details of the numerical solution are in Appendix 2 and in House (2014).

4.1. Quantitative model

The numerical model is cast in discrete time with time intervals of size Δ. There are $J+1$ possible capital stocks $\overline{k}, k_1, k_2, \ldots, k_J$ with $k_j = e^{-\delta \Delta j}\overline{k}$. The lowest possible capital stock is k_J. Let $V_{j,t}$ be the value of having capital stock j at time t and let $\overline{V}_t$ be the value of having the reset level $\overline{k}$ at time t.

The numerical solution uses a method developed jointly by Robert King, Julia Thomas and Marcelo Veracierto.[10] The key simplifying assumption of the method is to assume that firms draw idiosyncratic fixed costs of adjustment each period. Thus, instead of facing the fixed cost F each period, firm i faces the stochastic fixed cost $\varepsilon_{i,t}$ where $\varepsilon_{i,t} \sim \Psi(\varepsilon)$, $E[\varepsilon_{i,t}] = F$, $\varepsilon_{i,t} \geq 0$ and $\varepsilon_{i,t}$ is *i.i.d.* across periods and across firms. Given a time interval Δ, the discount factor is $\beta = e^{-r\Delta}$. The value for a firm with cost draw ε, capital stock $k = k_j$ at time t is

$$V_{j,t}(\varepsilon) = \max\{\Delta A_t k_j^{\alpha} + \beta E_t[v_{j+1,t+1}], \Delta A_t \overline{k}^{\alpha} + \beta E_t[v_{1,t+1}] - \varepsilon - p_t(\overline{k} - k_j)\}, \tag{13}$$

where

$$v_{j,t} = \int_0^{\infty} V_{j,t}(\varepsilon) d\Psi(\varepsilon). \tag{14}$$

The marginal firms with capital stock j have critical cost draw

$$\hat{\varepsilon}_{j,t} = \Delta A_t[\overline{k}^{\alpha} - k_j^{\alpha}] + \beta E_t[v_{1,t+1} - v_{j+1,t+1}] - p_t(\overline{k} - k_j). \tag{15}$$

Firms with $\varepsilon > \hat{\varepsilon}_{j,t}$ do not adjust while firms with $\varepsilon < \hat{\varepsilon}_{j,t}$ adjust thus the probability that a firm with capital of vintage j adjusts is $\Psi(\hat{\varepsilon}_{j,t})$. These adjustment hazards endogenously respond to aggregate conditions according to the changes in the cutoff $\hat{\varepsilon}_{j,t}$. When a firm adjusts, its investment is $\overline{k} - k_j$. Let $f_{j,t}$ be the number of firms with capital of vintage j. Aggregate investment I_t is then,

$$I_t = \sum_{j=1}^{J} f_{j,t} \Psi(\hat{\varepsilon}_{j,t})[\overline{k} - k_j]. \tag{16}$$

The measures $f_{j,t}$ satisfy

$$f_{j,t} = f_{j-1,t-1}(1 - \Psi(\hat{\varepsilon}_{j-1,t-1})) \tag{17}$$

for $2 \leq j \leq J$and

$$f_{1,t} = \sum_{j=2}^{J} \Psi(\hat{\varepsilon}_{j,t-1}) f_{j,t-1} \tag{18}$$

for $j = 1$.

To close the model, assume an isoelastic supply curve of the form

$$p_t = Z_t \cdot (I_t/\overline{I})^{1/\xi}. \tag{19}$$

Here ξ is the elasticity of investment supply, $\overline{I}$ is steady state investment and Z_t is a cost shock with mean 1. Finally, Z and A follow autoregressive processes

$$A_{t+1} = (1 - \rho_A) + \rho_A A_t + e_{A,t+1}, \tag{20}$$

$$Z_{t+1} = (1 - \rho_Z) + \rho_Z Z_t + e_{Z,t+1}. \tag{21}$$

The model is solved using a linear approximation in the neighborhood of the non-stochastic steady state (the steady state associated with $A_t = Z_t = 1$ for all t). The Anderson–Moore algorithm is used to compute rational expectations equilibria.

I choose parameter which are plausible but which also accentuate the potential role for fixed adjustment costs. Baseline parameter values are summarized in Table 1. The supply elasticity ξ is 1; the autoregressive parameters ρ_Z and ρ_A are set to imply a 6-month half-life of shocks which, together with the variances of the innovations e_Z and e_A, imply a long-run variance of one percent. A 6-month half-life is a typical duration for transitory shocks used in New Keynesian DSGE sticky-price models

[10] This solution technique has been used by many researchers working in this area. Dotsey et al. (1999) use this technique to analyze price setting in a menu cost model. Thomas (2002); Veracierto (2002) and Gourio and Kashyap (2007) use it to analyze investment. King and Thomas (2006) use the same technique to analyze labor adjustment.

though it is shorter duration for shocks in RBC models which often use much more persistent disturbances. I focus on shorter-lived shocks because these allow me to clearly illustrate the behavior of the model. (House (2014) includes additional discussion of how changes in the persistence of the shocks influence the model.)

The choice of the fixed cost F is the matter of debate in the literature. In the model, any choice of F implies a corresponding adjustment horizon T. In principle one could use data on adjustment events to discipline the choice of the fixed adjustment cost. For instance, Cooper et al. (1999) define an investment spike as any increase in plant-level capital of 20 percent or more. In their data, 18 percent of firms experience an investment spike in any given year implying an average adjustment horizon of roughly 5.5 years. Using a similar definition of an investment spike, Gourio and Kashyap (2007) find that in their data, firms have adjustment horizons of roughly 6.7 years. This horizon is sensitive to the definition of an investment spike however. If one uses a 30 percent cutoff in the Doms and Dunne (1998) data, then plants have roughly a 12.5 year adjustment horizon. The calibrations used in the literature are similarly diverse. The calibration in Thomas (2002) corresponds to an average adjustment horizon of roughly 3.4 years while the calibration in Bachmann et al. (2013) implies an average adjustment horizon of roughly 10 years. For the baseline calibration, F is chosen to imply a steady state adjustment horizon of ten years. The 10 year horizon is chosen for two reasons. First, it is in the range of plausible values suggested by the data. Second, it is intentionally at the higher end of calibrations in the literature.

The curvature parameter is $\alpha = 0.77$ which implies fairly low returns to scale at the plant level. This calibration, suggested by Bachmann et al. (2013), again accentuates the potential role of fixed costs in the model. Finally, $r = 0.04$.

By definition, long-lived capital goods have low depreciation rates. For purposes of this discussion, a capital good is said to be long-lived if it has an annual depreciation rate of no more than five percent. Examples of such goods include manufacturing structures, commercial office buildings, electrical transmission and distribution apparatus, telecommunications structures, and so forth. Most structures are long-lived capital goods. (Structures typically have depreciation rates between two and four percent; see Fraumeni (1997).) Because structures make up roughly thirty percent of all non-residential investment, long-lived investments are a nontrivial fraction of aggregate investment. In the numerical experiments, I consider five depreciation rates: 0.20, 0.10, 0.05, 0.02 and 0.01. (Note, each time the depreciation rate is changed in the model, the fixed cost F is re-calibrated to maintain the 10-year adjustment horizon.) The baseline annual depreciation rate is 5 percent. (See House (2014) for additional discussion of the effects of alternate parameter choices.)

4.2. Quantitative analysis

The numerical model can now assess the accuracy of the analysis from Section 3. I consider temporary supply and demand shocks, and the effect of changes in the distribution of capital across firms. I also compare simulated data from the fixed-cost model to simulated data from a standard neoclassical model.

4.2.1. Supply and demand shocks

Consider a positive innovation to Z_t of one percent. This increases the cost of investment and shifts the supply schedule back. The left-hand side panels of Fig. 2 show the model's reaction. The top panel shows aggregate investment. The middle panel shows the price. As expected, investment prices barely change. For $\delta = 0.10$ and $\delta = 0.20$, prices rise by less than 10 basis points. For lower depreciation rates the price change is even smaller. For $\delta = 0.01$ and $\delta = 0.02$, the price increase is roughly 1 basis point (1/100$^{\text{th}}$ the size of the impulse). Since prices change only slightly, most adjustment occurs through changes in aggregate investment. In each case, the drop in investment is almost 1.00 percent. For $\delta = 0.01$ and $\delta = 0.02$, it is 0.99 percent. This is exactly what would be predicted if demand were nearly flat.

The right-hand side of Fig. 2 shows a demand shock – specifically, the response to a temporary increase in capital productivity A_t. Since supply is unchanged and since the supply elasticity is 1.00, prices and investment are identical. As predicted, the changes in prices and investment are small. For $\delta = 0.20$ and $\delta = 0.10$, prices and investment rise by roughly 15 basis points and 8 basis points, respectively. For $\delta = 0.02$ and $\delta = 0.01$, the increases are 3 basis points and 2.5 basis points. In each case, the price is essentially unaffected.[11]

4.2.2. A non-uniform initial distribution

Fig. 3 shows the equilibrium when the system begins with an out-of-steady-state distribution.[12] The initial distribution has many firms with five-year-old capital. To make the illustration stark, the initial density of firms with capital between 4.5 and 5.5 years old is twice the density elsewhere. The perturbed distribution, as well as the steady state distribution, is shown in the upper left panel. Notice that the steady state distribution is close to a pure uniform distribution (the stochastic fixed costs imply that the distribution of vintages is not quite uniform). Since firms adjust roughly once every 40 quarters (10 years), the measure of firms at each vintage is approximately 0.025 in the steady state (the measure of all firms is 1.00).

The alternate distribution has twice as many firms with five-year old capital so the density for these firms is roughly 0.045 while the density at every other vintage is slightly lower. With twice as many firms with five-year-old capital,

[11] The near-infinite elasticity of investment demand implies that after-tax prices are constant for temporary shocks. A temporary investment subsidy increases pre-tax prices by the amount of the subsidy. House and Shapiro (2008) use this property to estimate ξ following the 2002 bonus depreciation policy. While their analysis uses a neoclassical model, the estimates are valid in a model with fixed-costs of adjustment.

[12] This experiment is inspired by the analysis in Gourio and Kashyap (2007).

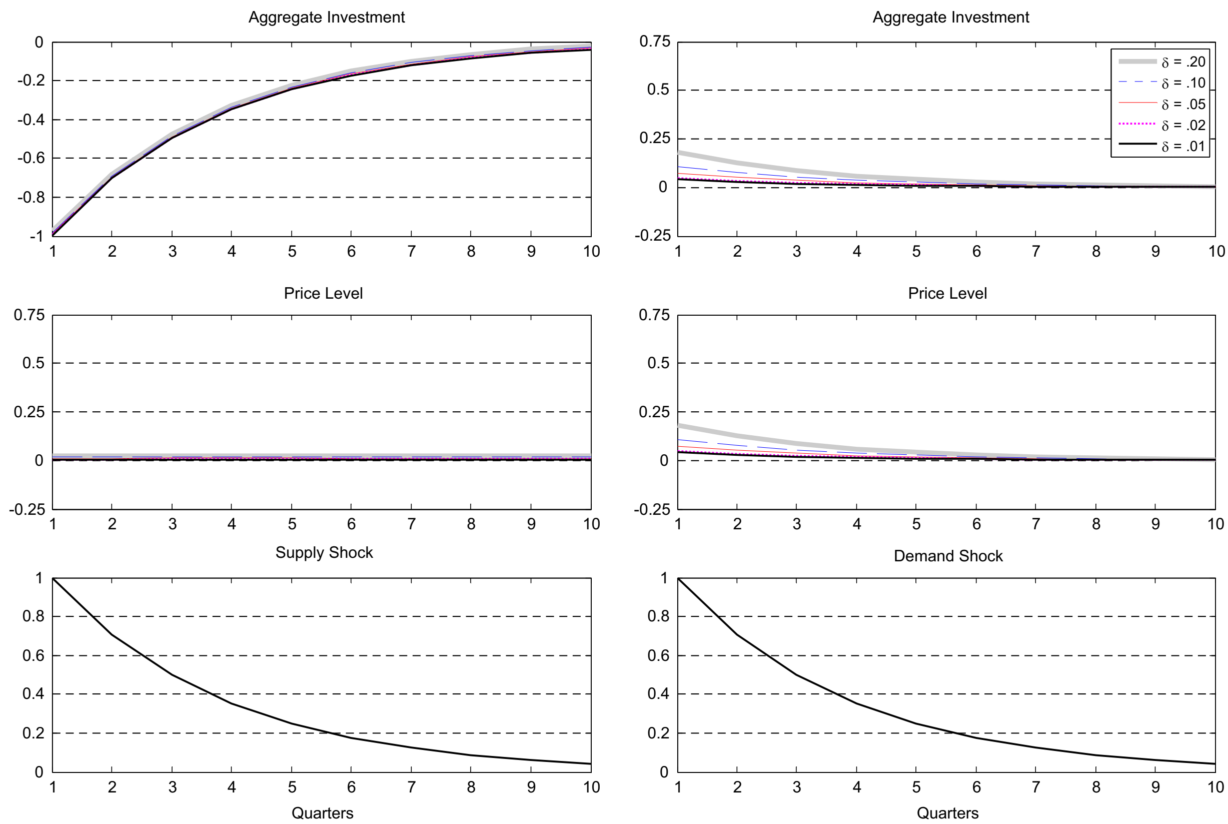

Fig. 2. Equilibrium response to investment supply and demand shocks. *Notes*: The left-hand side panels show the model's reaction to an investment supply shock (a shock to Z in Eq. (19)). The right-hand side panels show the model's reaction to an investment demand shock (a shock to A in Eq. (13)). The reactions are all quoted as percent deviations from steady state.

one would expect that, in roughly five years, prices and investment would rise as these firms adjust. If the firms could not change the timing of their investments, total investment would roughly double as the number of firms adjusting goes from roughly 0.022 to roughly 0.045. Since the elasticity of investment supply is 1.00, prices and aggregate investment would rise by 100 percent.

The upper right panel shows the response of aggregate investment for different depreciation rates. The conventional supply and demand prediction that prices and investment should rise as the mass of firms adjusts is present in the figure but is quantitatively negligible relative to the magnitude of the distributional change. Instead of an increase of 100 percent, investment rises by less than one-half of one percent when $\delta = 0.20$. For $\delta = 0.05$, the equilibrium increase in investment is only 10 basis points.

The lower left panel shows results for different supply elasticities. Each line corresponds to a different ξ. The depreciation rate is set to its baseline value $\delta = 0.05$. It is remarkable how little influence the elasticity has on the equilibrium. With $\xi = 20$, the maximum change in aggregate investment is roughly 1 percent. This is quite small in comparison with the 100 percent increase in firms with five-year-old capital. Even for $\xi = 100$ aggregate investment reacts by less than 4 percent.

4.3. Comparison with neoclassical investment models

In this section, a standard neoclassical investment model is used to compare the equilibrium outcomes with the fixed-cost model. The neoclassical model has flow production $A_t k_t^{\alpha}$. The supply curve for both models is given by (19) and the parameters are both set to the baseline values in Table 1. Both models are subjected to the same sequence of shocks.

Fig. 4 presents simulated data from the neoclassical model and the fixed-cost model. The upper panels show results for aggregate investment while the lower panels show prices. The panels on the left show simulated time series. The thin black line is the fixed-cost model while the thick gray line is the neoclassical model. (The two lines in the upper left panel are difficult to distinguish because they lie almost on top of each other.) While the paths for aggregate investment are essentially identical, there are noticeable differences in the price series. The middle panels show the impulse response to a cost shock like the one considered in the left panels of Fig. 2. The investment responses are identical while the price responses display small differences. The panels on the right show scatterplots of 500 years of quarterly data. Each dot represents a data point from the neoclassical model and the corresponding data point from the fixed-cost model. Again, the

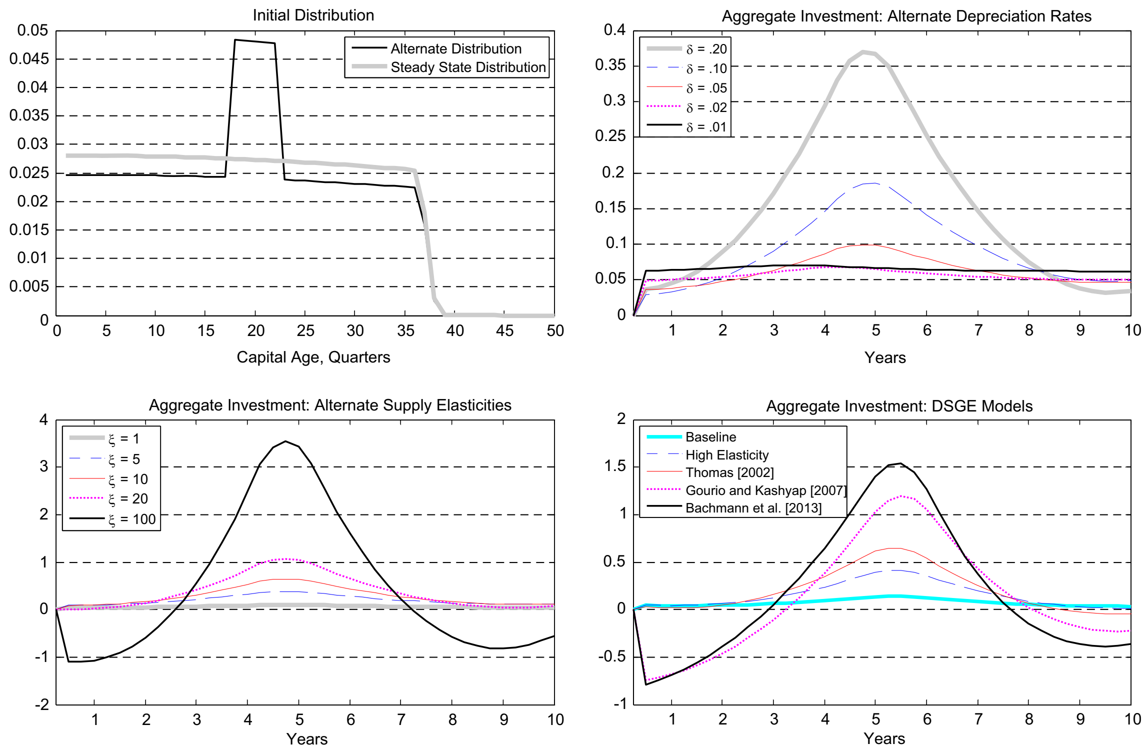

Fig. 3. Equilibrium starting from an out-of-steady-state distribution. *Notes*: The figure shows the percent deviations of aggregate investment for fixed cost models conditional on beginning in an out-of-steady state distribution. The upper left panel shows the initial distribution as well as the steady state distribution. The upper right panel shows the reaction for different depreciation rates. The lower left panel shows the reaction for different supply elasticities. The lower right panel shows the reaction for five DSGE models as described in the text.

Table 1
Baseline parameters.

Parameter	Baseline value
Discount rate, annual (r)	0.04
Depreciation rate, annual (δ)	0.05
Curvature of profit function (α)	0.77
Steady state adjustment horizon (T) (years)	10.00
Elasticity of aggregate investment supply (ξ)	1.00
Half-life of demand shock (years)	0.50
Half-life of supply shock (years)	0.50

investment data are virtually the same while the price data display some small differences. This pattern is robust to wide variations in the parameters.

The difference in the equilibrium prices deserves some additional discussion. Recall the approximation in Section 3.2. Because the intertemporal elasticity of investment demand is so high, we concluded that there would not be large predictable changes in the value of capital (the price). Moreover, since we are looking at transitory shocks, the price would be approximately unaffected. As the approximation suggests, for both models, the computed price changes are quite small. They are not zero however. The extent to which prices deviate from zero reflects the imperfections in the approximation itself. The exact equilibrium price change reflects changes in the long-run value of capital. Unlike investment demand, which is characterized by a (nearly) flat demand curve, the demand for capital is downward sloping and the shape of the demand curve is model dependent. In the neoclassical model, the price reflects the discounted marginal product of capital. In the fixed-cost model, the price is more closely tied to the discounted average total product of capital in the range of inaction where the firm optimally chooses not to adjust. In addition, because our approximation says that there cannot be predictable changes in the price, the prices are all (near) random walks. While the price reaction for any one shock is small, the price change is highly persistent and thus the unconditional variance of the price is non-negligible. These differences account for the observed patterns in the simulated prices.

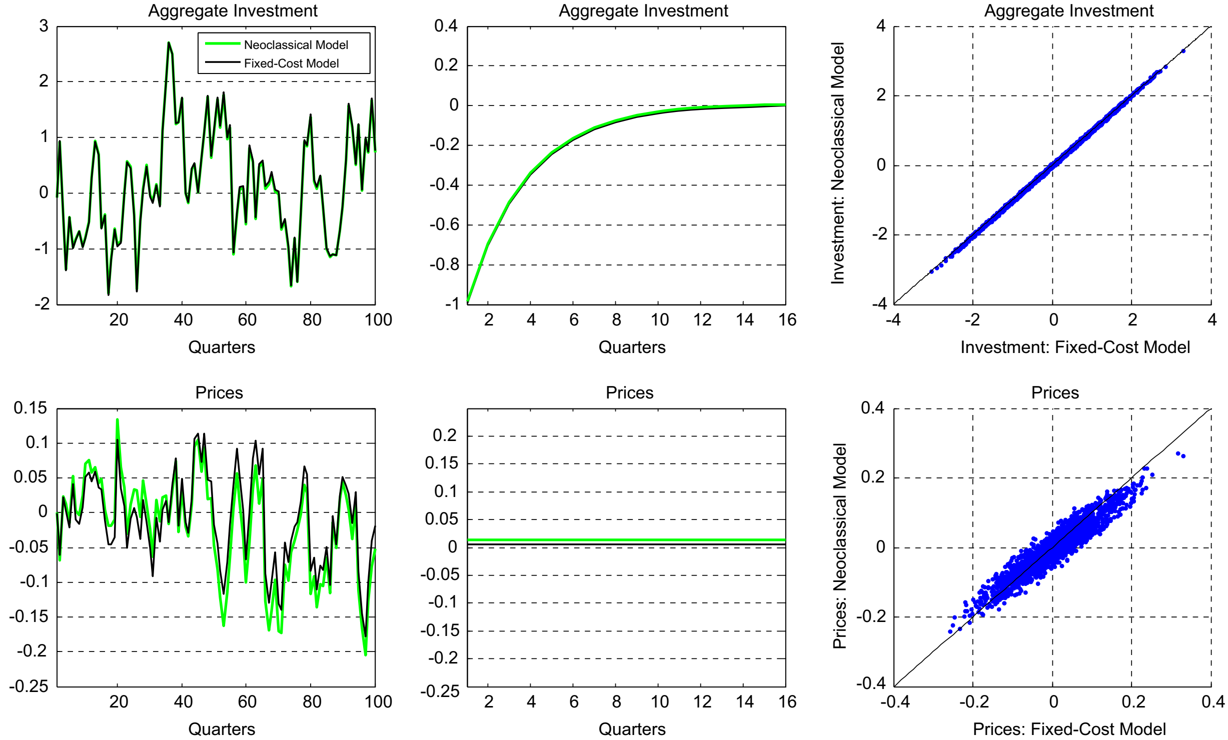

Fig. 4. Comparing the fixed-cost model with the neoclassical model, baseline parameter values. *Notes*: The parameter values for the fixed-cost model are given in Table 1. The neoclassical model is described in the text. Parameter values are identical to those in the fixed cost model. The scatter-plot shows 500 years of simulated data. Both models experienced identical shocks.

4.4. Relation to DSGE models

Most of the well-known work in this area numerically analyzes calibrated DSGE models. Superficially, the investment supply and demand framework analyzed here – the supply side in particular – seems fundamentally different from the DSGE models. In fact, this apparent difference is an illusion. One can show that there is a family of one-good DSGE models with fixed adjustment costs which can be mapped directly onto the supply-demand model in this paper. Every one-good DSGE model in this family can be represented by a version of the investment supply and demand model here. Because my framework is a generalization of these models, the properties that I identify (namely the high intertemporal elasticity of investment demand) are present in DSGE models.[13]

Consider the following conventional one-good model. A representative agent maximizes $E_t \sum_{j=0}^{\infty} \beta^j \{C_{t+j}{}^{1-(1/\sigma)} - aN_{t+j}{}^{1+(1/\eta)}\}$ subject to $X_t F(\{k_j\}_t) \cdot N_t^{\theta} = C_t + I_t$ where $\{k_j\}_t$ is the current collection of firm-level capital distributed over firms j, C_t is consumption, N_t is labor, X_t is aggregate productivity and $F(\cdot)$ is a production function defined over the firm-level capital stocks; σ is the intertemporal elasticity of substitution for consumption, η is the Frisch labor supply elasticity, and θ is the elasticity of output with respect to labor. The constant interest rate in my model corresponds to the time discount factor β. The price is the marginal (utility) cost of an additional unit of investment. In a one-good model, this is the marginal utility of consumption $C_t^{-(1/\sigma)}$. Fluctuations in the stochastic discount factor in the DSGE model are captured by fluctuations in A_t.

The supply curve in the DSGE model is the increasing relationship between the marginal utility of consumption and aggregate investment given capital and productivity. As discussed earlier, while consumption smoothing is not *necessary* for irrelevance results like those in Thomas (2002), consumption smoothing is a natural source of increasing marginal costs of investment in standard macroeconomic DSGE models. The supply schedule in the DSGE model is $C_t^{-(1/\sigma)} = S(I_t, X_t, \{k_j\}_t)$ which is a special case of the supply curve in Section 3. Near the steady state, the elasticity of supply is

$$\xi = \frac{\eta}{1+\eta(1-\theta)} \theta \frac{Y}{I} + \sigma \frac{C}{I}. \tag{22}$$

[13] The description of the family of DSGE models as well as the formal mapping between the DSGE models and the investment supply and demand framework are in Appendix 3.

Table 2
Calibrations for DSGE models.

Parameter	Baseline	High elasticity	Thomas (2002)	Gourio and Kashyap (2007)	Bachmann et al. (2013)
Capital elasticity of production (γ)	0.325	0.325	0.325	0.216	0.180
Labor elasticity of production (θ)	0.580	0.580	0.580	0.385	0.640
Frisch labor supply elasticity (η)	0.5	1	∞	∞	∞
Intertemporal elasiticity of substitution (σ)	0.2	1	1	1	1
Implied ξ	2.83	8.71	15.42	10.21	18.16
Implied α	0.77	0.77	0.77	0.35	0.50

Note: All models have an investment to GDP ratio of 0.145, an annual depreciation rate of 0.10 and an annual subjective time discount rate of 0.04.

Table 3
DSGE model implied business cycle volatilities.

Series	Data	Baseline	High elasticity	Thomas (2002)	Gourio and Kashyap (2007)	Bachmann et al. (2013)
Panel A: RBC model (no fixed costs)						
GDP	1.66	1.12	1.57	2.00	1.71	2.28
Consumption	0.78	0.41	0.74	0.89	0.72	0.86
Investment	4.97	5.32	6.77	9.02	7.93	11.38
Real Wage	0.64	1.43	1.14	0.89	0.72	0.86
Employment	1.61	0.33	0.44	1.20	1.06	1.55
Panel B: Fixed cost models (flexible reset capital stock)						
GDP	1.66	1.17	1.60	2.12	1.77	2.45
Consumption	0.78	0.38	0.66	0.83	0.59	0.77
Investment	4.97	5.90	7.55	10.34	9.23	13.20
Real Wage	0.64	1.40	1.12	0.83	0.59	0.77
Employment	1.61	0.25	0.51	1.40	1.26	1.82

Note: All models have an investment to GDP ratio of 0.145, an annual depreciation rate of 0.10 and an annual subjective time discount rate of 0.04. The data are from Stock and Watson (1999).

If $\sigma = 0.2$, $\eta = 1/2$, and $\theta = 2/3$, and if the investment to GDP ratio is roughly 0.15 then the elasticity implied by (22) is roughly 3. If $\sigma = \eta = 1$, the implied elasticity is 9. Based on the results in Fig. 3, these elasticities are not high enough to allow micro-level heterogeneity to play a noticeable role in DSGE models.

To explore the effects of variations in the parameters, I consider five DSGE models with lumpy investment.[14] The models are in the family outlined above. Firm-level production is $y_{j,t} = X_t k_{j,t}^{\gamma} n_{j,t}^{\theta}$ and aggregate output is $Y_t = \sum_j y_{j,t}$.[15] I consider a baseline calibration, a calibration with a relatively high supply elasticity and three calibrations inspired by prominent papers in the fixed-costs literature: Thomas (2002), Gourio and Kashyap (2007) and Bachmann et al. (2013).[16] The calibrations are summarized in Table 2. The table also reports the implied supply elasticity (ξ) and the implied curvature (α) for each calibration. The elasticity is calculated from (22). The implied curvature is $\alpha = \gamma/(1-\theta)$. Each model has an investment to GDP ratio of 0.145, an average adjustment horizon of 10 years, an annual depreciation rate of 0.10, and a subjective time discount rate of 0.04. The lower right panel in Fig. 3 shows equilibrium aggregate investment for the five DSGE models when they begin with the out-of-steady-state capital distribution considered in Section 4.2.

As emphasized by Gourio and Kashyap (2007), the equilibrium is clearly influenced by the parameter values. For the baseline case, investment increases by roughly 15 basis points after five years. Not surprisingly, the calibration with the higher supply elasticity ($\xi = 8.71$) exhibits a greater response – roughly 40 basis points. The calibration based on Thomas' (2002) has a higher supply elasticity ($\xi = 15.42$) and the peak response is 65 basis points. The calibration based on Gourio and Kashyap (2007) implies a more modest supply elasticity ($\xi = 10.21$) but also features a low curvature, ($\alpha = 0.35$) so the marginal product of capital changes more as capital depreciates, reducing the elasticity of investment demand somewhat (recall Eq. (9)). The peak response for this calibration is 1.20 percent. The calibration in Bachmann et al. (2013) has both a high supply elasticity ($\xi = 18.16$) and a relatively low curvature ($\alpha = 0.50$). For this calibration, the peak response is 1.55 percent – roughly 10 times the baseline response.

While parametric variations clearly cause differences in the equilibrium, in all cases the quantitative importance of lumpy investment is extremely small relative to the perturbation. In the initial distribution, the number of firms with 5-year old capital is 100 percent greater than the number of firms with other vintages. In contrast, the maximum increase in investment in the DSGE models is never greater than 2 percent and for most, the impact is less than 1 percent.

[14] For comparability with DSGE models in the literature, I allow firms to choose their reset capital stocks.

[15] This specification implies that total output is $Y_t = X_t F(\{k_j\}_t) \cdot N_t^{\theta}$ as above.

[16] I describe these calibrations as "inspired" by these studies because while I use the same parameter values they use, I do not adopt all of the elements featured in the previous studies. As a consequence, the results here are not *reproductions* of the earlier results.

Table 3 computes business cycle volatilities for HP-filtered simulated data from the five DSGE models (Panel A) as well as five DSGE models with the same calibrations but without fixed adjustment costs (Panel B). The table also includes empirical volatilities taken from Stock and Watson (1999). Comparing the models with fixed costs in Panel A to the models without fixed costs in Panel B, it is clear that the addition of the fixed adjustment costs to do not make pronounced changes in the statistical performance of the models at business cycle frequencies. The main mismatch between the model generated volatilities and those observed in the data is that investment volatility is too high while consumption volatility is too low. This is a common property of RBC models which is typically addressed by introducing a convex adjustment cost applied to aggregate investment.

Readers familiar with the literature on fixed adjustment costs will notice that the business cycle statistics reported in Table 3 do not match the statistics reported in the earlier papers. This is because there are other distinguishing features of the earlier models that are not included in the simulation. (E.g., Bachmann et al. include a maintenance expense for firms that do not adjust; the model here uses quarterly time periods while Thomas, and Gourio and Kashyap consider annual models, etc.) The performance of the models also differs according to the solution method. The model here and the models in Thomas (2002) and Gourio and Kashyap (2007) are solved with linear methods while Bachmann et al. (2013) use discrete-state dynamic programming and the Krusell–Smith algorithm.[17]

4.5. *Discussion*

This section briefly discusses some remaining issues. In particular, the assumption of a fixed reset capital stock, the source of the irrelevance results, whether idiosyncratic shocks influence the conclusions and finally whether there are circumstances where the two models will differ.

4.5.1. *The source of the irrelevance results*

One of the main contributions of this paper is to reveal the source of the irrelevance results documented by other papers in the fixed cost literature. The similarity between neoclassical models and fixed-cost models, and the quantitative irrelevance of the distribution of capital across firms is caused by the combination of an upward sloping marginal cost of investment and an extremely high elasticity of investment demand. Previous researchers found the irrelevance results seemed to depend critically on consumption smoothing behavior in general equilibrium. In those models, consumption smoothing implied an increasing marginal cost of aggregate investment which, according to the analysis here, is sufficient to imply irrelevance.

While consumption smoothing is sufficient to generate irrelevance, the model in this paper shows that irrelevance results can emerge in settings without consumption smoothing and without general equilibrium. Specific reasons for an upward sloping investment supply curve are not essential – anything which causes an increasing marginal cost of investment will produce similar results. The key property which eliminates the role of the distribution of capital across firms is the high elasticity of investment demand.

4.5.2. *Capital adjustment at the intensive margin*

Researchers refer to changes in the number of firms experiencing an investment spike as changes in the *extensive* margin of investment. Changes in the average size of spikes are referred to as changes in the *intensive* margin. In my model, firms have a fixed reset capital stock $\bar{k}$. Thus, essentially all variation in aggregate investment is due to variation in the extensive margin. In reality, firms make adjustments at both the intensive and extensive margins.

The key insight from the analysis in Section 3 is that investment demand is extremely elastic. This property effectively eliminated predictable price changes in equilibrium and, by doing so, also eliminated any role for the distribution of capital to influence the equilibrium. Allowing firms to vary their reset capital stock – to make changes at the intensive margin – would not change the conclusions. If firms could change their reset capital stock, the elasticity of investment demand would be even higher and the approximation in Section 3 would be even more accurate.

4.5.3. *Idiosyncratic shocks, credit constraints and one-hoss shays*

The model intentionally abstracts from a variety of complicating real-world features to expose the mechanisms at work. Undoubtedly many firms base their investment decisions on factors other than price. For example, many firms face binding credit constraints when making investment decisions. Idiosyncratic demand or supply considerations also surely play a role in determining investment timing for particular firms. Some firms simply have to replace capital because of an unforeseen event like a fire or a flood which causes their existing capital stock to fail suddenly – a "one-hoss shay" depreciation process.

[17] Another important distinction between the models is the assumption on the distribution of the stochastic fixed adjustment costs. Bachmann et al. (2013) use a wide uniform distribution which introduces an increasing marginal cost of aggregate investment makes their specification similar to an RBC model with convex adjustment costs. The model here considers a more tightly concentrated distribution of stochastic fixed costs and is thus closer to a pure fixed cost model. See also the discussion in Gourio and Kashyap (2007) on this point.

As long as some firms are free to re-time their investments as the model assumes, complications like those mentioned above should leave the basic result intact. Firms that are free to re-time their investments should continue to effectively eliminate predictable movements in investment prices through intertemporal arbitrage while other firms invest as their circumstances dictate.[18]

4.5.4. Differences between neoclassical models and fixed-cost models

The preceding analysis suggests that, at the aggregate level, neoclassical models and fixed-cost models should often behave similarly in equilibrium. A question which naturally arises is whether there are any circumstances when the two models differ.

An obvious difference between the two is in their implications for firm-level investment patterns. In typical neoclassical investment models, firm-level investment is simply a scaled-down version of aggregate investment. In fixed-cost models, firm-level investment is starkly different from aggregate investment. Thus, the distinction between the two families of models will matter a lot for any study focused on firm-level investment patterns.

Another potentially interesting difference between the two modeling frameworks is in their reaction to "uncertainty shocks." It is well known that models with irreversibilities (like fixed cost models) react to changes in uncertainty about economic conditions (see for instance Bloom 2009). In particular, if firms anticipate that uncertainty will be resolved in the near future they will "wait and see" and reduce investment until the resolution. Thus, uncertainty shocks may be one example of shocks that have different consequences in fixed-costs settings than in neoclassical settings.

Finally, features that limit the intertemporal elasticity of investment demand may create environments in which the distribution of capital plays a stronger role. Planning costs or models with "sticky investment" may provide such features by constraining the firms' ability to freely choose the timing of investment.

5. Conclusion

The study of investment is of central importance to understanding business cycles and economic activity. The drive to base aggregate theories on solid micro-foundations as well as the desire to match firm-level investment patterns has led to the development of complex models of investment behavior at the firm level. Investment models featuring fixed costs of adjustment are attractive because they imply that investment at the plant-level will be infrequent, as seen in micro data sets.

This paper analyzed the approximate equilibrium behavior of a dynamic investment model with fixed adjustment costs. The analysis shows that for sufficiently long-lived capital goods, the elasticity of intertemporal substitution for the timing of investment is extremely high. As the depreciation rate approaches zero, this elasticity approaches infinity. The near-infinite elasticity of intertemporal substitution eliminates virtually any role for microeconomic heterogeneity in governing investment demand. This high intertemporal elasticity of investment demand is a property that conventional neoclassical models of investment demand and models with fixed costs have in common. Thus, even though simple neoclassical investment models are starkly at odds with the micro data, they capture virtually all of the relevant aggregate investment dynamics embodied in models with fixed investment adjustment costs. This finding is highly robust and explains why researchers working in the DSGE tradition have found little role for fixed costs in numerical trials.

Because the differences between the models are small for plausible depreciation rates and vanish in the low-depreciation limit, the behavior of conventional investment models will closely track the equilibrium behavior of models with fixed adjustment costs and thus should be just as useful for analyzing aggregate investment. Whether firms in reality exhibit the high intertemporal price sensitivity implied by the model is unknown. Study of this issue is left to future research.

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Appendix A. Supplementary materials

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jmoneco.2014.07.011.

[18] Khan and Thomas (2003) demonstrate that idiosyncratic shocks have little impact on the equilibrium in a DSGE model with fixed investment adjustment costs.

References

Abel, A.B., 1982. Dynamic effects of permanent and temporary tax policies in a q model of investment. J. Monet. Econ. 9, 353–373.
Abel, A.B., Eberly, J.C., 1994. A unified model of investment under uncertainty. Am. Econ. Rev. 84, 1369–1384.
Adda, J., Cooper, R., 2000. Balladurette and Juppette: a discrete analysis of scrapping subsidies. J. Polit. Econ. 108, 778–806.
Bachmann, R., Caballero, R., Engel, E., 2013. Aggregate implications of lumpy investment: new evidence and a DSGE model. Am. Econ. J.: Macroecon. 5, 29–67.
Barsky, R.B., House, C.L., Kimball, M.S., 2007. Sticky-price models and durable goods. Am. Econ. Rev. 97, 984–998.
Berger, D., Vavra, J., 2012. Consumption dynamics during recessions. Working paper. University of Chicago.
Bertola, G., Caballero, R., 1990. Kinked adjustment costs and aggregate dynamics. In: Blanchard, O., Fischer, S. (Eds.), NBER Macroeconomics Annual, MIT Press, Cambridge MA, pp. 237–296.
Bloom, N., 2009. The impact of uncertainty shocks. Econometrica 77, 623–685.
Caballero, R., 1993. Durable goods: an explanation for their slow adjustment. J. Political Econ. 101, 351–384.
Caballero, R., Engel, E., 1999. Explaining investment dynamics in U.S. manufacturing: a generalized (S,s) approach. Econometrica 67, 783–826.
Caballero, R., Engel, E., Haltiwanger, J., 1995. Plant-level adjustment and aggregate investment dynamics. Brookings Pap. Econ. Act. 1995, 1–54.
Caballero, R., Leahy, J.V., 1996. Fixed costs: the demise of marginal q. NBER working Paper No. 5508.
Caplin, A., Leahy, J.V., 1991. State dependent pricing and the dynamics of money and output. Q. J. Econ. 106, 683–708.
Caplin, A., Leahy, J.V., 1997. Aggregation and optimization with state-dependent pricing. Econometrica 65, 601–627.
Caplin, A., Leahy, J.V., 2004. How important is discrete adjustment in aggregate fluctuations?. In: Ito, T., Rose, A.K. (Eds.), Growth and Productivity in East Asia, NBER-East Asia Seminar on Economics, 13. University of Chicago Press, Chicago, pp. 351–376.
Caplin, A., Leahy, J.V., 2006. Equilibrium in a durable goods market with lumpy adjustment. J. Econ. Theory 128, 187–203.
Caplin, A., Spulber, D., 1987. Menu costs and the neutrality of money. Q. J. Econ. 102, 703–725.
Cooper, R., Haltiwanger, J., 1993. The aggregate implications of machine replacement: theory and evidence. Am. Econ. Rev. 83, 360–382.
Cooper, Russell, Haltiwanger, John, 2006. On the nature of capital adjustment costs. Rev. Econ. Stud. 73, 611–634.
Cooper, R., Haltiwanger, J., Power, L., 1999. Machine replacement and the business cycle: lumps and bumps. Am. Econ. Rev. 89, 921–946.
Danziger, L., 1999. A dynamic economy with costly price adjustments. Am. Econ. Rev. 89, 878–901.
Dixit, A.K., Pindyck, R.S., 1994. Investment under uncertainty. Princeton University Press, Princeton NJ.
Doms, M., Dunne, T., 1998. Capital adjustment patterns in manufacturing plants. Rev. Econ. Dyn. 1, 409–429.
Dotsey, M., King, R., Wolman, A., 1999. state dependent pricing and the general equilibrium dynamics of money and output. Q. J. Econ. 104, 655–690.
Eberly, J., 1994. Adjustment of consumers' durables stocks: evidence from automobile purchases. J. Polit. Econ. 102, 403–436.
Fiori, G., 2012. Lumpiness, capital adjustment costs and investment dynamics. J. Monet. Econ. 58, 371–382.
Fraumeni, B.M., 1997. The measurement of depreciation in the U.S. national income and product accounts. Surv. Curr. Bus., 7–23.
Gertler, M., Leahy, J.V., 2008. A Phillips curve with an Ss foundation. J. Polit. Econ. 116, 533–572.
Gourio, F., Kashyap, A., 2007. Investment spikes: new facts and a general equilibrium exploration. J. Monet. Econ. 54 (Suppl. 1), S1–S22.
Hayashi, F., 1982. Tobin's marginal q and average q: a neoclassical interpretation. Econometrica 50, 213–224.
Hendel, I., Lizzeri, A., 1999. Adverse selection in durable goods markets. Am. Econ. Rev. 89, 1097–1115.
Hendel, I., Lizzeri, A., 2002. The role of leasing under adverse selection. J. Polit. Econ. 89, 1097–1115.
House, C.L., 2014. Fixed costs and long-lived investments: Supplemental quantitative analysis. Supplemental Technical Paper. Available online at ⟨http://www-personal.umich.edu/~chouse/⟩ or from the author by request.
House, C.L., Leahy, J.V., 2004. An sS model with adverse selection. J. Polit. Econ. 112, 581–614.
House, C.L., Ozdenoren, E., 2008. Durable goods and conformity. RAND J. Econ. 39, 452–468.
House, C.L., Shapiro, M.D., 2008. Temporary investment tax incentives: theory with evidence from bonus depreciation. Am. Econ. Rev. 93, 437–768.
House, C.L., Zhang, J., 2012. Layoffs, lemons and temps. NBER Working Paper No. 17962.
Khan, A., Thomas, J., 2003. Nonconvex factor adjustments in equilibrium business cycle models: do nonlinearities matter? J. Monet. Econ. 50, 331–360.
Khan, A., Thomas, J., 2008. Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics. Econometrica 76, 395–436.
King, R.G., Thomas, J., 2006. Partial adjustment without apology. Int. Econ. Rev. 47, 779–809.
Krusell, P., Smith, A., 1997. Income and wealth heterogeneity, portfolio choice and equilibrium asset returns. Macroecon. Dyn. 1, 387–422.
Krusell, P., Smith, A., 1998. Income and wealth heterogeneity in the macroeconomy. J. Polit. Econ. 106, 867–898.
Miao, J., Wang, J., 2014. Lumpy investment and corporate tax policy. J. Money, Credit Bank.. (forthcoming-a).
Miao, J., Wang, J., 2014. A Q-theory model with lumpy investment. Econ Theory. (forthcoming-b).
Rios-Rull, J.V., 1999. Computation of equilibria in heterogeneous agent models. In: Marimon, R., Scott, A. (Eds.), Computational Methods for the Study of Dynamic Economies, Oxford University Press, Oxford.
Stock, J.H., Watson, M.W. 1999. Business cycle fluctuations in US macroeconomic time series. In: Taylor, J.B., Woodford, M. (Eds.), Handbook of Macroeconomics, 1st ed., vol. 1, chapter 1. Elsevier, pp. 3–64.
Stolyarov, D., 2002. Turnover of used durables in a stationary equilibrium: are older goods traded more? J. Polit. Econ. 110, 1390–1413.
Summers, L.H., 1981. Taxation and corporate investment: a q-theory approach. Brookings Pap. Econ. Act. 1981, 67–127.
Thomas, J., 2002. Is lumpy investment relevant for the business cycle? J. Polit. Econ. 110, 508–534.
Veracierto, M.L., 2002. Plant-level irreversible investment and equilibrium business cycles. Am. Econ. Rev. 92, 181–197.